CIS 6331 Homework 9
Due: Friday, April 14 by class time

Note: in this homework, use the definition of flow that includes the skew-symmetry condition.

1. Consider a flow network in which vertices, as well as edges, have capacities. In addition to the original edge capacity constraint, there is now a new vertex capacity constraint: the total positive flow entering any vertex \( u \) cannot exceed its capacity \( c(u) \). Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum flow problem.

2. Suppose that during an execution of Relabel-to-Front, \( \text{Discharge}(u) \) is called twice for some particular node \( u \).

**Question:** Prove or disprove that if an edge \((u,v)\) is inadmissible at the end/exit of the first \( \text{Discharge}(u) \), then it is still inadmissible at the beginning/entry of the second \( \text{Discharge}(u) \). Clearly indicate whether you prove or disprove.

3. Let \( G = (V,E) \) be a flow network with source \( s \), sink \( t \), and integer capacities. Suppose we are given a maximum flow \( f \) in \( G \), and suppose the capacity of a single edge \((u,v) \in E\) is increased by 1. Give an \( O(V + E) \)-time algorithm to update the maximum flow.

4. Let \( G = (V,E) \) be a flow network with source \( s \), sink \( t \), and integer capacities. Suppose we are given a maximum flow \( f \) in \( G \), and suppose the capacity of a single edge \((u,v) \in E\) is decreased by 1. Give an \( O(V + E) \)-time algorithm to update the maximum flow.