We proved the following theorem in class.

**Theorem 5.** If $T(n)$ is asymptotically nondecreasing and $f(n)$ is smooth, then $T(n) = O(f(n)|n \text{ a power of } b)$ implies $T(n) = O(f(n))$.

1. Show that Theorem 5 would not hold if $T(n)$ is not asymptotically nondecreasing. (Give a counterexample.)

2. Show that Theorem 5 would not hold if $f(n)$ is nondecreasing but not smooth (even if $T(n)$ is asymptotically nondecreasing). (Give a counterexample.)

3. Prove **Theorem 6**: If $T(n)$ is asymptotically nondecreasing and $f(n)$ is smooth, then $T(n) = \Omega(f(n)|n \text{ a power of } b)$ implies $T(n) = \Omega(f(n))$. 