# CIS 6331 Homework 9

Due: Friday, April 14 by class time

**Note: in this homework, use the definition of flow that includes the skew-symmetry condition.**

1. Consider a flow network in which vertices, as well as edges, have capacities. In addition to the original edge capacity constraint, there is now a new vertex capacity constraint: the total positive flow entering any vertex $u$ cannot exceed its capacity $c(u)$. Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum flow problem.

2. Suppose that during an execution of Relabel-to-Front, Discharge($u$) is called **twice** for some particular node $u$.
   **Question : Prove or disprove** that if an edge $(u,v)$ is **inadmissible** at the end/exit of the first Discharge($u$), then it is still **inadmissible** at the beginning/entry of the second Discharge($u$). **Clearly indicate whether you prove or disprove.**

3. Let $G=(V,E)$ be a flow network with source *s*, sink *t*, and integer capacities. Suppose we are given a maximum flow $f$ in $G$, and suppose the capacity of a single edge $(u,v)\in E$ is **increased** by 1. Give an $O(V+E)$-time algorithm to update the maximum flow.

4. Let $G=(V,E)$ be a flow network with source *s*, sink *t*, and integer capacities. Suppose we are given a maximum flow $f$ in $G$, and suppose the capacity of a single edge $(u,v)\in E$ is **decreased** by 1. Give an $O(V+E)$-time algorithm to update the maximum flow.