1. Give the EDGES of the minimum spanning tree of the weighted graph in Figure 1 in the order they would be output by Prim’s algorithm starting at vertex $v_1$.

![Figure 1](image1.png)

2. Give the EDGES of the shortest path tree of the weighted graph in Figure 2 in the order they would be output by Dijkstra’s shortest path algorithm starting at vertex $v_1$.

![Figure 2](image2.png)

3. Consider the following graph $G$ with solid and dotted edges:

![Graph](image3.png)

The solid edges form a spanning tree $T$ of graph $G$. Each of the solid edges has a weight. Assign weights to the dotted edges, $(1,2)$, $(2,3)$, $(4,5)$, $(3,6)$ and $(4,7)$, such that:

- Each of the edge weights is a positive INTEGER;
- Tree $T$ is a MINIMUM spanning tree of $G$ and NO other tree is a MINIMUM spanning tree of $G$;
- Each of the edge weights of the dotted edges is as small as possible.

For instance, if you assign the edge weight 1 to edge $(2,3)$, then replacing edge $(3,5)$ in $T$ with edge $(2,3)$ will give a spanning tree with less weight than $T$. Thus edge $(2,3)$ must have a weight greater than 1. If you assign the edge weight 2 to edge $(2,3)$, then replacing edge $(3,5)$ in $T$ with edge $(2,3)$ will give a
different spanning tree with weight equal to \( T \). This new tree would also be a minimum spanning tree. Thus edge \((2,3)\) must have a weight greater than 2.

4. Consider the following graph \( G \) with solid and dotted edges:

![Graph Image]

The solid edges form a spanning tree \( T \) of graph \( G \). Each of the solid edges has a weight. Assign weights to the dotted edges, \((2,5)\), \((3,6)\), \((6,8)\), \((7,8)\) and \((6,9)\), such that:

- Each of the edge weights is a positive INTEGER;
- Tree \( T \) is a SHORTEST PATH tree of \( G \) and NO other tree is a SHORTEST PATH tree of \( G \);
- Each of the edge weights of the dotted edges is as small as possible.

For instance, the distance from \( v_1 \) to \( v_5 \) in \( T \) is 6. If you assign the edge weight 3 to edge \((2,5)\), then replacing edge \((1,4)\) in \( T \) with edge \((2,5)\) will give a spanning tree whose distance from \( v_1 \) to \( v_5 \) is 5. Thus edge \((2,5)\) must have a weight greater than 3. If you assign the edge weight 4 to edge \((2,5)\), then replacing edge \((1,4)\) in \( T \) with edge \((2,5)\) will give a spanning tree whose distance from \( v_1 \) to \( v_5 \) is 6. This new tree would also be a shortest path tree of \( G \). Thus edge \((2,5)\) must have weight greater than 4.

5. Let \( T \) be a minimum spanning tree of an edge weighted graph \( G \). Let \( e \) be an edge of \( G \) whose weight is larger than the weight of any other edge of \( G \) and such that \( G - \{e\} \) is connected. (\( G - \{e\} \) is the graph \( G \) with edge \( e \) deleted.) Prove that \( e \) is not an edge of \( T \).