CSE 6361 Homework 3

Due: Thursday, September 10 by class time

1. A function $T(n)$ satisfies the following recurrence:

$$T(n) = \begin{cases} 
c, & \text{if } n \leq 1 \\
3T([n/4]) + n, & \text{if } n > 1
\end{cases}$$

where $c$ is a positive constant. Prove that $T(n)$ is asymptotically nondecreasing.

2. Determine the tight asymptotic complexity of the following function. Give your answer in $O$ notation with a proof.

$$T(n) = \begin{cases} 
b, & \text{if } n \leq 3 \\
T([n/2]) + T([n/4]) + cn, & \text{if } n > 3
\end{cases}$$

3. Determine the tight asymptotic complexity of the following function. Give your answer in $O$ notation with a proof.

$$T(n) = \begin{cases} 
b, & \text{if } n \leq 3 \\
T([n/2]) + 2T([n/4]) + cn, & \text{if } n > 3
\end{cases}$$

4. Use the master method to solve the following recurrences.

(a) $T(n) = 4T(n/2) + n^2$.
(b) $T(n) = 4T(n/2) + n^2 \log^2 n$.
(c) $T(n) = 4T(n/2) + n^3$.

5. The running time of an algorithm $A$ is described by the recurrence $T(n) = 7T(n/2) + n^2$. A competing algorithm $A'$ has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for $a$ such that $A'$ is asymptotically faster than $A$. 