



OpenGL Transformation Composition

- A global modeling transformation matrix
(`GL_MODELVIEW`, called it M here)
- The user is responsible to reset it if necessary

`glMatrixMode(GL_MODELVIEW)`

`glLoadIdentity()`

-> $M = \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$



OpenGL Transformation Composition

- Matrices for performing user-specified transformations are multiplied to the model view global matrix
- For example,

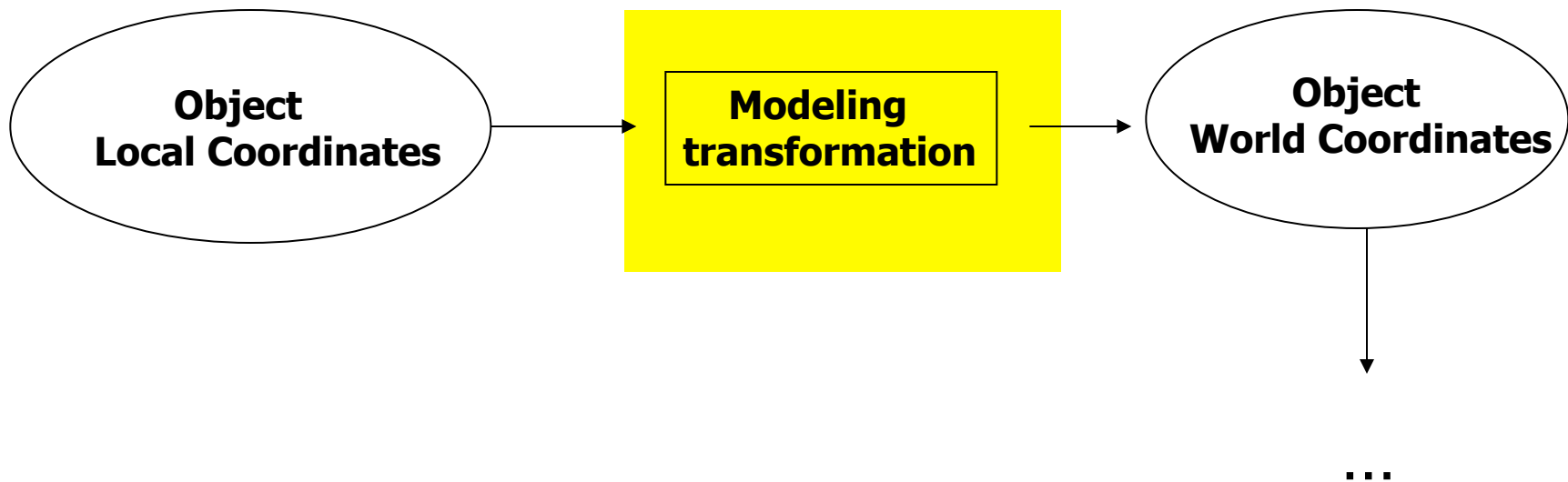
$$\text{glTranslated}(1,1,0); \quad M = M \times \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- All the vertices P will go through the transformation (modeling transformation)

$$P' = M \times P$$



Transformation Pipeline





OpenGL Transformation



- OpenGL **postmultiplies** each new transformation matrix

$$M = M \times M_{\text{new}}$$

- Example: perform translation, then rotation

0) $M = \text{Identity}$

1) translation $T(tx, ty, 0) \rightarrow M = M \times T(tx, ty, 0)$

2) rotation $R(\theta) \rightarrow M = M \times R(\theta)$

3) Now, transform a point $P \rightarrow P' = M \times P$

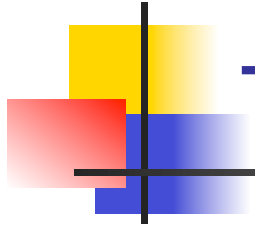
$$= \boxed{T(tx, ty, 0) \times R(\theta) \times P}$$

Wrong!!!

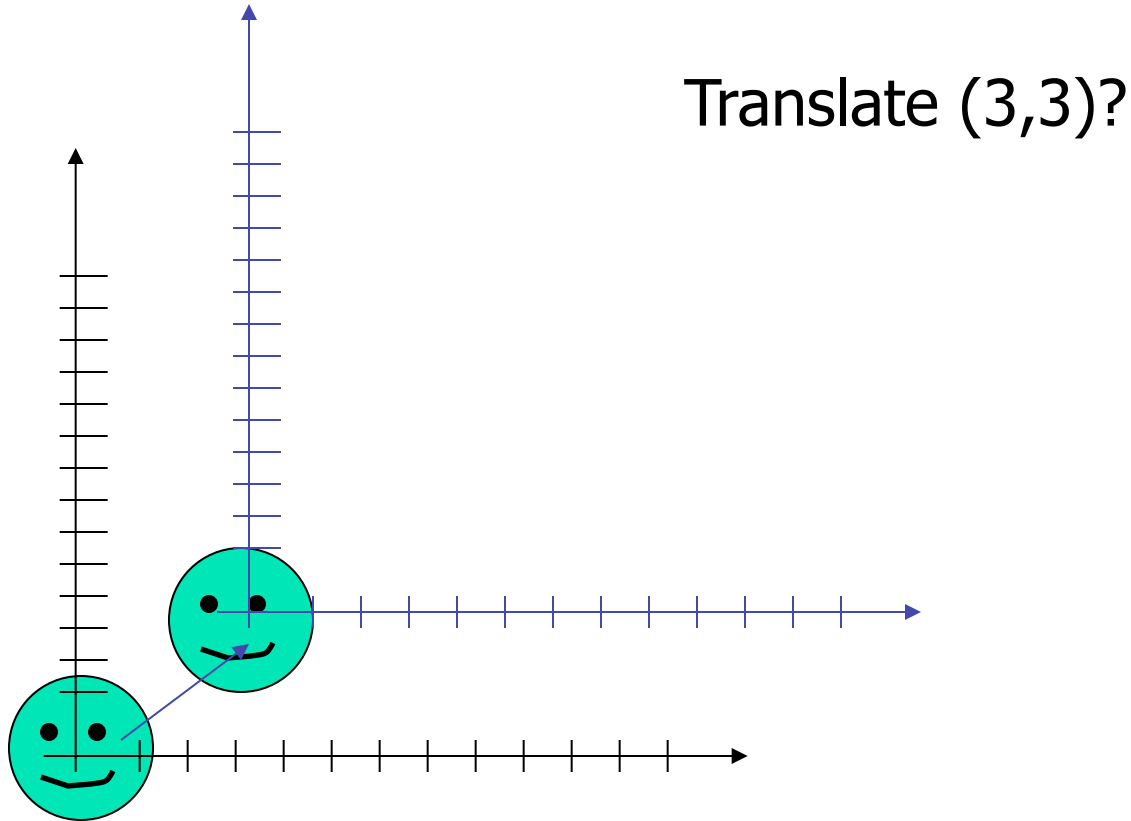


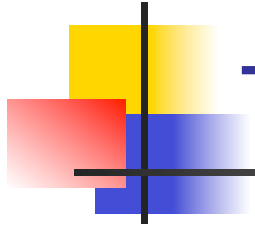
OpenGL Transformation

- When use OpenGL, we need to think of object transformations as moving its local coordinate frame
- All the transformations are performed **relative to the current coordinate frame origin and axes**

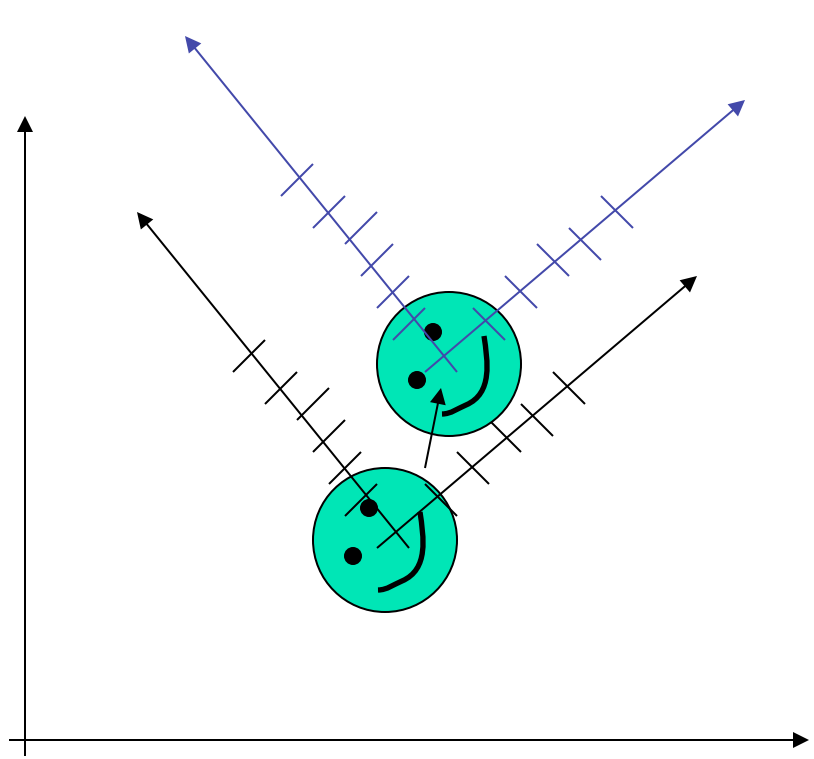


Translate Coordinate Frame

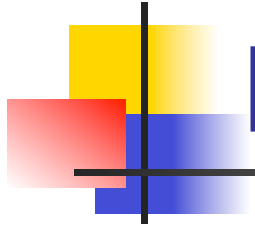




Translate Coordinate Frame (2)

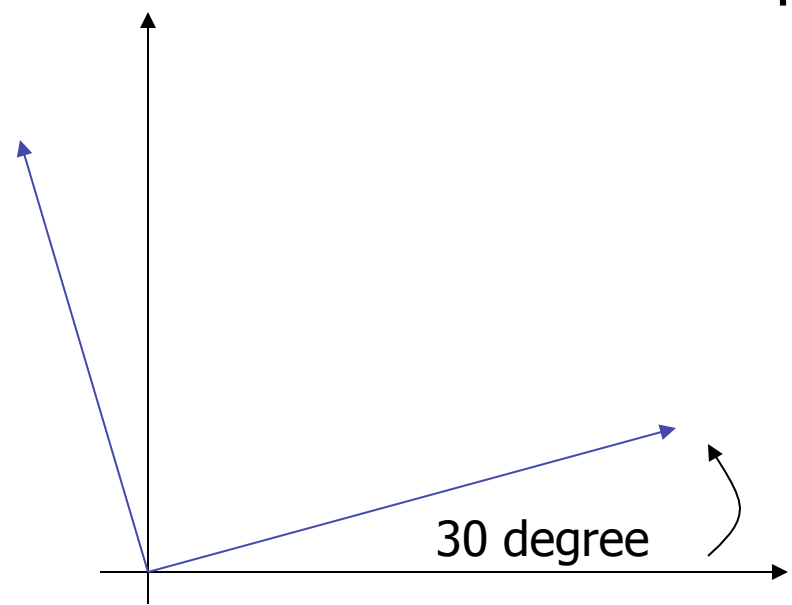


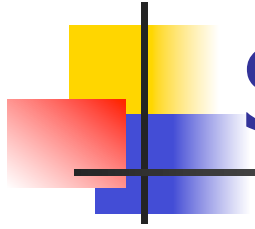
Translate (3,3)?



Rotate Coordinate Frame

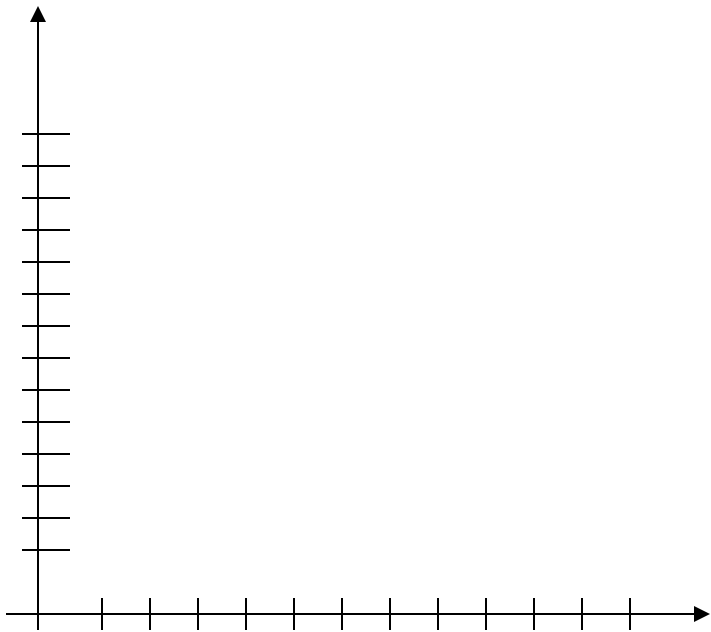
Rotate 30 degree?

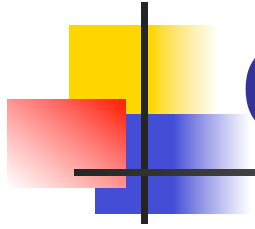




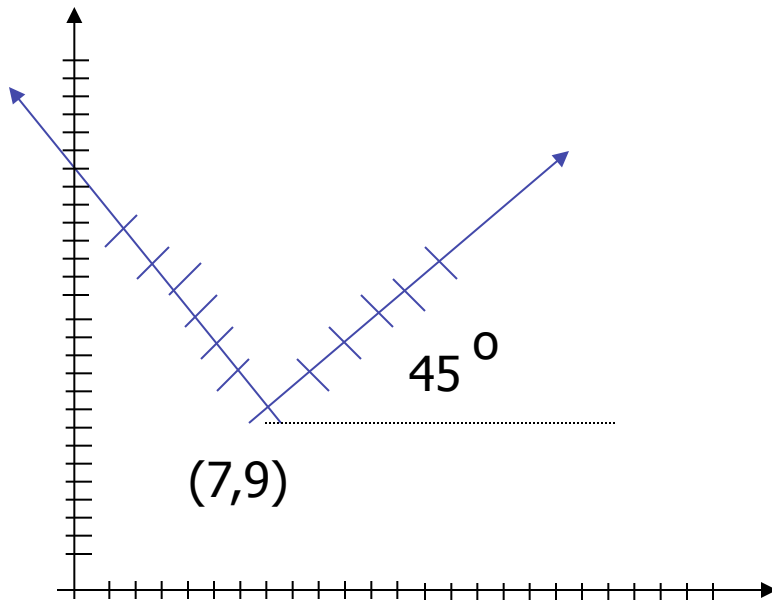
Scale Coordinate Frame

Scale (0.5,0.5)?





Compose Transformations



Transformations?

Answer:

1. Translate(7,9)
2. Rotate 45
3. Scale (2,2)



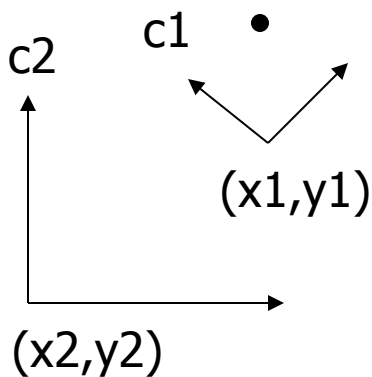
OpenGL Transformation

- Think of transformation as moving coordinate frames
- Call OpenGL transformation functions in that order
- OpenGL will actually perform the transformations in the reverse order

Transform Coordinates

Coordinate system transformation

- Transform an object from coordinate system C1 with the origin at (x_1, y_1) or (x_1, y_1, z_1) in 3D, to coordinate system C2 with the origin (x_2, y_2) or (x_2, y_2, z_2) in 3D



1. Find the transformation sequence to move C2 to C1 (so C2 will align with C1)
 - Move the origin of C2 to coincide with the origin of C1
 - Rotate the basis vectors of C2 so that they coincide with C1's.
 - Scale the unit if necessary
2. Apply the above transformation sequence to the object in the opposite order