## CSE 5243 INTRO. TO DATA MINING

Mining Frequent Patterns and Associations: Basic Concepts (Chapter 6) Huan Sun, CSE@The Ohio State University

Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods
$\square$ Basic Concepts
$\square$ Efficient Pattern Mining Methods
$\square$ Pattern Evaluation
$\square$ Summary

## Pattern Discovery: Basic Concepts

$\square$ What Is Pattern Discovery? Why Is It Important?
$\square$ Basic Concepts: Frequent Patterns and Association Rules
$\square$ Compressed Representation: Closed Patterns and Max-Patterns

## What Is Pattern Discovery?

$\square$ Motivation examples:
$\square$ What products were often purchased together?
$\square$ What are the subsequent purchases after buying an iPad?
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$\square$ Patterns represent intrinsic and important properties of datasets
$\square$ Pattern discovery: Uncovering patterns from massive data sets


## Pattern Discovery: Why Is It Important?

$\square$ Finding inherent regularities in a data set
$\square$ Foundation for many essential data mining tasks

- Association, correlation, and causality analysis
$\square$ Mining sequential, structural (e.g., sub-graph) patterns
$\square$ Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
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$\square$ Broad applications
- Market basket analysis, cross-marketing, catalog design, sale campaign analysis, Web log analysis, biological sequence analysis


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- (relative) support, s\{X\}: The fraction of transactions that contains $X$ (i.e., the probability that a transaction contains $X$ )
- Ex. $s\{$ Beer $\}=3 / 5=60 \%$
$\square$ Ex. s\{Diaper\} $=4 / 5=80 \%$
$\square$ Ex. s\{Beer, Eggs $\}=1 / 5=20 \%$


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$\square$ All the frequent 1-itemsets:

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- Beer: 3/5 (60\%); Nuts: 3/5 (60\%)

■ Diaper: 4/5 (80\%); Eggs: 3/5 (60\%)
$\square$ All the frequent 2-itemsets:
$\square$ \{Beer, Diaper\}: 3/5 (60\%)
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- None



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- Do these itemsets (shown on the left) form the complete set of frequent $k$ itemsets (patterns) for any $k$ ?
- Observation: We may need an efficient method to mine a complete set of frequent patterns


## From Frequent Itemsets to Association Rules

$\square$ Comparing with itemsets, rules can be more telling
$\square$ Ex. Diaper $\rightarrow$ Beer

- Buying diapers may likely lead to buying beers


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$\square$ How strong is this rule? (support, confidence)
$\square$ Measuring association rules: $X \rightarrow Y(\mathrm{~s}, \mathrm{c})$

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- Ex. s\{Diaper, Beer\} $=3 / 5=0.6$ (i.e., 60\%)
$\square$ Confidence, $c$ : The conditional probability that a transaction containing $X$ also contains $Y$
- Calculation: $c=\sup (X \cup Y) / \sup (X)$



## Mining Frequent Itemsets and Association Rules

$\square$ Association rule mining
$\square$ Given two thresholds: minsup, minconf
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- Let minsup $=50 \%$
- Freq. 1-itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3

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- Let minconf $=50 \%$
- Beer $\rightarrow$ Diaper ( $60 \%, 100 \%$ )
$\square$ Diaper $\rightarrow$ Beer ( $60 \%, 75 \%$ )


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$\square$ Given two thresholds: minsup, minconf

- Find all of the rules, $X \rightarrow Y(\mathrm{~s}, \mathrm{c})$
- such that, $\mathrm{s} \geq$ minsup and $c \geq$ minconf
- Let minsup $=50 \%$
- Freq. 1 -itemsets: Beer: 3, Nuts: 3, Diaper: 4, Eggs: 3
- Freq. 2-itemsets: \{Beer, Diaper\}: 3
- Let minconf $=50 \%$
$\square \quad$ Beer $\rightarrow$ Diaper ( $60 \%$, 100\%)Diaper $\rightarrow$ Beer (60\%, 75\%)

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ㅁ Observations:

- Mining association rules and mining frequent patterns are very close problems
- Scalable methods are needed for mining large datasets


## Challenge: There Are Too Many Frequent Patterns!

$\square$ A long pattern contains a combinatorial number of sub-patterns
$\square$ How many frequent itemsets does the following TDB $_{1}$ contain?
$\square \operatorname{TDB}_{1:} \quad T_{1}:\left\{a_{1}, \ldots, a_{50}\right\} ; T_{2}:\left\{a_{1}, \ldots, a_{100}\right\}$
$\square$ Assuming (absolute) minsup $=1$

- Let's have a try

1-itemsets: $\left\{\mathrm{a}_{1}\right\}: 2,\left\{\mathrm{a}_{2}\right\}: 2, \ldots,\left\{\mathrm{a}_{50}\right\}: 2,\left\{\mathrm{a}_{51}\right\}: 1, \ldots,\left\{\mathrm{a}_{100}\right\}: 1$,
2-itemsets: $\left\{a_{1}, a_{2}\right\}: 2, \ldots,\left\{a_{1}, a_{50}\right\}: 2,\left\{a_{1}, a_{51}\right\}: 1 \ldots, \ldots,\left\{a_{99}, a_{100}\right\}: 1$,
..., ..., ..., ...
99-itemsets: $\left\{a_{1}, a_{2}, \ldots, a_{99}\right\}: 1, \ldots,\left\{a_{2}, a_{3}, \ldots, a_{100}\right\}: 1$
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- Assuming (absolute) minsup = 1
- Let's have a try

1-itemsets: $\left\{\mathrm{a}_{1}\right\}: 2,\left\{\mathrm{a}_{2}\right\}: 2, \ldots,\left\{\mathrm{a}_{50}\right\}: 2,\left\{\mathrm{a}_{51}\right\}: 1, \ldots,\left\{\mathrm{a}_{100}\right\}: 1$,
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100-itemset: $\left\{a_{1}, a_{2}, \ldots, a_{100}\right\}: 1$
$\square$ The total number of frequent itemsets:

## A too huge set for any one to compute or store!

$$
\binom{100}{1}+\binom{100}{2}+\binom{100}{3}+\cdots+\binom{100}{100}=2^{100}-1
$$

## Expressing Patterns in Compressed Form: Closed Patterns

$\square$ How to handle such a challenge?
$\square$ Solution 1: Closed patterns: A pattern (itemset) $X$ is closed if $X$ is frequent, and there exists no super-pattern $Y$ J $X$, with the same support as $X$

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$\square$ Let Transaction DB TDB ${ }_{1}: T_{1}:\left\{a_{1}, \ldots, a_{50}\right\} ; T_{2}:\left\{a_{1}, \ldots, a_{100}\right\}$
$\square$ Suppose minsup $=1$. How many closed patterns does TDB 1 contain?

- Two: $P_{1}$ : " $\left\{a_{1}, \ldots, a_{50}\right\}: 2 " ; P_{2}$ : "\{ $\left.a_{1}, \ldots, a_{100}\right\}: 1 "$


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$\square$ Closed pattern is a lossless compression of frequent patterns
$\square$ Reduces the \# of patterns but does not lose the support information!
$\square$ You will still be able to say: "\{ $\left.a_{2}, \ldots, a_{40}\right\}: 2 "$ ", "\{ $\left.a_{5}, a_{51}\right\}$ : 1 "


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$\square$ Difference from close-patterns?
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- One: P: "\{a $\left.a_{1}, \ldots, a_{100}\right\}: 1 "$
$\square$ Max-pattern is a lossy compression!
$\square$ We only know $\left\{a_{1}, \ldots, a_{40}\right\}$ is frequent
$\square$ But we do not know the real support of $\left\{a_{1}, \ldots, a_{40}\right\}, \ldots$, any more!
$\square$ Thus in many applications, close-patterns are more desirable than max-patterns


# Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods 

$\square$ Basic ConceptsEfficient Pattern Mining Methods
$\square$ The Apriori Algorithm

- Application in Classification
$\square$ Pattern Evaluation
$\square$ Summary


## Efficient Pattern Mining Methods

$\square$ The Downward Closure Property of Frequent Patterns
$\square$ The Apriori Algorithm
$\square$ Extensions or Improvements of AprioriMining Frequent Patterns by Exploring Vertical Data Format
$\square$ FPGrowth: A Frequent Pattern-Growth Approach
$\square$ Mining Closed Patterns

## The Downward Closure Property of Frequent Patterns

- Observation: From TDB T $_{1}: T_{1}:\left\{a_{1}, \ldots, a_{50}\right\} ; T_{2}:\left\{a_{1}, \ldots, a_{100}\right\}$
- We get a frequent itemset: $\left\{a_{1}, \ldots, a_{50}\right\}$
- Also, its subsets are all frequent: $\left\{a_{1}\right\},\left\{a_{2}\right\}, \ldots,\left\{a_{50}\right\},\left\{a_{1}, a_{2}\right\}, \ldots,\left\{a_{1}, \ldots\right.$, $\left.a_{49}\right\}, \ldots$
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- There must be some hidden relationships among frequent patterns!
$\square$ The downward closure (also called "Apriori") property of frequent patterns
- If \{beer, diaper, nuts\} is frequent, so is \{beer, diaper\}
- Every transaction containing \{beer, diaper, nuts\} also contains \{beer, diaper\}
- Apriori: Any subset of a frequent itemset must be frequent $\mathbb{\Sigma}$

A sharp knife for pruning!

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$\square$ Efficient mining methodology A sharp knife for pruning!
- If any subset of an itemset $S$ is infrequent, then there is no chance for $S$ to be frequent-why do we even have to consider S!?


## Apriori Pruning and Scalable Mining Methods

$\square$ Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated!

- (Agrawal \& Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
$\square$ Scalable mining Methods: Three major approaches
$\square$ Level-wise, join-based approach:
- Apriori (Agrawal \& Srikant@VLDB'94)
$\square$ Vertical data format approach:
- Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
$\square$ Frequent pattern projection and growth:
- FPgrowth (Han, Pei, Yin @SIGMOD’00)


## Apriori: A Candidate Generation \& Test Approach

$\square$ Outline of Apriori (level-wise, candidate generation and test)
$\square$ Initially, scan DB once to get frequent 1-itemset

- Repeat
- Generate length- $(k+1)$ candidate itemsets from length-k frequent itemsets
- Test the candidates against DB to find frequent ( $k+1$ )-itemsets
- Set $\mathrm{k}:=\mathrm{k}+1$
- Until no frequent or candidate set can be generated
$\square$ Return all the frequent itemsets derived


## The Apriori Algorithm (Pseudo-Code)

$\mathrm{C}_{\mathrm{k}}$ : Candidate itemset of size k
$F_{k}$ : Frequent itemset of size $k$
$\mathrm{K}:=1$;

While $\left(F_{k}!=\varnothing\right.$ ) do \{ $/ /$ when $F_{k}$ is non-empty
$C_{k+1}:=$ candidates generated from $F_{k i} / /$ candidate generation Derive $F_{k+1}$ by counting candidates in $\mathrm{C}_{k+1}$ with respect to TDB at minsup; $\mathrm{k}:=\mathrm{k}+1$
\}
return $\cup_{k} F_{k} \quad / /$ return $F_{k}$ generated at each level

## The Apriori Algorithm—An Example



## The Apriori Algorithm—An Example

| Database TDB |  | minsup $=2$ | $\begin{gathered} \hline \text { Itemset } \\ \hline\{\mathrm{A}\} \end{gathered}$ | $\frac{\sup }{2}$ |  |  | sup |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $F_{1}$ |  |  | Itemset |  |
| Tid | Items |  | $1^{\text {st }} \operatorname{scan}$ | \{B\} | 3 | \{A\} | 2 |
| 10 | A, C, D |  |  | \{C\} | 3 | \{B\} | 3 |
| 20 | B, C, E | \{D |  | 1 |  | \{C\} | 3 |
| 30 | A, B, C, E | \{E\} |  | 3 |  | \{E\} | 3 |
| 40 | B, E |  |  |  |  |  |  |


$F_{2}$| Itemset | sup |
| :---: | :---: |
|  | $\{\mathrm{A}, \mathrm{C}\}$ |
| $\{\mathrm{B}, \mathrm{C}\}$ | 2 |
|  | $2 \mathrm{~B}, \mathrm{E}\}$ |
|  | 3 |
| $\{\mathrm{C}, \mathrm{E}\}$ | 2 |


| $C_{2}$ | Itemset | sup | $2^{\text {nd }} \operatorname{scan} C_{2}$ | Itemset |
| :---: | :---: | :---: | :---: | :---: |
|  | $\{\mathrm{A}, \mathrm{B}\}$ | 1 |  | \{A, B $\}$ |
|  | $\{\mathrm{A}, \mathrm{C}\}$ | 2 |  | \{A, C $\}$ |
|  | $\{\mathrm{A}, \mathrm{E}\}$ | 1 |  | $\{\mathrm{A}, \mathrm{E}\}$ |
|  | \{B, C $\}$ | 2 |  | $\{\mathrm{B}, \mathrm{C}\}$ |
|  | \{B, E\} | 3 |  | $\{\mathrm{B}, \mathrm{E}\}$ |
|  |  |  |  | \{C, E\} |

> |  | $C_{3}$ |
| :---: | :---: |
|  | Itemset |
|  | $\{\mathrm{B}, \mathrm{C}, \mathrm{E}\}$ |
|  | $3^{\text {rd }} \mathrm{scan}$ |
|  | $F_{3}$Itemset sup <br>  $\{\mathrm{B}, \mathrm{C}, \mathrm{E}\}$ |

## Apriori: Implementation Tricks

$\square$ How to generate candidates?
$\square$ Step 1: self-joining $F_{k}$
$\square$ Step 2: pruning

## Apriori: Implementation Tricks

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self-join
$\square$ Step 1: self-joining $F_{k}$
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$\square$ Example of candidate-generation
$\square F_{3}=\{a b c, a b d, a c d, a c e, b c d\}$
$\square$ Self-joining: $F_{3}{ }^{*} F_{3}$
■ abcd from abc and abd
$\square$ acde from acd and ace

## Apriori: Implementation Tricks

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$\square$ Self-joining: $F_{3}{ }^{*} F_{3}$

- abcd from abc and abd

- acde from acd and ace
$\square$ Pruning:
$\square$ acde is removed because ade is not in $F_{3}$
$\square C_{4}=\{a b c d\}$


## Candidate Generation: An SQL Implementation

$\square$ Suppose the items in $F_{k-1}$ are listed in an order
$\square$ Step 1: self-joining $F_{k-1}$ insert into $C_{k}$
select p.item ${ }_{1}$, p.item $_{2}, \ldots$, p.item $_{k-1}$, q.item $_{k-1}$ from $F_{k-1}$ as $p, F_{k-1}$ as $q$

${\text { where } \text { p.item }_{I}=\text { q.item }}_{1^{\prime}} \ldots$, p.item $_{k-2}=$ q.item ${ }_{k-2}$ p.item $_{k-1}<$ q.item ${ }_{k-1}$
$\square$ Step 2: pruning
for all itemsets $\mathbf{c}$ in $\mathbf{C}_{k}$ do
for all ( $k-1$ )-subsets $s$ of $c$ do if ( $s$ is not in $F_{k-1}$ ) then delete $c$ from $C_{k}$

## Apriori Adv/Disadv

$\square$ Advantages:
$\square$ Uses large itemset property
$\square$ Easily parallelized
$\square$ Easy to implement
$\square$ Disadvantages:
$\square$ Assumes transaction database is memory resident
$\square$ Requires up to $m$ database scans

## Classification based on Association Rules (CBA)

$\square$ Why?
$\square$ Can effectively uncover the correlation structure in data
$\square$ AR are typically quite scalable in practice

- Rules are often very intuitive
- Hence classifier built on intuitive rules is easier to interpret
$\square$ When to use?
$\square$ On large dynamic datasets where class labels are available and the correlation structure is unknown.
- Multi-class categorization problems
$\square$ E.g. Web/Text Categorization, Network Intrusion Detection


# Mining Frequent Patterns, Association and Correlations: 

 Basic Concepts and Methods$\square$ Basic Concepts
$\square$ Efficient Pattern Mining Methods
$\square$ Pattern Evaluation
$\square$ Summary

## Summary

## Basic Concepts

- What Is Pattern Discovery? Why Is It Important?
- Basic Concepts: Frequent Patterns and Association Rules
- Compressed Representation: Closed Patterns and Max-PatternsEfficient Pattern Mining Methods
- The Downward Closure Property of Frequent Patterns
- The Apriori Algorithm
- Extensions or Improvements of Apriori
- Mining Frequent Patterns by Exploring Vertical Data Format
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Closed Patterns
- Pattern Evaluation
- Interestingness Measures in Pattern Mining
- Interestingness Measures: Lift and $\chi^{2}$
- Null-Invariant Measures
- Comparison of Interestingness Measures


## Recommended Readings (Basic Concepts)

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$\square$ J. Han, H. Cheng, D. Xin, and X. Yan, "Frequent Pattern Mining: Current Status and Future Directions", Data Mining and Knowledge Discovery, 15(1): 55-86, 2007

## Recommended Readings <br> (Efficient Pattern Mining Methods)

$\square$ R. Agrawal and R. Srikant, "Fast algorithms for mining association rules", VLDB'94
$\square$ A. Savasere, E. Omiecinski, and S. Navathe, "An efficient algorithm for mining association rules in large databases", VLDB'95
$\square$ J. S. Park, M. S. Chen, and P. S. Yu, "An effective hash-based algorithm for mining association rules", SIGMOD'95
$\square$ S. Sarawagi, S. Thomas, and R. Agrawal, "Integrating association rule mining with relational database systems: Alternatives and implications", SIGMOD'98
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$\square$ M. J. Zaki and Hsiao, "CHARM: An Efficient Algorithm for Closed Itemset Mining", SDM'02
$\square$ J. Wang, J. Han, and J. Pei, "CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets", KDD'03
$\square$ C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, "Frequent Pattern Mining Algorithms: A Survey", in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014

## Recommended Readings (Pattern Evaluation)

$\square$ C. C. Aggarwal and P. S. Yu. A New Framework for Itemset Generation. PODS'98
$\square$ S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97
$\square$ M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94
$\square \quad$ E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03
$\square$ P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02
$\square$ T. Wu, Y. Chen and J. Han, Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework, Data Mining and Knowledge Discovery, 21(3):371397, 2010

