Barrier Coverage With Wireless Sensors

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ABSTRACT

In old times, castles were surrounded by moats (deep trenches filled with water, and even alligators) to thwart or discourage intrusion attempts. One can now replace such barriers with stealthy and wireless sensors. In this paper, we develop theoretical foundations for laying barriers of wireless sensors. We define the notion of k-barrier coverage of a belt region using wireless sensors. We propose efficient algorithms using which one can quickly determine, after deploying the sensors, whether a region is k-barrier covered. Next, we establish the optimal deployment pattern to achieve k-barrier coverage when deploying sensors deterministically. Finally, we consider barrier coverage with high probability when sensors are deployed randomly. We introduce two notions of probabilistic barrier coverage in a belt region – weak and strong barrier coverage. While weak barrier-coverage with high probability guarantees the detection of intruders as they cross a barrier of *stealthy* sensors, a sensor network providing strong barrier-coverage with high probability (at the expense of more sensors) guarantees the detection of all intruders crossing a barrier of sensors, even when the sensors are *not* stealthy. Both types of barrier coverage require significantly less number of sensors than full-coverage, where every point in the region needs to be covered. We derive critical conditions for weak k-barrier coverage, using which one can compute the minimum number of sensors needed to provide weak k-barrier coverage with high probability in a given belt region. Deriving critical conditions for strong k-barrier coverage for a belt region is still an open problem.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication networks**]: Network Architecture and Design—*network topology*; G.3 [**Probability and Statistics**]: Stochastic Processes

General Terms

Algorithms, Theory

Figure 1: A Castle with a moat to discourage intrusion.

Keywords

Wireless sensor networks, barrier coverage, coverage, network topology, localized algorithms, random geometric graphs, critical conditions.

1. INTRODUCTION

In the days of castles and forts, a popular defense mechanism from intruders were moats (a deep and wide trench that is usually filled with water). One such castle with moat around it is shown in Figure 1. Some forts used to put even alligators in those water. Other defense mechanisms were mires, fire barriers, and even forests. These mechanisms are no longer viable in today's world of high-technology.

Wireless sensor networks can replace such barriers today at the building level and at the estate level, where barriers can be more than a kilometer long [1]. Efforts are currently underway to extend the scalability of wireless sensor networks so that they can be used to monitor one of the largest international borders [1]. Intrusion detection and border surveillance constitute a major application category for wireless sensor networks. A major goal in these applications is to detect intruders as they cross a border or as they penetrate a protected area. This type of coverage is referred to as *barrier coverage*, where the sensors form a barrier for the intruders. A given belt region is said to be *k-barrier covered* with a sensor network if all crossing paths through the region are k-covered[1], where a crossing path is any path that crosses the width of the region completely. This is in contrast to the other type of coverage, where every point in the deployment region is covered, referred to as *full coverage* in this paper.

[1]A path is said to be k-covered if it intersects with the sensing disks of at least k distinct sensors. This is in contrast with the notion when every point in the path is covered by at least k distinct sensors.

By their very nature, the deployments for barrier coverage are expected to be in long (sometimes very long, as in international borders) thin belts (a region bounded by two parallel curves) as opposed to in regular structures such as squares and disks [3, 11]. Further, since the goal is only to detect intruders before they have crossed the border as opposed to detecting them at every point in their trajectory, using the results on full coverage is often an overkill. Therefore, the traditional work on coverage [9, 12, 25] are not directly applicable to barrier coverage. A natural question then is **how does one determine the minimum number of sensors to deploy to have k-barrier coverage in a given belt region?** And, **how does one determine, after deploying sensors in a region, whether the region is indeed k-barrier covered?**

In this paper, we establish equivalence conditions between k-barrier coverage and the existence of k node-disjoint paths between two vertices in a graph. With such a condition, efficient (global) algorithms already existing to test the existence of k node-disjoint paths can now be used to test whether or not a given region is k-barrier covered by a network of wireless sensors. We also establish that it is *not* possible to locally come up with a yes/no answer to the question of whether the given region is k-barrier covered. This should be contrasted with the fact that for full k-coverage, it *is* possible to locally come up with a *no* answer to the question of whether the given region is fully k-covered [9].

Next, we prove that when deploying sensors deterministically, the optimal deployment pattern to achieve k-barrier coverage is to deploy k rows of sensors on the shortest path across the length of the belt region such that consecutive sensors' sensing disks abut each other. This should be contrasted with the fact that optimal deployment pattern to achieve full k-coverage for general values of k are not known.

Finally, we consider barrier coverage with high probability. We introduce two notions of barrier coverage with high probability – weak and strong barrier coverage. Let i be a crossing path through a belt region and let $L(i)$ denote the set of all crossing paths congruent to i. Then, a belt region is said to be *weakly k-barrier covered* with high probability if and only if[2]

$$\forall i : \lim \Pr[\forall j \in L(i) : j \text{ is } k\text{-covered}] = 1,$$

and it is said to be *strongly k-barrier covered* with high probability if and only if

$$\lim \Pr[\forall i : i \text{ is } k\text{-covered}] = 1,$$

which is also equivalent to the following condition

$$\lim \Pr[\forall i : \forall j \in L(i) : j \text{ is } k\text{-covered}] = 1.$$

The conditions for both the weak barrier coverage and strong barrier coverage will be same when both of these events need to have probabilities exactly 1, i.e. make deterministic guarantee. Otherwise, the conditions may be different.

To provide weak barrier coverage in a belt region with high probability, one is likely to require significantly less sensors than that required for strong barrier coverage with high probability. Also, if the sensors are stealthy, then having weak barrier coverage with high probability may be enough to detect all intruders with high probability. Finally, if the intruders are known to traverse in groups (when they will follow congruent or nearly congruent paths), weak barrier coverage will guarantee detection with high probability. With the three notions of coverage, weak k-barrier coverage, strong k-barrier coverage, and full k-coverage, a deployer now has more design freedom to trade the number of sensors with the quality of surveillance desired.

We derive critical conditions for weak k-barrier coverage in this paper, using which one can compute the minimum number of sensors needed to provide weak k-barrier coverage with high probability in a given belt region when the sensors are deployed with Poisson distribution or with random uniform distribution. We prove that our conditions hold not only for rectangular belt regions, but also for arbitrary belt regions (a long and narrow region bounded by two uniformly separated curves such as a pair of concentric circles). Deriving critical conditions for strong k-barrier coverage for a belt region is still an open problem. We provide details in Section 3.2 on why standard percolation theory results do not directly yield critical conditions for strong k-barrier coverage in long belt regions.

Our critical conditions can be used to design efficient sleep-wakeup schemes for a sensor network providing continuous weak k-barrier coverage. Because sensors can not locally determine whether or not the region is k-barrier covered (a result established in this paper), it is not possible to design local and deterministic sleep/wakeup algorithms to increase network lifetime and still maintain barrier coverage of the region with an arbitrary sensor network topology. However, it is possible to design a purely local, but randomized sleep/wakeup algorithm to increase the network lifetime by a given factor, while guaranteeing that the region is weakly k-barrier covered with high probability at all times.

Randomized Independent Sleeping (RIS) scheme proposed in [12] is one such scheme. In this algorithm, time is divided in intervals and in every interval each sensor is active with probability p, independently of every other sensor. With this scheme, the network will last $(1/p)$-times the lifetime of individual sensors. If the number of sensors to be deployed is chosen using our critical conditions for weak k-barrier coverage, then the RIS scheme will increase the network lifetime by the desired factor, $(1/p)$, while guaranteeing the continuous weak k-barrier coverage of the region with high probability.

The rest of the paper is organized as follows. In Section 2, we formally define the network model, key assumptions and the conditions for k-barrier coverage. In Section 3, we describe key contributions of this paper and discuss some related work. In Section 4, we prove equivalence conditions that lead to efficient algorithms for determining whether a given belt region is k-barrier covered. In Section 5, we establish the optimal deployment pattern for achieving k-barrier coverage when deploying sensors deterministically. In Section 6, we derive critical conditions for weak k-barrier coverage with high probability in an arbitrary belt region. In Section 7, we provide some results from numerical computation and simulation. Section 8 concludes the paper.

2. THE NETWORK MODEL

Model of Deployment. We consider a long, narrow region, referred to as a *belt*, where sensors are deployed randomly with Poisson distribution of rate n. As proved in [8, Page 39] for a region of unit area, as n becomes larger and larger, Poisson distribution of sensors with rate n is equivalent to random uniform distribution of n sensors, where each sensor has an equal likelihood of being at any location within the deployed

[2] We define these notions more formally in Section 2.

region, independently of the other sensors. Therefore, all the results we prove here for Poisson distribution also hold for uniform distribution.

ASSUMPTION 2.1. **[Disc-based sensing]** *We assume a disc-based sensing model where each active sensor has a sensing radius of r; any object within the disc of radius r centered at an active sensor is reliably detected by it. The sensing disk of a sensor located at location u is denoted by $D_r(u)$.*

We would like to note that the results of Section 4 and that of Section 5 will continue to hold if sensing is directional and does not follow the disk model.

DEFINITION 2.1. **[RIS scheme [12]]** *Time is divided in regular intervals and in each interval, each sensor is active with a probability of p, independently of all the other sensors.*

DEFINITION 2.2. **[Sensor network $N(n,r)$]** *A sensor network where sensors are distributed with Poisson distribution of rate n and each sensor has a sensing radius of r is denoted by $N(n,r)$. If each sensor in a sensor network $N(n,r)$ sleeps according to the RIS scheme [12] so that each sensor is active with probability p, then the sensor network is denoted by $N(n,p,r)$.*

DEFINITION 2.3. **[Belt of dimension $s \times (1/s)$]** *A rectangular region is said to be a **belt of dimension** $s \times (1/s)$, if it has length s and width $1/s$.*

Figure 3 illustrates such a belt. Note that even if s approaches to ∞, the area of the region always remains 1.

DEFINITION 2.4. **[$d(u,v)$]** *Let the Euclidean distance between points u and v be denoted by $d(u,v)$. If l is a line or a path, then $d(u,l) = \min\{d(u,v) : v \in l\}$.*

DEFINITION 2.5. **[Belt of dimension $(\lambda_1, \lambda_2, (1/s))$]** *Two curves l_1 and l_2 are **uniformly separated** with separation $1/s$ if $d(l_1,y) = d(x,l_2) = 1/s$ for all points $x \in l_1$ and all points y in l_2. A region bounded by two curves l_1 and l_2, which are uniformly separated with separation $1/s$ and are of lengths λ_1 and λ_2 respectively, is referred to as a belt of dimensions $(\lambda_1, \lambda_2, (1/s))$, in which case $1/s$ is referred to as the belt's **width** and λ_1 and λ_2 its **lengths**.*

A belt as defined in Definition 2.5 occurs between railroad tracks. Such a belt also occurs if sensors are dropped from a moving vehicle. Figure 2 illustrates an example of such a belt with dimensions $(2\pi r_1, 2\pi r_2, r_1 - r_2)$, which is the region between the circumference of two concentric circles of radii r_1 and r_2.

ASSUMPTION 2.2. **[Small Width]** *We assume that the width of the belt, $1/s$, is in the same order of magnitude as the sensing radius, r, i.e. $\exists m, M : m \leq r/s \leq M$.*

In practice, most of the barrier coverage deployments are expected to satisfy Assumption 2.2.

ASSUMPTION 2.3. *We also assume that $n \to \infty$ as $s \to \infty$.*

With assumptions 2.2 and 2.3, it follows that the parameters r and n are actually functions of s and should have been denoted as $r(s)$ and $n(s)$. However, we write n, r in place of $n(s)$, $r(s)$ to improve the clarity of presentation. The same convention applies to any other parameter that is potentially a function of s. Also, if some parameter is a function of n, then it is also a function of s because n is a function of s.

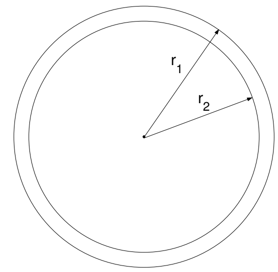

Figure 2: A belt region with dimension $(2\pi r_1, 2\pi r_2, r_1 - r_2)$, which is the region between the circumference of two concentric circles with radii r_1 and r_2.

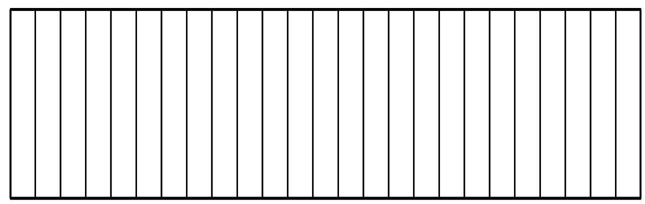

Figure 3: A belt region showing some crossing paths that are congruent (also parallel in this case) to the width of the belt. Note that the total number of crossing paths that are congruent to the width is uncountable.

DEFINITION 2.6. **[Intruder]** *An intruder is any person or object that is subject to detection by the sensor network as it crosses the barrier.*

DEFINITION 2.7. **[Stealthiness]** *A sensor network is said to satisfy the **stealthiness assumption** if no intruder is aware of the locations of the sensors.*

DEFINITION 2.8. **[k-coverage of a Path]** *A path (i.e. line or curve) l is said to be k-**covered** if $l \cap D_r(u) \neq \emptyset$ for at least k active sensors u. We denote this event by $A_k(l)$. (In contrast, a path is said to be **"fully" k-covered** if every point in it is covered by at least k sensors. This paper is concerned only with k-coverage.)*

Thus, if an intruder moves along a k-covered path, it will be detected by at least k sensors.

DEFINITION 2.9. **[Crossing line (or Crossing path)]** *A line segment (or path) in a belt region is said to be a **crossing line** (or **crossing path**) if it crosses the complete width of the region. A crossing line is **orthogonal** if its length equals the belt's width.*

Figure 3 illustrates orthogonal crossing lines.

DEFINITION 2.10. **[k-barrier Coverage]** *A belt region with a sensor network deployed over it is said to be **k-barrier covered** if and only if all crossing paths through the belt are k-covered by the sensor network.*

We use $\Pr[T]$ to denote the probability that event T occurs; and $\Pr[\overline{T}]$, the probability that T does not occur. We use $\mathbb{E}[X]$ to denote the expected value of a random variable X.

DEFINITION 2.11. **[With high probability (whp)]** *We say that event $T(n)$ occurs* ***with high probability (whp)*** *if*

$$\lim_{n\to\infty} Pr[T(n)] = 1.$$

We use the concept of *congruency* in the next definition. Two curves in the Euclidean plane are said to be congruent iff one can be transformed into another by an isometry [5]. An isometry is a (Euclidean) distance preserving transformation. Of all possible isometric transformations, we only consider translation and rotation.

Note that by the definition of congruency and by the definition of an orthogonal crossing line (Definition 2.9), all orthogonal crossing lines in a belt region (whether of dimension $s \times (1/s)$ or of dimension $(\lambda_1, \lambda_2, (1/s))$) are congruent to each other.

DEFINITION 2.12. **[k-barrier coverage modulo l]** *Let B be a belt region with a sensor network deployed over it. Let l be a crossing path through B and let $L(l)$ denote the set of all crossing paths* ***congruent*** *to l. B is said to be* ***k-barrier covered modulo*** *l if and only if*

$$Pr[\forall i \in L(l) : A_k(i)] = 1,$$

i.e. every path in $L(l)$ is k-covered by the sensor network.

Note that congruent crossing paths in a rectangular belt will be parallel to each other as in Figure 3. But, if the belt region is non-rectangular, then congruent paths need not be parallel. For example, orthogonal crossing paths in a belt region such as the one shown in Figure 2 will all be congruent to each other, but not mutually parallel.

DEFINITION 2.13. **[Weak k-barrier coverage whp]** *Let B_s be a belt region of dimension $s \times (1/s)$ or $(\lambda_1, \lambda_2, (1/s))$ with a sensor network $N(n, r)$ deployed over it. Let l be a crossing path through B_s. Then, B_s is said to be* ***weakly k-barrier covered whp*** *if and only if*[3]

$$\forall l : \lim_{s\to\infty} Pr[B_s \text{ is } k\text{-barrier covered modulo } l] = 1. \quad (1)$$

DEFINITION 2.14. **[Strong k-barrier coverage whp]** *Let B_s be a belt region of dimension $s\times(1/s)$ or $(\lambda_1, \lambda_2, (1/s))$ with a sensor network $N(n, r)$ deployed over it. Let i be a crossing path through B_s. Then, B_s is said to be* ***strongly k-barrier covered whp*** *if and only if*

$$\lim_{s\to\infty} Pr[\forall i : A_k(i)] = 1. \quad (2)$$

To see the difference between weak barrier coverage and strong barrier coverage *whp*, note that (2) is equivalent to the following condition:

$$\lim_{s\to\infty} \Pr[\forall l : B_s \text{ is } k\text{-barrier covered modulo } l] = 1.$$

[3]Although this definition is intuitively clear, it may be mathematically ambiguous. For rectangular belts B_s, this issue can be addressed as follows. Let B_s be the belt region $[0, s] \times [0, 1/s]$. In particular, B_1 is the B_s with $s = 1$. Let L_1 be the set of all crossing paths in B_1. For each crossing path $l \in L_1$, define $l_s = \{(x * s, y * 1/s) : (x, y) \in l\}$, which is a crossing path in B_s naturally corresponding to l. Now, (1) can be more precisely stated as

$$\forall l \in L_1 : \lim_{s\to\infty} \Pr[B_s \text{ is } k\text{-barrier covered modulo } l_s] = 1.$$

For non-rectangular belts B_s, the issue can be addressed similarly by introducing a natural one-one mapping between B_1 (the B_s with $s = 1$) and B_s.

Also note that

$$\lim_{s\to\infty} \Pr[\forall i : A_k(i)] = 1 \Leftrightarrow \lim_{s\to\infty} \Pr[\exists i : \overline{A_k(i)}] = 0,$$

but

$$\forall l \quad : \quad \lim_{s\to\infty} \Pr[B_s \text{ is } k\text{-barrier covered modulo } l] = 1$$
$$\not\Rightarrow \lim_{s\to\infty} \Pr[\exists i : \overline{A_k(i)}] = 0,$$

which means that strong barrier coverage *whp* is a stronger condition than weak barrier coverage *whp*. A consequence of this distinction between strong and weak barrier coverage *whp* is that if a region is strongly barrier covered *whp*, then even if the intruders can see the location of sensors, *whp* they can not find an uncovered path through the region. On the other hand, if the region is weakly barrier covered *whp*, then all intruders will be detected *whp* if they can not see the sensors. However, if the region is weakly barrier covered *whp* and the network does not satisfy the stealthiness assumption (Definition 2.7), then an intruder may be able to find an uncovered path through the region.

3. SUMMARY OF CONTRIBUTIONS AND RELATED WORK

3.1 Summary of Contributions

In this section, we summarize our main results. We divide them in three categories:

Algorithms for k-barrier Coverage:

We establish the following three key results on the issue of how to determine whether a given belt region is k-barrier covered with a sensor network:

1) We establish that it is not possible to locally come up with a "yes" or a "no" answer to the question of whether a given belt region is k-barrier covered. This is in contrast to the results known for the case of full coverage, where it is possible to locally come up with a "no" answer to the analogous question [9].

2) We prove (in Theorem 4.1) that the condition for an open belt region (such as the one shown in Figure 3) to be k-barrier covered can be reduced to problem of determining whether there exist k node-disjoint paths between a pair of vertices in a graph. One can now use existing algorithms for testing the existence of k node-disjoint paths between two vertices to globally test k-barrier coverage.

3) We prove (in Theorem 4.2) that the condition for a closed belt region (such as the one shown in Figure 7) to be k-barrier covered can be reduced to the problem of determining whether there exist k node-disjoint cycles, each of which loops around the entire belt region[4]. Notice that the problem of finding node-disjoint cycles that go around the entire belt region is not the same as finding node-disjoint cycles in graphs (which could be local cycles). The good news is that it can be decided in polynomial time whether there exist k node-disjoint cycles that go around the entire closed belt, although it appears to be similar to the multi-commodity flow problem, which is known to be NP-Complete.

The equivalence conditions we have established are different from other known results on the relation between coverage and connectivity [23]. (See Section 4.4).

[4]We formally define node-disjoint cycles that go around the entire belt in Section 4.3.

Optimal Configuration for Deterministic Deployment:

For k-barrier coverage, we prove in Theorem 5.1 that the optimal configuration for achieving k-barrier coverage in a belt region is to deploy k rows of sensors on the shortest path across the length of the region, where each line has consecutive sensors' sensing disks abut each other. This is in contrast to the fact that the analogous problem of determining an optimal configuration for achieving full k-coverage for general values of k is still an open problem.

Critical Conditions for Weak k-barrier Coverage for Randomized Deployments:

If in a Poisson distributed sensor network with rate n, each sensor sleeps according to the RIS sleep/wakeup scheme [12] so that it is active with probability p at any given time, then the distribution of the active sensors follows Poisson distribution of rate np [20]. Assume that sensors are Poisson distributed with rate np over a belt region. We establish a critical condition for the belt region to be weakly k-barrier covered *whp*. Such a condition will allow us to easily compute the number of sensors necessary to ensure weak k-barrier coverage of the region with high probability.

DEFINITION 3.1. **[$\phi(np)$]** *We use $\phi(np)$ to denote an arbitrary, slowly and monotonically increasing function that goes to infinity, where $\phi(np) = o(\log\log(np))$.*

DEFINITION 3.2. *We define*

$$c(s) = 2npr/(s\log(np)) \quad (3)$$

$$f_k(n) = \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)} \quad (4)$$

The following two results establish a critical condition for weak k-barrier coverage in a belt region:

1) Let $N(n,p,r)$ be a Poisson distributed sensor network over a belt of dimension $(\lambda_1, \lambda_2, (1/s))$. We prove (in Theorem 6.5) that if

$$c(s) \geq 1 + f_k(n)$$

for sufficiently large s, then the belt region is weakly k-barrier covered *whp* (as $s \to \infty$).

2) Again, let $N(n,p,r)$ be a Poisson distributed sensor network over a belt of dimension $(\lambda_1, \lambda_2, (1/s))$. We prove (in Theorem 6.4) that if

$$c(s) \leq 1 - f_2(n)$$

then *whp* there exists an orthogonal crossing line in the region that is not 1-covered as $s \to \infty$. This implies that in order for a belt region to be weakly barrier-covered *whp*, it is necessary that $c(s) > 1 - f_2(n)$.

Notice that since $c(s) \stackrel{s\to\infty}{\longrightarrow} 1$ in both of the results above, the critical value of the function $c(s)$ is 1 for the case of weak k-barrier coverage of a belt region of dimension $(\lambda_1, \lambda_2, (1/s))$. Roughly speaking, the critical condition indicates that in order to ensure barrier coverage *whp*, there must be at least $\log(np)$ active sensors in each orthogonal crossing line's r-neighborhood.

3.2 Related Work

Most of the existing work on coverage focus on full-coverage [9, 12, 25] and that too in regular regions rather than in a thin belt region. The proofs and the conditions developed for full-coverage do not readily carry over to the case of barrier coverage in thin belt regions.

The concept of barrier coverage first appeared in [6] in the context of robotic sensors. Simulations were performed in [10] to find the optimal number of sensors to be deployed to achieve barrier coverage. To the best of our knowledge, ours is the first work to address the theoretical foundation for determining the minimum number of sensors to be deployed (using critical conditions) to achieve barrier coverage in belt regions.

Full-coverage in one dimension and barrier-coverage in a square region were addressed in [14]. It is pointed out in this work that percolation theory results can be used to establish critical conditions for the existence of a giant cluster of overlapping sensing disks. It was concluded that beyond the critical threshold, no crossing path will exist because a giant cluster of overlapping sensing disks exists. However, as pointed out earlier, deployments for barrier coverage are expected to be in thin belt regions as opposed to square regions and the percolation theory results developed for square regions are not directly applicable to thin belt regions. For instance, the crossing probability (which, in a sense is equivalent to strong barrier coverage) in rectangular regions approaches 0 at the percolation threshold, as the ratio of width to length approaches 0 (which is the case in our $s \times (1/s)$ model with $s \to \infty$). For details, we refer the reader to [13]. Also, notice that for barrier coverage even in a square region, all one needs is a set of sensors whose sensing disks overlap and cover the entire length of the region. It does not need to be a giant component, as is demanded by the percolation theory.

The work on maximal exposure paths in [15, 16, 22] focus on devising algorithms to find a least covered crossing path through the region between a given set of initial and final points. The problems addressed in these work are complementary to our algorithm for determining whether a belt region is k-barrier covered. Once it is found out using our algorithm that the region is not k-barrier covered, the *Maximal Breach Path* algorithm [15] or its localized version [22] can be executed for those sets of initial and final points that the intruders are most likely to follow in the protected region, to find the least covered paths. It may be too prohibitive to use *Maximal Breach Path* algorithms to determine whether a region is k-barrier covered. We also note that the work on maximal exposure paths do not address the issue of deriving critical conditions, although they do observe the existence of critical thresholds in their experiments.

Another work related to ours is [7]. This work addresses the issue of intruder tracking in regular regions such as a square. The focus of this work is the following problem — Given a value of l, what is the minimum number of sensors needed so that if the nodes are independently and uniformly distributed, the average length of an uncovered path traveled by an intruder that starts at a random (uniformly chosen) location within the field, will be less than l? In other words, the question addressed in this work is — Under what condition does the largest uncovered region have a diameter of less than a given value of l? Although this is an important problem for tracking applications, it does not address the problem of k-barrier coverage. For instance, a region may be k-barrier covered, and yet the largest hole may be as long as the length of the entire region (for example, see Figure 5).

As can be seen from the discussion of some related work above, a lot of interesting work have come close to the problem of barrier coverage, but none have addressed the issue

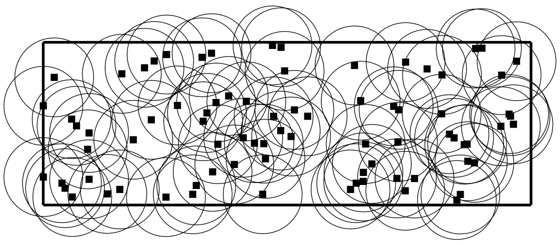

Figure 4: What is the largest value of k such that this region k-barrier covered?

of deriving critical conditions for barrier coverage in a *belt region*, which is a more realistic model for sensor deployments for barrier coverage than a square or a disk. Also, no existing work, to the best of our knowledge, has addressed the issue of developing efficient algorithms for determining whether a given belt region is k-barrier covered.

4. ALGORITHMS FOR K-BARRIER COVERAGE

Looking at the sensor deployment in Figure 5, one can easily conclude that the region is 3-barrier covered. However, if we look at the sensor deployment in Figure 4, it would be harder to see for what value of k this region is k-barrier covered. Therefore, it is desirable to have an efficient algorithm for determining whether or not a given belt region is k-barrier covered.

We first establish in Section 4.1 that it is not possible to determine locally if a given region is not k-barrier covered. We then derive conditions, using which one can design efficient global algorithms to determine whether a given region is k-barrier covered. Divide belt regions into two categories — open belts and closed belts. We show the following: (1) The problem of determining whether an open belt region is k-barrier covered can be reduced to the problem of determining whether two nodes in a graph are k-connected (in Section 4.2). (2) The problem of determining whether a closed belt region is k-barrier covered can be reduced to the problem of determining whether there exist k node-disjoint cycles each of which loops around the entire belt region (in Section 4.3). Such reductions will enable us to use existing graph theoretic algorithms for k node-disjoint paths to determine if an open or closed belt is k-barrier covered. Finally, in Section 4.4, we discuss how the conditions we establish in Sections 4.2 and 4.3 are different from a similar sounding result developed in [23].

The results of this section do not depend on either Assumption 2.2 or Assumption 2.3 described in Section 2. Also, the belt region considered in this section need not be of the form of Definition 2.3 or of Definition 2.5. Finally, the results of this section will continue to hold even if sensing is directional.

4.1 Non-locality of k-barrier Coverage

We first define what we mean by local algorithms. This definition is based on a model proposed in [19].

DEFINITION 4.1. **[Local Algorithms]** *Assume that each computation step takes one unit of time and so does every message to get from one node to its directly connected neighbors. With this model, an algorithm is called* ***local*** *if its computation time is $O(1)$, in terms of the number of nodes n in the system.*

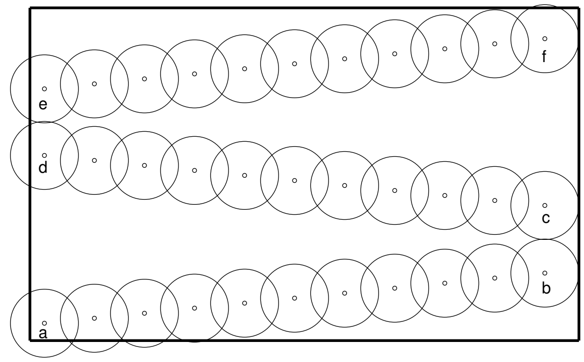

Figure 5: The above region is 3-barrier covered since there does not exist any path that crosses the complete width of the region without being detected by at least three sensors.

In [9], it was established that sensors *can* locally determine if a given region is *not* fully k-covered. (If any point on the perimeter of a sensor's sensing disk is covered by less than k sensors, then this sensor can locally conclude that the region is *not* fully k-covered.) However, in the case of k-barrier coverage, individual nodes can neither locally say "yes" nor "no" to the question of whether a given region is k-barrier covered. To see this, consider sensors deployed as in Figure 5. Assume that the communication range of each sensor is exactly twice its sensing range so that the sensors whose sensing disks overlap can communicate with each other.

The region is not 1-barrier covered iff there is at least one inactive sensor in each of the three rows. No sensor can locally determine whether at least one sensor in each of the three rows is inactive. Therefore, it is not possible to locally determine whether the belt region is not 1-barrier covered, in general.

As a result of this non-locality property, one cannot possibly design a deterministic local algorithm that allows sensors to locally decide whether to go to sleep or remain active, and still guarantees that the belt region is continuously k-barrier covered.

4.2 Open Belt Regions

Corresponding to a sensor network deployed in a belt region, we derive a *coverage graph* $CG = \langle V, E \rangle$, where V is the set of all sensor locations plus two virtual nodes u and v (see Figure 6). The set of edges E is derived as follows: Each pair of sensors whose sensing disks overlap are connected by an edge. Additionally, the sensors whose sensing disks intersect with the left boundary are connected to node u and the sensors whose sensing disks intersect with the right boundary are connected to node v. The resulting coverage graph for the sensor network in Figure 5 is shown in Figure 6.

The following theorem establishes that the conditions for a region to be k-barrier covered and the conditions for the corresponding coverage graph to have k-connectivity between nodes u and v are equivalent.

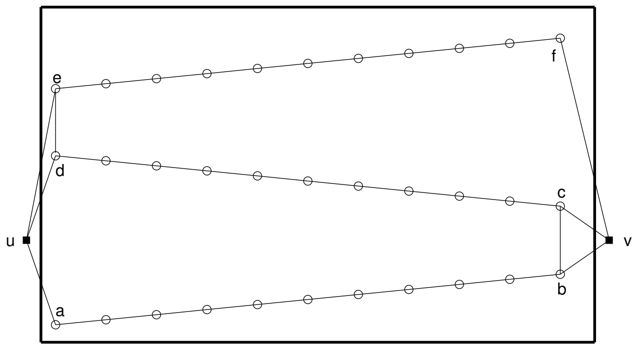

Figure 6: Coverage graph CG of the sensor network represented by Figure 5.

ASSUMPTION 4.1. *Let B be the belt region in consideration. If two sensing disks D_1 and D_2 have overlap, then $(D_1 \cup D_2) \cap B$ is a connected sub-region in B.*

THEOREM 4.1. *An open belt region B that satisfies Assumption 4.1 is k-barrier covered iff u and v are k-connected in the corresponding coverage graph, CG.*

PROOF. Let us first prove the "if" part. Assume that u and v are k-connected in the corresponding coverage graph CG. Then, by definition, there exist k node-disjoint paths in CG that connect u to v. These paths define k disjoint sets of sensors, each of which provides 1-barrier coverage for the belt. This is because sensing disks of neighboring sensors overlap with each other and, in addition, the sensor next to u (or to v) has its sensing disk intersecting the belt's left (or right) boundary. Therefore, the sensing disks of the sensors in each set cover the entire length of the belt and thereby provide 1-barrier coverage. This last claim relies on Assumption 4.1. Since there are k such sets (of sensors) which are mutually disjoint, the belt region is k-covered.

Now, we prove the "only if" part. Assume that u and v are not k-connected in CG. By Menger's Theorem [24, page 167], there exist $(k-1)$ vertices in $V - \{u, v\}$, removal of which will make u and v disconnected in CG. Let us denote one such set of $(k-1)$ vertices by W. Let the coverage subgraph induced by the vertex set $V - W$ be called CG'.

Since u and v are disconnected in CG', there exists a crossing path P in the belt region that is not covered by any sensor (corresponding to any vertex) in $V - W$. This path, P, may be covered by some or all of the sensors in W. Since $|W| = k-1$, P is covered by at most $k-1$ sensors in V. The existence of such a P means that the belt region is not k-barrier covered — it is at most $(k-1)$-barrier covered. □

Algorithm for an Open Belt:

After proving the equivalence between k-barrier coverage and k-connectivity between u and v, we can now use the algorithms developed for determining whether two vertices in a given graph are k-connected to determine whether a given belt region is k-barrier covered. According to [21], the best known-algorithm for testing whether u and v are k-connected has $O(k^2|V|)$ complexity.

4.3 Closed Belt Regions

The equivalence condition established in Theorem 4.1 does not work when the belt region is looped around as the one

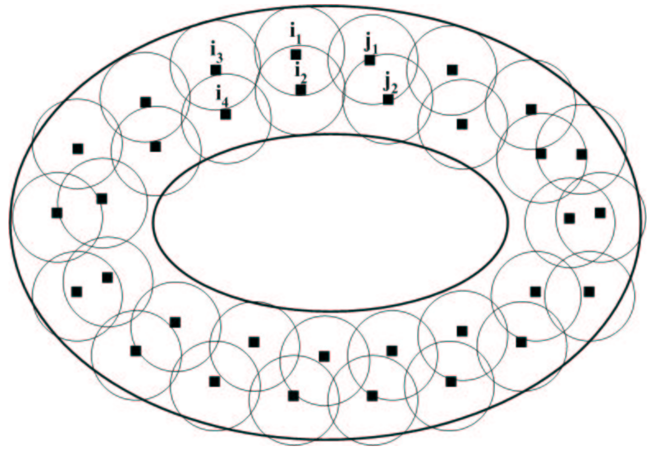

Figure 7: A sensor network deployed over a closed belt region.

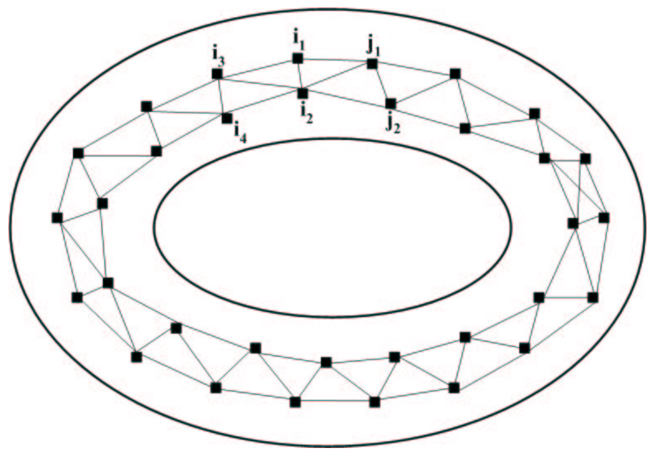

Figure 8: Coverage graph CG of the sensor network represented by Figure 7.

shown in Figure 7. In this section, we prove a theorem analogous to Theorem 4.1 for closed belt regions.

Construct a coverage graph CG as described in Section 4.2, except that the set of nodes V no longer contains virtual nodes u and v. (The coverage graph corresponding to the sensor network in Figure 7 appears in Figure 8.) We first make precise what we mean by node-disjoint cycles that go around the entire belt. This definition is taken from [17].

DEFINITION 4.2. **[Disjoint Essential Cycles]** *Let G be a graph embedded on some surface. A cycle C of G is called essential if C is non-contractible on the surface. A set of essential cycles are disjoint if they do not share a vertex in G.*

In Figure 8, $i_1 - i_2 - i_3 - i_1$ is a cycle, but not essential; the edges of this cycle can be "contracted." The cycle that starts at i_1, goes through i_3 and comes back to i_1 through j_1 after looping the entire belt is an essential cycle; this cycle can *not* be contracted on the belt's surface. In Figure 8, there exist two disjoint essential cycles. We refer the reader to [17] for more details on essential cycles and to [18] for more details on graphs embedded on surfaces.

THEOREM 4.2. *A closed belt region B that satisfies Assumption 4.1 is k-barrier covered iff there exist k node-disjoint essential cycles in the corresponding coverage graph, CG.*

PROOF. The "if" part can be proved in the same way as in Theorem 4.1. The "only if" part follows from Theorem 76.2 in [21] when applied to CG. □

Algorithm for a Donut-shaped Belt:
A polynomial time algorithm for determining whether there exist k node-disjoint cycles in the coverage graph corresponding to a sensor network deployed over a closed belt region follows from the proof of Theorem 76.2 in [21].

4.4 Difference Between Our Results and Other Known Results

The equivalence conditions we established in Theorems 4.1 and 4.2 are different from the result on the relation between full-coverage and connectivity established in Theorem 3 of [23] in several ways:

1. **Goal:** The goal of Theorems 4.1 and 4.2 are to derive conditions that one can use to determine whether a belt region is k-barrier covered. The goal of Theorem 3 in [23] is to establish conditions such that k-full coverage of a region will imply k-connectivity among all the sensors if the communication range is at least twice the sensing range.

2. **Result:** The equivalence conditions in Theorems 4.1 and 4.2 imply that if one uses a communication radius at least twice the sensing radius and if the region is k-barrier covered, then there will exist k node-disjoint paths between the two shorter sides of the belt region. This is not the same condition as the existence of k node-disjoint paths between every pair of sensor nodes as is implied by Theorem 3 in [23].

3. **Proofs:** The proof of Theorems 4.1 and 4.2 are very different than those of Theorem 3 in [23].

5. OPTIMAL CONFIGURATION FOR DETERMINISTIC DEPLOYMENTS

It is well known that the optimal configuration for achieving full 1-coverage is to deploy sensors on a triangular lattice [4]. However, to the best of our knowledge, the problem of determining an optimal configuration for achieving full k-coverage for general values of k is still an open problem.

For k-barrier coverage, we prove in the following theorem that the optimal configuration for achieving k-barrier coverage in a belt region is to deploy k rows of sensors along a shortest path (line or curve) across the length of the region, where each path has consecutive sensors' sensing disks abutting each other. For instance, for a rectangular belt region such as the one shown in Figure 3, the shortest path across the length of the region is a line parallel to its length. So, the optimal configuration to achieve k-barrier coverage in this region is to deploy k rows of sensors parallel to the length such that consecutive sensors are separated by a distance of $2r$. For the belt region in Figure 2, an optimal configuration will be k rows of sensors along the circumference of the inner circle.

THEOREM 5.1. *Consider a belt region. Let s denote the length of the shortest path across the length of the region. Then, the number of sensors necessary and sufficient to achieve k-barrier coverage in this region is $k * \lceil s/2r \rceil$, assuming sensors are deployed to satisfy Assumption 4.1.*

PROOF. The sufficient part of the theorem is obvious. For the necessary part, first consider an open belt. By Theorem 4.1, for the region to be k-barrier covered, it is necessary that the two shorter sides of the belt region are connected via k node-disjoint paths in the coverage graph. Each such path entails at least $\lceil s/2r \rceil$ sensors. Since the k paths are node-disjoint, a total of $k * \lceil s/2r \rceil$ sensors at least are needed. Similar arguments can be made for a closed belt using Theorem 4.2. □

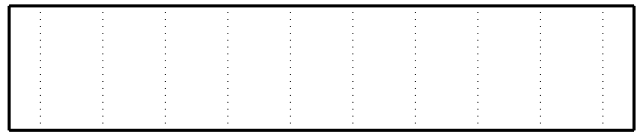

Figure 9: An $s \times (1/s)$ belt region. The dotted lines represent virtual crossing lines. The number of such lines is ℓ and the separation between neighboring lines is $t = s/\ell$.

6. CRITICAL CONDITIONS FOR WEAK K-BARRIER COVERAGE

In this section, we develop critical conditions for weak k-barrier coverage in a belt region. We first establish a key lemma (Lemma 6.1) in Section 6.1 to move from the continuous domain to the discrete domain. Then, we establish critical conditions for the k-coverage of orthogonal crossing lines in a rectangular $s \times (1/s)$ belt region (sufficient condition for coverage *whp* in Section 6.2 and sufficient condition for non-coverage *whp* in Section 6.3). We then extend these results when the region of deployment is a belt of dimension $(\lambda_1, \lambda_2, (1/s))$ in Section 6.4 (Theorem 6.3 and Theorem 6.4). Finally, we extend the results to the k-coverage of any set of congruent crossing paths in a belt of dimension $(\lambda_1, \lambda_2, (1/s))$ in Section 6.5 (Theorem 6.5). Theorems 6.5 and 6.4 together provide critical conditions for weak k-barrier coverage in an arbitrary belt when the model of deployment is Poisson or random uniform.

6.1 Finite Set of Orthogonal Crossing Lines

Let L_ℓ for any positive integer ℓ be the set of ℓ regularly-spaced orthogonal crossing lines in an $s \times (1/s)$ belt region, as illustrated in Figure 9, with any two consecutive lines a distance of s/ℓ apart. The L_ℓ in the following lemma refers to this set.

LEMMA 6.1. *All orthogonal crossing lines in an $s \times (1/s)$ belt region are k-covered by a sensor network with a sensing radius of r if all orthogonal crossing lines in L_ℓ are k-covered by the same network with a sensing radius of $r' = r - s/(2\ell)$.*

PROOF. Assume that all lines in L_ℓ are k-covered by a sensor network with a sensing radius of $r' = r - s/(2\ell)$. Let i be an arbitrary orthogonal crossing line in the region, and let i' be an orthogonal crossing line in L_ℓ that is closest to i. Obviously, i and i' (which are parallel to each other, if not identical) are apart by a distance no more than $s/2\ell$. By assumption, i' is k-covered and, thus, intersects at least k active sensors' sensing discs $D_{r'}(u)$. Let u be any of such sensors, and let a be any point in the intersection of i' and $D_{r'}(u)$. Note that $d(u,a) < r'$. Let v be the point on i that is closest to a. Then, $d(a,v) \leq s/(2\ell)$. From triangle inequality,

$$d(u,v) \leq d(u,a) + d(a,v) < r' + \frac{s}{2\ell} = r.$$

Therefore, v is covered by u and so is line i. Since there are at least k such sensors u, i is k-covered using a sensing radius of r. This proves the lemma. □

With this lemma, when wanting to show that all orthogonal crossing lines in the protected region are k-covered by a sensor network with a sensing radius of r, we will only have to show that all orthogonal crossing lines in L_ℓ, with an appropriate value of ℓ and with a reduced sensing radius of $r - s/(2\ell)$, are k-covered. This result also helps in simulation because whenever we need to show that all orthogonal crossing lines (uncountable) are covered using a sensing radius of r, we will only need to show that all crossing lines in L_ℓ (finite) are covered using a sensing radius of $r - s/(2\ell)$.

6.2 Sufficient Condition for k-Coverage of Orthogonal Crossing Lines

In this section, we prove a sufficient condition for the coverage of all orthogonal crossing lines in a rectangular belt region. Note that orthogonal crossing lines in a rectangular belt region are not only congruent, but also parallel to each other.

Let $N(n,p,r)$ be as defined in Definition 2.2, $c(s)$ be as defined in (3), and $\phi(np)$ be as defined in Definition 3.1. Let

$$\ell = (np)\phi(np). \tag{5}$$

And again, let L_ℓ be the set of ℓ orthogonal crossing lines as defined in Section 6.1.

The following lemma indicates a sufficient condition for all crossing lines in L_ℓ to be k-covered *whp*.

LEMMA 6.2. *Let $N(n,p,r)$ be a Poisson distributed sensor network over an $s\times(1/s)$ belt region. If $c(s) = 2npr/(s\log(np))$ satisfies*

$$c(s) = 1 + \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)}, \tag{6}$$

for sufficiently large s, then all orthogonal crossing lines in L_ℓ are k-covered whp as $s \to \infty$.

PROOF. Since the probability of a crossing line to be k-covered partly depends on whether it is close to either of the two vertical sides, we partition L_ℓ into two sets: I and S. Set I contains all the inner crossing lines which are at least a distance of r away from either of the belt's two vertical sides. Set S contains the remaining crossing lines, which are less than a distance of r away from a side. We will follow the following approach for both the subregions.

Let $A_k(i)$ denote the event that the crossing line i is k-covered. For $Z \in \{I, S\}$, we will obtain a lower bound on $\Pr[\bigwedge_{i\in Z} A_k(i)]$ and show it to approach 1 as $s \to \infty$. Let $X_k(i)$ be a random variable assuming a value of 1 if the crossing line i is not k-covered, and 0 otherwise. In other words, $X_k(i)$ is an indicator of the event $\overline{A_k(i)}$. Let $X_{k,Z} = X_k(1) + X_k(2) + \cdots + X_k(|Z|)$. Now, $\mathbb{E}[X_k(i)] = \Pr[\overline{A_k(i)}]$. Further, since $X_{k,Z}$ is a nonnegative integral valued random variable, $\Pr[X_{k,Z} > 0] \leq \mathbb{E}[X_{k,Z}]$, and therefore, we have

$$\begin{aligned} \Pr[\textstyle\bigwedge_{i\in Z} A_k(i)] &= \Pr[X_{k,Z} = 0] \\ &= 1 - \Pr[X_{k,Z} > 0] \\ &\geq 1 - \mathbb{E}[X_{k,Z}]. \end{aligned} \tag{7}$$

By showing $\mathbb{E}[X_{k,Z}] \to 0$, we will prove the k-coverage of all crossing lines in Z, *whp*.

We first apply the above approach to prove the k-coverage of all orthogonal crossing lines in the interior, I. Let $P_j(i)$ denote the probability that exactly j sensors cover crossing line i. Since sensors are deployed with Poisson distribution, for any line $i \in I$, we have

$$P_j(i) = \exp\left(\frac{-2npr}{s}\right)\left(\frac{\left(\frac{2npr}{s}\right)^j}{j!}\right). \tag{8}$$

This is because sensors are distributed in the r-neighborhood of the crossing line i, whose area is $2r/s$, with a Poisson distribution of rate $2npr/s$. Using the definition of c from (3), we can simplify (8) to the following, when $j > 0$:

$$\begin{aligned} P_j(i) &= (np)^{-c}\left(\frac{(c\log(np))^j}{j!}\right) \\ &\leq (np)^{-c}(c\log(np))^j \\ &= (np)^{-c}(\alpha)^j, \end{aligned} \tag{9}$$

where

$$\alpha = c\log(np). \tag{10}$$

Now, the event $\overline{A_k(i)}$ occurs iff i is covered by less than k sensors. Thus,

$$\Pr[\overline{A_k(i)}] = \sum_{j=0}^{k-1} P_j(i) \leq (np)^{-c}\sum_{j=0}^{k-1}\alpha^j \approx (np)^{-c}\alpha^{k-1} \tag{11}$$

and, therefore,

$$\mathbb{E}[X_{k,I}] = \sum_{i=1}^{|I|}\mathbb{E}[X_k(i)] \leq \ell(np)^{-c}\alpha^{k-1}. \tag{12}$$

We claim that $\mathbb{E}[X_{k,I}] \to 0$ as $s \to \infty$. To verify this, take the logarithm of both sides of (12) and simplify it using (5) and (6) as follows:

$$\log(\mathbb{E}[X_{k,I}]) \leq -\phi(np) + \log(\phi(np)) + (k-1)\log(c). \tag{13}$$

Since $-\phi(np)$ dominates the other two terms, $\log(\mathbb{E}[X_{k,I}])$ goes to $-\infty$ making $\mathbb{E}[X_k]$ to approach 0, as $s \to \infty$. Thus, from (7), we conclude $\Pr[\bigwedge_{i\in I} A_k(i)] \to 1$ as $s \to \infty$.

Next, we prove the k-coverage *whp* of all orthogonal crossing lines in the side region, S. Let $P_j(i)$ be as defined above. Since the r-neighborhood of any orthogonal crossing line $i \in S$ is at least r/s, we obtain the following in place of (8)

$$P_j(i) \leq \exp\left(\frac{-npr}{s}\right)\left(\frac{\left(\frac{npr}{s}\right)^j}{j!}\right). \tag{14}$$

In place of (9), we obtain

$$P_j(i) \leq (np)^{\frac{-c}{2}}\left(\frac{\alpha}{2}\right)^j, \tag{15}$$

where α is as defined in (10); and in place of (11), we obtain

$$\Pr[\overline{A_k(i)}] \leq (np)^{\frac{-c}{2}}\left(\frac{\alpha}{2}\right)^{k-1} \tag{16}$$

Since the total number of orthogonal crossing lines in S is $2r\ell/s$, we obtain the following in place of (12):

$$\mathbb{E}[X_{k,S}] \leq \frac{2r\ell}{s}(np)^{-c}\left(\frac{\alpha}{2}\right)^{k-1} \leq \phi(np)(np)^{-c}\alpha^k \qquad (17)$$

where notice that $2r\ell/s$ can be written as $c\log(np)\phi(np)$ using (5) and (3). Take the logarithm of both sides of (17) and simplify it using (6) as follows:

$$\begin{aligned}\log(\mathbb{E}[X_{k,S}]) &\leq \log(\phi(np)) - \log(np) - \phi(np) \\ &\quad +k\log(c) + \log\log(np). \qquad (18)\end{aligned}$$

Observe that the right hand side of (18) approaches $-\infty$, and hence $\mathbb{E}[X_{k,S}] \to 0$, as $s \to \infty$. Thus, from (7), we conclude $\Pr[\bigwedge_{i\in S} A_k(i)] \to 1$ as $s \to \infty$. This completes the proof. □

Now, let us consider the same sensors deployed on the long belt, but with the original sensing radius of r. We will now use Lemma 6.2 together with Lemma 6.1 to establish a sufficient condition for the k-coverage *whp* of all orthogonal crossing lines in the protected region, in the following theorem.

THEOREM 6.1. *Let $N(n,p,r)$ be a Poisson distributed sensor network over an $s\times(1/s)$ belt region. If $c(s) = 2npr/(s\log(np))$ satisfies*

$$c(s) \geq 1 + \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)} \qquad (19)$$

for sufficiently large s, then all the orthogonal crossing lines in the region are k-covered ***whp*** *as $s \to \infty$.*

PROOF. First, assume that condition (19) is satisfied with equality. Let L_ℓ be the set of orthogonal crossing lines introduced in Section 6.1. Let $r' = r - s/(2\ell)$ be a reduced sensing radius; let $c'(s) = 2npr'/(s\log(np))$; and $\ell = np\phi(np)$ as defined in (5). It is easy to verify that

$$\begin{aligned} c'(s) &= \frac{2np\,(r - s/(2\ell))}{s\log(np)} \\ &= c(s) - \frac{2/\phi(np)}{\log(np)} \\ &= 1 - \frac{\phi'(np) + (k-1)\log\log(np)}{\log(np)}, \qquad (20)\end{aligned}$$

where $\phi'(np) = \phi(np) - 2/\phi(np)$. Note that $\phi'(np)$ shares $\phi(np)$'s property of being asymptotically monotonically increasing, approaching infinity, and in $o(\log\log(np))$. Applying Lemma 6.2 now ensures the k-coverage *whp* of all crossing lines in L_ℓ when the reduced sensing radius r' is used; and, applying Lemma 6.1 ensures the k-coverage *whp* of all crossing lines in the protected region when the original sensing radius r is used.

Now suppose the inequality in (19) holds. Then there exists an $r_l \leq r$ for which $c_l(s) = 2npr_l/(s\log(np))$ satisfies

$$c_l(s) = 1 + \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)}$$

and so, by the first part of this proof, all the orthogonal crossing lines in the region are k-covered *whp* using this smaller sensing radius r_l. All the crossing lines in the region are evidently covered when the original, larger sensing radius r is used. □

6.3 Sufficient Condition for Non-coverage of Orthogonal Crossing Lines

In this section, we prove a sufficient condition for the existence of an uncovered orthogonal crossing path in a rectangular belt region.

If $P(s)$ denotes the probability that all the orthogonal crossing lines in the protected region are k-covered by Poisson distributed sensors of rate np, in view of Theorem 6.1, a necessary condition for k-coverage *whp* may take the following form: *If $c(s) < f(s)$ for sufficiently large s, then* $\lim_{s\to\infty} P(s) < 1$. In the next theorem, we establish a condition under which it is not just $\lim_{s\to\infty} P(s) < 1$, but $\lim_{s\to\infty} P(s) = 0$. Such a result is stronger than a mere necessary condition when we are dealing with probabilities. This is because if the probability of the event of non-coverage is close to one then we expect that if the condition for non-coverage is satisfied, then there will exist a non-covered orthogonal crossing line, *whp*. Whereas, if we were to prove a necessary condition for coverage, then all we could claim is that if the necessary condition is not satisfied, then sometimes there may exist a non-covered orthogonal crossing line, but not always.

In the following theorem and its proof, $c(s)$ and $\phi(np)$, as well as ℓ and L_ℓ, are all the same as defined in Section 6.2.

THEOREM 6.2. *Let $N(n,p,r)$ be a Poisson distributed sensor network over an $s\times(1/s)$ belt region. If $c(s) = 2npr/(s\log(np))$ satisfies*

$$c(s) \leq 1 - \frac{\phi(np) + \log\log(np)}{\log(np)} \qquad (21)$$

for sufficiently large s, then there exists a non-k-covered orthogonal crossing line in the region ***whp*** *as $s \to \infty$.*

PROOF. First assume that the "=" in condition (21) holds. That is,

$$c(s) = 1 - \frac{\phi(np) + \log\log(np)}{\log(np)} \qquad (22)$$

Consider the set of interior crossing lines $I \subseteq L_\ell$ as defined in the proof of Lemma 6.2. We show that *whp* there exists a non-1-covered crossing line in I.

For any crossing line $i \in I$, let $A(i)$ denote the event that i is 1-covered; and $\overline{A(i)}$, its negation. Also, let X_i be the indicator random variable of event $\overline{A(i)}$, i.e. $X_i = 1$ if i is *not* 1-covered and 0, otherwise. Let X be the number of lines in I which are not 1-covered. Then, $X = X_1 + X_2 + \cdots + X_\kappa$, where $\kappa = |I|$. We will show that $X > 0$ *whp* using Corollary 4.3.4 of [2], which states that *whp* $X > 0$ if

$$\mathbb{E}[X] \to \infty \text{ and } \Delta = o(\mathbb{E}^2[X]), \qquad (23)$$

where $\mathbb{E}[X]$ denotes the expected value of X and

$$\Delta = \sum_{u\sim v} \Pr[\overline{A(u)} \wedge \overline{A(v)}],$$

where $u \sim v$ means $u \neq v$ and $\overline{A(u)}$ and $\overline{A(v)}$ are not independent.

We first show $\mathbb{E}[X] \to \infty$. From the first equality of (9) and the fact $\mathbb{E}[X_i] = \Pr[\overline{A(i)}] = P_0(i)$, we obtain

$$\mathbb{E}[X_i] = \Pr[\overline{A(i)}] = P_0(i) = (np)^{-c}, \qquad (24)$$

and

$$\mathbb{E}[X] = \sum_{i=1}^{\kappa} \mathbb{E}[X_i] = \kappa(np)^{-c}, \quad (25)$$

where

$$\kappa = |I| = (1 - 2r/s)\ell. \quad (26)$$

Taking the logarithm of $\kappa(np)^{-c}$ and simplifying it using (22) and the relation $\ell = (np)\phi(np)$ yields

$$\log\left(\kappa(np)^{-c}\right) = \log(1 - 2r/s) + \phi(np) + \log\log(np) + \log(\phi(np)). \quad (27)$$

As $s \to \infty$, the right hand side of (27) goes to infinity, thereby forcing $\mathbb{E}[X]$ to go to infinity.

Next, we show $\Delta = o(\mathbb{E}^2[X])$ by obtaining an upper bound on Δ and then showing the upper bound to be $o(\mathbb{E}^2[X])$. To this end, we first obtain an upper bound on $\Pr[\overline{A(i)} \wedge \overline{A(j)}]$:

$$\Pr[\overline{A(i)} \wedge \overline{A(j)}] \leq \Pr[\overline{A(i)}] = P_0(i) = (np)^{-c}. \quad (28)$$

There are no more than $2r\ell^2/s$ pairs of i and j such that $i \sim j$, for $|I| \leq \ell$ and, for any $i \in I$, at most $2r\ell/s$ lines satisfy the "$\sim$" relation with i. Therefore,

$$\Delta = \sum_{(i \sim j) \wedge (i,j \in I)} \Pr[\overline{A(i)} \wedge \overline{A(j)}] \leq \frac{2r\ell^2}{s}(np)^{-c}. \quad (29)$$

Using (25) and (29), we obtain an upper bound on $\Delta/\mathbb{E}]^2[X$:

$$\frac{\Delta}{\mathbb{E}^2[X]} \leq \frac{2r(np)^{-c}}{s(1-2r/s)^2(np)^{-2c}} \leq \frac{\log(np)(np)^{(c-1)}}{(1-2r/s)^2}. \quad (30)$$

In the last inequality, we have used $r/s = c(s)\log(np)/(np)$, a relation that follows from (3) and the fact $c(s) \leq 1$ implied by (22).

Taking the logarithm of the right hand side of (30) and simplifying it using (22) yields

$$-\phi(np) - 2\log(1 - 2r/s), \quad (31)$$

which goes to $-\infty$ as $s \to \infty$, thereby forcing the right hand side of (30) to approach 0. This proves $\Delta = o(\mathbb{E}^2[X])$. From this and the earlier proved result, $\mathbb{E}[X] \to \infty$, we conclude by Corollary 4.3.4 of [2] that $X > 0$ *whp* and, therefore, *whp* there exists a non-covered crossing line.

Now suppose the inequality in (21) holds. There exists an $r_u \geq r$ for which $c_u(s) = 2npr_u/(s\log(np))$ satisfies (22), and so by the first part of this proof *whp* there exists a non-1-covered orthogonal crossing line when using the sensing radius r_u. Thus, when the original, smaller sensing radius r is used, evidently there there will exist a non-1-covered orthogonal crossing line in the region. ☐

6.4 Coverage of Orthogonal Crossing Lines in a Belt

In this section, we extend the critical conditions for the k-coverage of orthogonal crossing lines (sufficient condition for coverage derived in Section 6.2 and sufficient condition for non-coverage derived Section 6.3) in rectangular belt regions to belt regions of dimension $(\lambda_1, \lambda_2, (1/s))$.

Recall the definition of a belt of dimension $(\lambda_1, \lambda_2, (1/s))$ from Section 2 (Definition 2.5). For ease of presentation, we assume in this paper that belts have a nominal total length of 2s; i.e. $\lambda_1 + \lambda_2 = 2s$. Under this assumption, the area of a belt with dimension $(\lambda_1, \lambda_2, (1/s))$ is 1.

Recall from Definition 2.9 that a crossing line over a belt of width $1/s$ is said to be *orthogonal* to the belt if its length is $1/s$ (i.e. it crosses the belt along a shortest path). Notice that the orthogonal crossing lines for a belt of dimension $(\lambda_1, \lambda_2, (1/s))$ need not be parallel to each other. For example, at most two orthogonal crossing lines (out of uncountably many of them) in the belt region shown in Figure 2 are parallel to each other. At the same time, since orthogonal crossing lines are the shortest paths through the belt region, we would like to establish a sufficient condition for their coverage *whp*, for use in applications. This is the subject of the following theorem.

THEOREM 6.3. *Let $N(n,p,r)$ be a Poisson distributed sensor network over a belt of dimension $(\lambda_1, \lambda_2, (1/s))$. If $c(s) = 2npr/(s\log(np))$ satisfies*

$$c(s) \geq 1 + \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)} \quad (32)$$

*for sufficiently large s, then all orthogonal crossing lines over the belt are k-covered **whp** as $s \to \infty$.*

PROOF. The proof is not much different from that of Theorem 6.1, so we will only give a sketch of it here.

First, let $\ell = (np)\phi(np)$ as in (5). We claim that if $N(n,p,r)$ satisfies (32), then $N(n,p,r')$ with $r' = r - s/(2\ell)$ and $c'(s) = 2npr'/(s\log(np))$, will satisfy

$$c'(s) \geq 1 + \frac{\phi'(np) + (k-1)\log\log(np)}{\log(np)}. \quad (33)$$

This claim can be easily proved in the same way as (20) was obtained in the proof of Theorem 6.1.

Second, we define a set of crossing lines L'_ℓ such that if (33) holds for all sufficiently large s then all crossing lines in L'_ℓ will be k-covered *whp* by $N(n,p,r')$. L'_ℓ is defined as follows. Let the two lines of the belt be l_1 and l_2, which have lengths λ_1 and λ_2, respectively. (Recall that $\lambda_1 + \lambda_2 = 2s$.) On the two lines, mark a total of 2ℓ points regularly spaced at a distance of s/ℓ. This results in $\ell\lambda_1/s$ marked points on line l_1 and $\ell\lambda_2/s$ points on line l_2. Connect each marked point to the nearest point on the other line with a line segment of length $1/s$. Let L'_ℓ be the set of all such line segments, which are each an orthogonal crossing line. Note that $|L'_\ell| \leq 2\ell$. Now, we divide L'_ℓ into two subsets, I' and S', just as we divided L_ℓ into I and S in the proof of Lemma 6.2, then $|I'| \leq 2\ell$.

In place of (8), we obtain the following

$$P_j(i) \leq \exp\left(\frac{-2npr}{s}\right)\left(\frac{\left(\frac{2npr}{s}\right)^j}{j!}\right), \quad (34)$$

because the r-neighborhood of an orthogonal crossing line may now be larger than $2r/s$. Corresponding to (9), we obtain

$$P_j(i) \leq (np)^{-c}(\alpha)^j, \quad (35)$$

where α is as defined in (10).

Since with the above inequalities, (11) continues to hold, we obtain the following in place of (12)

$$\mathbb{E}[X_k] \leq 2\ell(np)^{-c}\alpha^{k-1}, \quad (36)$$

and in place of (13), we obtain

$$\log(\mathbb{E}[X_k]) \leq -\phi(np) + \log(2\phi(np)) + (k-1)\log(c). \quad (37)$$

Since $-\phi(np)$ still dominates the other two terms, $\log(\mathbb{E}[X_k])$ goes to $-\infty$ making $\mathbb{E}[X_k]$ to approach 0, as $s \to \infty$. Thus, $\Pr[\bigwedge_{i \in I'} A_k(i)] \to 1$ as $s \to \infty$. The proof for crossing lines in S can be carried out in a similar manner.

Third, we claim that if all (orthogonal) crossing lines in L' are k-covered by $N(n, p, r')$, then all orthogonal crossing lines in the protected belt are k-covered by $N(n, p, r)$. To see this, we observe that for any orthogonal crossing line l in the belt, there is a crossing line l' in L'_ℓ such that l and l' are separated by a distance no more than $s/(2\ell)$. The proof of Lemma 6.1 can now be carried over here to prove the claim. From the above three claims, the theorem follows immediately. □

The following theorem establishes a sufficient condition for the existence of an uncovered crossing path in a belt of dimension $(\lambda_1, \lambda_2, (1/s))$.

THEOREM 6.4. *Let $N(n, p, r)$ be a Poisson distributed sensor network over a belt of dimension $(\lambda_1, \lambda_2, (1/s))$. If $c(s) = 2npr/(s \log(np))$ satisfies*

$$c(s) \leq 1 - \frac{\phi(np) + \log\log(np)}{\log(np)}, \tag{38}$$

*for sufficiently large s, then there exists a non-k-covered orthogonal crossing line in the belt **whp** as $s \to \infty$.*

PROOF. Again, the proof is not much different from that of Theorem 6.2, so we will only give a sketch.

Let L'_ℓ and I' be as defined in the proof of Theorem 6.3. Let X and $A(i)$ be as defined in the proof of Theorem 6.2 and let κ be as defined in (26). Then, $\kappa \leq |I'|$. Since (24) continues to hold here, we obtain the following in place of (25),

$$\mathbb{E}[X] = |I'|(np)^{-c} \geq \kappa(np)^{-c}. \tag{39}$$

As was shown in the proof of Theorem 6.2, the right hand side of (39) approaches ∞ as $s \to \infty$. Therefore, $\mathbb{E}[X] \to \infty$ as $s \to \infty$.

We further note that (28) continues to hold here. Now, given a crossing line $i \in L'_\ell$, there are at most $tr\ell/s$ crossing lines $j \in L'_\ell$ for some constant t such that $i \sim j$. This is because of our model assumption that the lengths λ_1 and λ_2 are both of the order s and the width is $1/s$. Since there are at most $2r\ell$ lines in L'_ℓ, total number of pairs of crossing lines in L'_ℓ that satisfy $i \sim j$ is at most $2tr\ell^2/s$. Therefore, we obtain the following in place of (29)

$$\Delta \leq \frac{2tr\ell^2}{s}(np)^{-c}, \tag{40}$$

and in place of (30), we obtain

$$\frac{\Delta}{\mathbb{E}[X]^2} \leq \frac{2tr(np)^{-c}}{s(np)^{-2c}} \leq t\log(np)(np)^{(c-1)} \tag{41}$$

Taking the logarithm of the right hand side of (41) and simplifying it using (38) yields

$$\log\left(\log(np)(np)^{(c-1)}\right) = \log(t) - \phi(np). \tag{42}$$

The right hand side of (42) still goes to $-\infty$ as $s \to \infty$, thereby forcing the right hand side of (41) to approach 0. This proves $\Delta = o(\mathbb{E}^2[X])$.

The rest of the proof is the same as in Theorem 6.2. □

6.5 Coverage of Any Set of Parallel Crossing Paths

In this section, we extend Theorem 6.3 to the k-coverage *whp* of any set of congruent crossing paths in Theorem 6.5. The sufficient condition for non-coverage established in Theorem 6.4 continues to hold when considering any set of congruent crossing paths and therefore it constitutes one of the two components of a critical condition for weak k-barrier coverage.

THEOREM 6.5. *Let $N(n, p, r)$ be a Poisson distributed sensor network over a belt B_s of dimensions $(\lambda_1, \lambda_2, (1/s))$. If $c(s) = 2npr/(s \log(np))$ satisfies*

$$c(s) \geq 1 + \frac{\phi(np) + (k-1)\log\log(np)}{\log(np)} \tag{43}$$

*for sufficiently large s, then the belt region B_s is weakly k-barrier covered **whp** as $s \to \infty$.*

PROOF. Recall the definition of weak k barrier coverage from (1). The basic difference between the claim made here and that in Theorem 6.3 is the following: Here we claim that for each set of congruent crossing paths, all the crossing paths in that set are k-covered *whp*. In Theorem 6.3, we considered only the set of orthogonal crossing lines. The proof here, though, is not much different from that of Theorem 6.3, so we will only make key observations.

As in the proof of Theorem 6.3 we divide the proof into three claims. For the first claim, there is no change from Theorem 6.3. For the second claim, there are two differences. The first is the following observation: Let $P_j(i)$ be as defined in the proof of Lemma 6.2. We observe that for any crossing path l in the belt region and any orthogonal crossing line l_o, $P_j(l) \leq P_j(l_o)$. This is because with Poisson distribution the rate of Poisson distribution depends only on the area of the region and not on the location of the region and the regions in consideration here are the r-neighborhoods of l and l_o, and the r-neighborhood of l is larger than that of l_o.

The second change is in the construction of L'_ℓ. Given a crossing path i, we construct a set $L_\ell(i)$ (corresponding to L'_ℓ) that comprises $O(\ell)$ crossing paths congruent to i. Envision the belt as having the left end and the right end. We first include in $L_\ell(i)$ the leftmost crossing path j that is congruent to i. Next, we consider all crossing paths that are congruent to i but not entirely contained in the (s/ℓ)-neighborhood of any path that is already in $L_\ell(i)$, and include the leftmost such crossing path in $L_\ell(i)$. We continue this process until the right end of the belt. Since there are at most $O(\ell)$ crossing paths in $L_\ell(i)$ for any crossing path i, the proof of the second claim in Theorem 6.3 can be carried over here.

For the third claim, we observe that Lemma 6.1 can be proved for the coverage of any set of congruent crossing paths in the same way as in the proof of Theorem 6.3, with L'_ℓ replaced by $L_\ell(i)$ constructed in the preceding paragraph. Notice that for any crossing path j that is congruent to i, there is a crossing path $l \in L_\ell(i)$ that is at most a distance of $s/(2\ell)$ from j. □

7. SIMULATION AND NUMERICAL COMPUTATION

In this section, we present some numerical computation and simulation results to help a deployer get a sense of how realistic are our critical conditions for weak k-barrier coverage. The critical conditions are asymptotic results that get more and more accurate as one considers a larger and larger deployment area (so that the number of sensors deployed gets larger and larger). However, deployment regions, in practice, will have a fixed dimension and therefore one might ask — What kind of confidence can one get in real deployments when using our critical conditions to determine the minimum number of sensors to deploy? This is precisely the motivation for presenting simulation results.

Consider a deployment scenario where a rectangular belt region of dimension $10\text{km} \times 100\text{m}$ is to be barrier-covered by sensors, each of which has a sensing radius of 10m. Since our model of a rectangular belt region was that of $s \times (1/s)$, the parameter s, which is the square root of the ratio of length to width, assumes a value of 10 (since the length is 100 times that of the width). With this scaling, the length of the region becomes $s = 10$, the width becomes $1/s = 0.1$, and the radius that was $1/10^{th}$ that of the width becomes $r = (1/10) * (1/s) = 0.01$. Let us suppose that the network is desired to last 10 times longer than the active lifetime of an individual sensor, which implies a duty cycle of 10%. Therefore, $p = 0.1$. We answer the following questions for this deployment scenario from analysis as well as from simulation:

1. What is the minimum number of sensors such that if more than this many are be deployed then the probability that the belt region is weakly 1-barrier covered is close to 1? How closely does theoretical prediction match simulation results?

2. What is the largest number of sensors such that if less than this many are deployed, the probability that the region is weakly 1-barrier covered is close to zero? Again, how closely does theoretical prediction match simulation results?

3. How does k (in weak k-barrier coverage) grow as the number of sensors deployed is increased?

Another interesting question is the following: We know that if a given belt region is weakly barrier covered *whp*, then all crossing paths in any set of congruent crossing paths are covered *whp*. There are uncountably many such sets of congruent crossing paths. Which set should we check? We first check the set of orthogonal crossing paths because they are the shortest crossing paths through the region. Later, we show that the probability of any other set of congruent crossing paths being barrier covered is only higher, as would be expected with Poisson or uniform deployment. We do not make use of Theorem 4.1 in checking whether a region is k-barrier covered as this would not allow us to compute the probability of the region being weakly k-barrier covered.

For determining the probability of the region being weakly barrier covered, we tile the belt region with a set, L, of ℓ orthogonal crossing lines each of which are equally spaced with a spacing of s/ℓ between two consecutive lines, where $\ell = np\phi(np)$. We use $\sqrt{\log\log(np)}$ for $\phi(np)$. We use a reduced sensing radius of $r' = r - s/(2\ell)$. Whenever all the crossing lines in this set L are covered with the sensing radius r', we can conclude by Lemma 6.1 that all the orthogonal crossing lines (uncountably many of them) in the rectangular belt region will be covered with the actual sensing radius, r. Therefore, the probability of coverage of all the crossing paths in the set L is a lower bound on the probability of the region being weakly barrier covered.

During the experiment, we vary the rate of Poisson distribution, n, from 10,000 to 100,000 in steps of 5,000. For each value of n, we generate an instance of a Poisson distributed random variable of rate n. This gives us the actual number of sensors to be deployed in this iteration of the experiment. Now, for each of these sensors, their x and y locations within the rectangular belt are generated randomly with uniform distribution[5]. Finally, each sensor is activated with a probability of $p = 0.1$. After the process of activation, we compute the number of lines in L that are 1-covered using the sensing radius of r'. This experiment of determining how many sensors to deploy, choosing the x and y location for each sensor, and activating each with a probability of p, is repeated 100 times for each value of n to get statistical validation. Then, we compute the following for each value of n,

$$\Pr[\text{All Crossing Lines in } L \text{ 1-Covered}] = \frac{\text{Number of times all crossing lines 1-covered}}{100},$$

which approximates the probability of weak 1-coverage.

We use (3) and (19) to define the following for a given value of p and r.

$$c_{high} = \min_{n} \left\{ c(s) : c(s) \geq 1 + \frac{\phi(np)}{\log(np)} \right\}.$$

With this definition of c_{high}, we expect by Theorem 6.1 that if $2npr/(s\log(np)) \geq c_{high}$, where $p = 0.1$, $s = 10$, and $r = 0.01$, then Pr[All Crossing Lines 1-Covered] should be close to 1. Figure 10 shows the results of simulation for 1-barrier coverage. When $c = c_{high}$, Pr[All Crossing Lines 1-Covered] is approximately 0.99 (close to 1), as predicted by the analysis (Theorem 6.1). At this value of c, $n = 62,000$. We see that the answer to Question 1 posed at the beginning of this section is $n = 62,000$. Further, our analytical result matches very well with our simulation result.

Similarly, for a given value of p and r, we define the following using (3) and (21).

$$c_{low} = \max_{n} \left\{ c(s) : c(s) \leq 1 - \frac{\phi(np) + \log\log(np)}{\log(np)} \right\}.$$

With this definition of c_{low}, we expect by Theorem 6.2 that if $2npr/(s\log(np)) \leq c_{low}$, where $p = 0.1$, $s = 10$, and $r = 0.01$, then Pr[All Crossing Lines 1-Covered] should be close to 0. We observe in Figure 10 that when $c \leq c_{low}$, Pr[All Crossing Lines 1-Covered] is zero, which is again predicted well by our analysis (Theorem 6.2). We observe that when $c = c_{low}$, $n = 21,000$. So, the answer to Question 2 posed at the beginning of this section is $n = 21,000$ from analysis. However, simulation suggests that Pr[All Crossing Lines 1-Covered] continues to be zero for values of $n \leq 30,000$. This suggests that although our condition for weak non-barrier coverage is the strongest possible asymptotically, one may be able to prove a slightly stronger condition (for the 2^{nd} order term) for weak non-barrier coverage than what we have proved in this paper.

Next, we consider the case of k-coverage (Question 3). We first derive the value of k predicted by the analysis as n is

[5]For a Poisson distributed random variable, the location of each sensor, conditioned on the knowledge of how many sensors are to be deployed, is randomly distributed with uniform distribution.

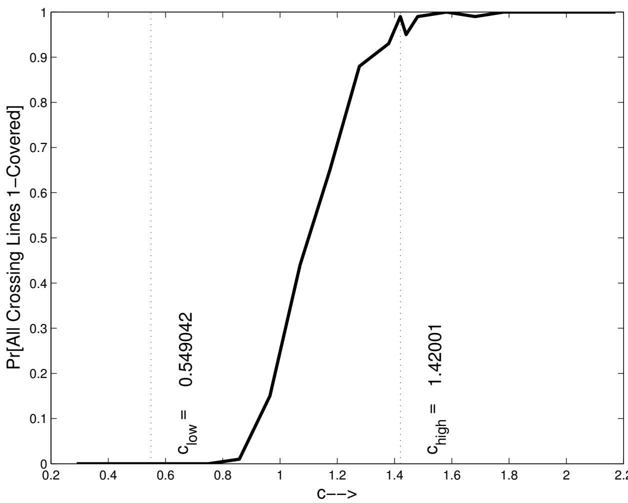

Figure 10: The variation in Pr[All Crossing Lines 1-Covered] for all orthogonal crossing lines when n sensors are deployed with Poisson distribution of rate n. The value of n varies from 10,000 to 100,000 in steps of 5,000. The value of c corresponds to the value of $2npr/(s\log(np))$, where $s = 10$, $r = 0.01$ and $p = 0.1$.

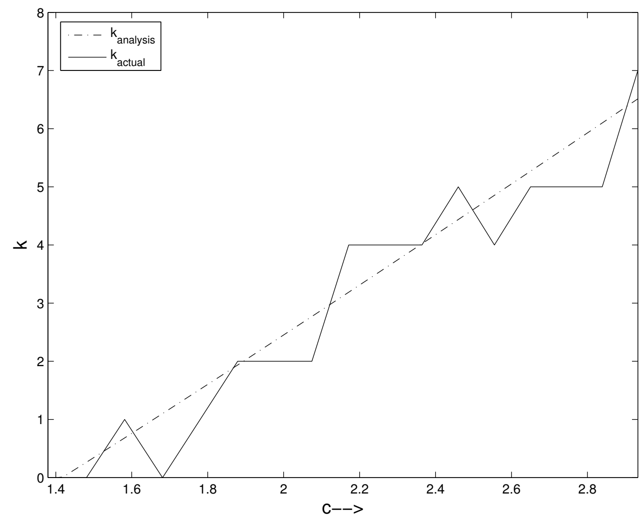

Figure 11: The variation of k for the k-coverage of orthogonal crossing lines when n sensors are deployed with Poisson distribution of rate n. The value of n varies from 60,000 to 140,000 in steps of 5,000. The value of c corresponds to the value of $2npr/(s\log(np))$, where $s = 10$, $r = 0.01$ and $p = 0.1$.

increased. We substitute k by $k_{analysis}$ in (6) to obtain

$$k_{analysis} = \frac{\log(np)}{\log\log(np)}\left(c - \left(1 + \frac{\phi(np)}{\log(np)}\right)\right).$$

The result of simulation appears in Figure 11, from which we observe that k_{actual}, the value of k that was actually observed in the simulation is close to $k_{analysis}$, the value of k predicted by the analysis.

Finally, we consider a set of slanted crossing lines that are parallel to each other. These crossing lines make an angle of $\arctan(r/w)$ with respect to the width. For simulation, we again consider a subset of these slanted crossing lines. We consider ℓ slanted crossing lines, which are parallel to each other and are spaced at a regular separation of s/ℓ. Let us denote this set of slanted crossing lines as L'. We use a sensing radius of $r' = r - s/(2\ell)$ to cover these slanted lines. We again apply Lemma 6.1[6] to ensure that all slanted crossing lines (making an angle of $\arctan(r/w)$ with respect to the width) are covered with a sensing radius of r, if the ℓ slanted lines in L' are covered using a sensing radius of r'. The graph in Figure 12 shows the results from simulation for the coverage of slanted crossing lines in L'. We observe from Figure 12 that the behavior of Pr[All Crossing Lines 1-Covered] is similar (and slightly better) to that observed for the orthogonal crossing lines, as expected.

We claimed at the beginning of Section 2 that our critical conditions also hold for random uniform distribution. We conducted experiments for random uniform distribution also and the results are very similar to what we have presented for Poisson distribution in this section. We have omitted the graphs for brevity.

We observe that for the case of barrier coverage, random placements need approximately $\log(n)$ more sensors than that needed for deterministic placement (compare Theorem 5.1 and Theorem 6.1), which conforms to the analogous results well known in the random graphs literature. For the example considered in this section, we will need 500 sensors to achieve 1-barrier coverage if deploying sensors deterministically. For random placement, we will need 6,200 sensors[7] to get 1-barrier coverage *whp*.

8. CONCLUSION

Detection of intruders breaching the perimeter of a building or an estate, or those crossing an international border is increasingly being seen as an important application for wireless sensor networks. We need a theoretical foundation to determine the minimum number of sensors to be deployed so that intruders crossing a barrier of sensors will always be detected by at least k active sensors. In this paper, we defined the concept of k-barrier coverage (arguably the weakest form of coverage in the area of wireless sensor networks) and derived several key results such as the optimal number of sensors needed to achieve k-barrier coverage, and efficient algorithms to determine whether a given belt region is k-barrier covered or not.

As the concept of barrier coverage is a relatively new concept, several problems still remain open in this space. One such problem is the derivation of critical conditions for strong k-barrier coverage for a belt region. Another open problem is that of topology control when a wireless sensor network has been deployed for barrier coverage. Also, the impact of barrier coverage on classification and tracking of intruders is not yet fully explored. In our future work, we plan to address these and other open problems in the area of barrier coverage.

[6] Lemma 6.1 can be easily proved for slanted crossing lines in the same manner as it has been proved for orthogonal crossing lines.

[7] $n * p = 62000 * 0.1 = 6200$.

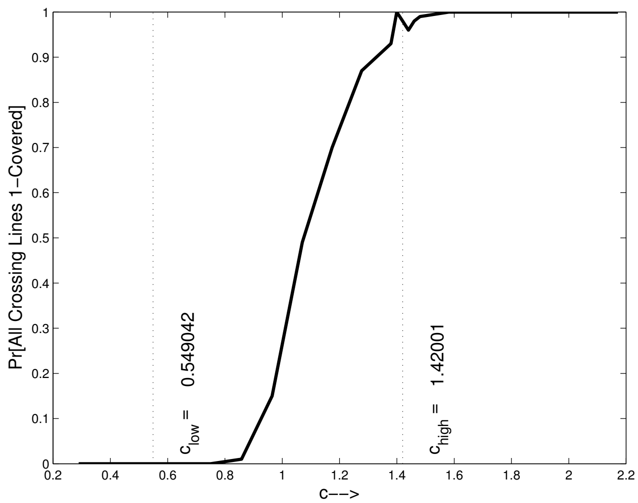

Figure 12: The variation in Pr[All Crossing Lines 1-Covered] for all crossing lines making an angle of $\arctan(r/w)$ with respect to the width, when n sensors are deployed with Poisson distribution of rate n. The value of n varies from 10,000 to 100,000 in steps of 5,000. The value of c corresponds to the value of $2npr/(s\log(np))$, where $s = 10$, $r = 0.01$ and $p = 0.1$.

Acknowledgments

We wish to thank Jozsef Balogh and Neil Robertson from the department of Mathematics at the Ohio State University for their insightful comments. We also thank anonymous referees and our shepherd whose comments helped us improve the presentation of this paper. This work was partially supported by DARPA contract OSU-RF #F33615-01-C-1901.

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