CSE 6331 Homework 1

Due Thursday, January 18 by class time

1. Order the following function by asymptotic dominance. That is, produce an order $f_1(n), f_2(n), \ldots$ such that $f_i = O(f_{i+1})$.

   (a) $f(n) = n! * n^2$
   Hint: use Stirling's formula, $n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \Theta\left(\frac{1}{n}\right)\right)$.

   (b) $f(n) = 2^{23}$

   (c) $f(n) = 1/n^2$

   (d) $f(n) = 4^{\sqrt{n}}$

   (e) $f(n) = n^{2n}$

   (f) $f(n) = \log_2 \log_2(n^5 + n^2)$

   (g) $f(n) = 19n^{1.5} + 3n^{2.1} + \sqrt{n}$

   (h) $f(n) = 27 \log_7(n) + \sqrt{\log_2(n)}$

   (i) $f(n) = (\log_2 n)^3$

   (j) $f(n) = 5^{\log_2 n} + \sqrt{n}$

   (k) $f(n) = 2^{(2^n)}$

2. Let $f(n)$ be a function defined for all positive integers n. Prove or disprove the following statement: If $f(n) = \Theta(n^2)$, then $f(n)$ is asymptotically monotonically nondecreasing (i.e., $f(n) \leq f(n+1)$ for all sufficiently large integers n). (Note: to disprove a statement, you need to give a concrete counterexample.)

3. Let $f(n)$ be a function defined for all positive integers n. Prove or disprove the following statement: If $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$. (Note: to disprove, you need to give a concrete counterexample.)

4. How many dollar signs ($'s) will the following procedure print? Give your answer in Θ notation. Justify your answer.

   Procedure dollar(n)
   $j \leftarrow 1$
   while $j < n$ do
   $j \leftarrow j + j$
   $k \leftarrow 2$
   while $k < n$ do
   $k \leftarrow k * k$
   print($)