


CSE 5243 INTRO. TO DATA MINING

Advanced Frequent Pattern Mining

(Chapter 7)

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Chapter 7 : Advanced Frequent Pattern Mining

- Mining Diverse Patterns 
- Constraint-Based Frequent Pattern Mining
- Sequential Pattern Mining
- Graph Pattern Mining
- Pattern Mining Application: Mining Software Copy-and-Paste Bugs
- Summary

Mining Diverse Patterns

- Mining Multiple-Level Associations
- Mining Multi-Dimensional Associations
- Mining Negative Correlations
- Mining Compressed and Redundancy-Aware Patterns

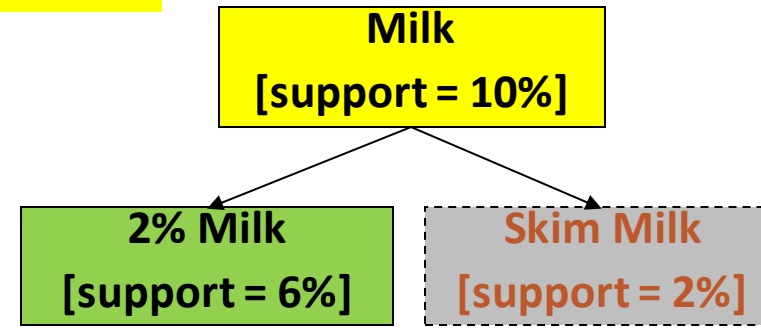
Mining Multiple-Level Frequent Patterns

- Items often form hierarchies
 - Ex.: Dairyland 2% milk; Wonder wheat bread
- How to set min-support thresholds?

Uniform support

Level 1
min_sup = 5%

Level 2
min_sup = 5%



- Uniform min-support across multiple levels (reasonable?)

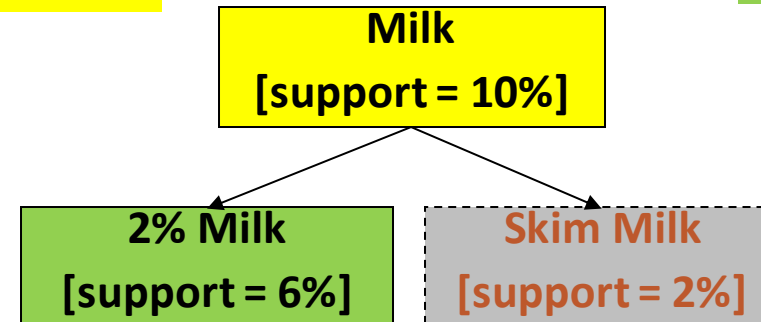
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Reduced support

Level 1
min_sup = 5%

Level 2
min_sup = 1%

- Uniform min-support across multiple levels (reasonable?)
- **Level-reduced min-support: Items at the lower level are expected to have lower support**

ML/MD Associations with Flexible Support Constraints

- Why flexible support constraints?
 - ▣ Real life occurrence frequencies vary greatly
 - Diamond, watch, pens in a shopping basket
 - ▣ Uniform support may not be an interesting model
- A flexible model
 - ▣ The lower-level, the more dimension combination, and the long pattern length, usually the smaller support
 - ▣ General rules should be easy to specify and understand
 - ▣ Special items and special group of items may be specified individually and have higher priority

Multi-level Association: Redundancy Filtering

- Some rules may be redundant due to “ancestor” relationships between items.
- Example
 - ▣ milk \Rightarrow wheat bread [support = 8%, confidence = 70%]
 - ▣ 2% milk \Rightarrow wheat bread [support = 2%, confidence = 72%]
 - ▣ Suppose the 2% milk sold is about $\frac{1}{4}$ of milk sold
- We say the first rule is an ancestor of the second rule.
- A rule is redundant if its support is close to the “expected” value, based on the rule’s ancestor.

Multi-Level Mining: Progressive Deepening

- A top-down, progressive deepening approach:
 - First mine high-level frequent items:
milk (15%), bread (10%)
 - Then mine their lower-level “weaker” frequent itemsets:
2% milk (5%), wheat bread (4%)
- Different min_support threshold across multi-levels lead to different algorithms:

Mining Multi-Dimensional Associations

- Single-dimensional rules (e.g., items are all in “product” dimension)
 - ▣ $\text{buys}(X, \text{“milk”}) \Rightarrow \text{buys}(X, \text{“bread”})$
- Multi-dimensional rules (i.e., items in ≥ 2 dimensions or predicates)
 - ▣ Inter-dimension association rules (*no repeated predicates*)
 - $\text{age}(X, \text{“18-25”}) \wedge \text{occupation}(X, \text{“student”}) \Rightarrow \text{buys}(X, \text{“coke”})$
 - ▣ Hybrid-dimension association rules (*repeated predicates*)
 - $\text{age}(X, \text{“18-25”}) \wedge \text{buys}(X, \text{“popcorn”}) \Rightarrow \text{buys}(X, \text{“coke”})$

Mining Rare Patterns vs. Negative Patterns

- Rare patterns
 - ▣ Very low support but interesting (e.g., buying Rolex watches)
 - ▣ How to mine them? Setting individualized, group-based min-support thresholds for different groups of items

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- Rare patterns
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- Negative patterns
 - ▣ **Negatively correlated: Unlikely to happen together**
 - ▣ Ex.: Since it is unlikely that the same customer buys both a **Ford Expedition** (an SUV car) and a **Ford Fusion** (a hybrid car), buying a **Ford Expedition** and buying a **Ford Fusion** are likely negatively correlated patterns
 - ▣ How to define negative patterns?

Defining Negative Correlated Patterns

- A (relative) support-based definition
 - If itemsets A and B are both frequent but rarely occur together, i.e., $\text{sup}(A \cup B) \ll \text{sup}(A) \times \text{sup}(B)$
 - Then A and B are negatively correlated

Defining Negative Correlated Patterns

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Does this remind you the definition of *lift*?

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- Is this a good definition for large transaction datasets?
- Ex.: Suppose a store sold two needle packages A and B 100 times each, but only one transaction contained both A and B
 - ▣ When there are in total 200 transactions, we have
 - $s(A \cup B) = 0.005, s(A) \times s(B) = 0.25, s(A \cup B) \ll s(A) \times s(B)$
 - ▣ But when there are 10^5 transactions, we have
 - $s(A \cup B) = 1/10^5, s(A) \times s(B) = 1/10^3 \times 1/10^3, s(A \cup B) > s(A) \times s(B)$

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 - ▣ What is the problem?—Null transactions: The support-based definition is not null-invariant!

Does this remind you the definition of *lift*?

Defining Negative Correlation: Need Null-Invariance in Definition


- A good definition on negative correlation should take care of the null-invariance problem
 - Whether two itemsets A and B are negatively correlated should not be influenced by the number of null-transactions

Which measure should we use? Recall last lectures....

Defining Negative Correlation: Need Null-Invariance in Definition

- A good definition on negative correlation should take care of the null-invariance problem
 - Whether two itemsets A and B are negatively correlated should not be influenced by the number of null-transactions
- A **Kulczynski measure-based** definition
 - If itemsets A and B are frequent but
$$(s(A \cup B)/s(A) + s(A \cup B)/s(B))/2 < \epsilon,$$
where ϵ is a negative pattern threshold, then A and B are negatively correlated
- For the same needle package problem:
 - No matter there are in total 200 or 10^5 transactions
 - If $\epsilon = 0.02$, we have
$$(s(A \cup B)/s(A) + s(A \cup B)/s(B))/2 = (0.01 + 0.01)/2 < \epsilon$$

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- Finding **all** the patterns in a database **autonomously?** — unrealistic!
 - ▣ The patterns could be too many but not focused!

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 - ▣ User directs what to be mined using a **data mining query language** (or a graphical user interface)

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 - ▣ The patterns could be too many but not focused!
- Data mining should be an **interactive** process
 - ▣ User directs what to be mined using a **data mining query language** (or a graphical user interface)
- Constraint-based mining
 - ▣ User flexibility: provides **constraints** on what to be mined
 - ▣ System optimization: explores such constraints for efficient mining—**constraint-based mining**

Categories of Constraints

CONSTRAINT 1 (ITEM CONSTRAINT). *An item constraint specifies what are the particular individual or groups of items that should or should not be present in the pattern.* □

For example, a dairy company may be interested in patterns containing only dairy products, when it mines transactions in a grocery store.

CONSTRAINT 2 (LENGTH CONSTRAINT). *A length constraint specifies the requirement on the length of the patterns, i.e., the number of items in the patterns.* □

For example, when mining classification rules for documents, a user may be interested in only frequent patterns with at least 5 keywords, a typical length constraint.

Categories of Constraints

CONSTRAINT 3 (MODEL-BASED CONSTRAINT). *A model-based constraint looks for patterns which are sub- or super-patterns of some given patterns (models).* □

For example, a travel agent may be interested in what other cities that a visitor is likely to travel if s/he visits both Washington and New York city. That is, they want to find frequent patterns which are super-patterns of {Washington, New York city}.

CONSTRAINT 4 (AGGREGATE CONSTRAINT). *An aggregate constraint is on an aggregate of items in a pattern, where the aggregate function can be SUM, AVG, MAX, MIN, etc.* □

For example, a marketing analyst may like to find frequent patterns where the average price of all items in each pattern is over \$100.

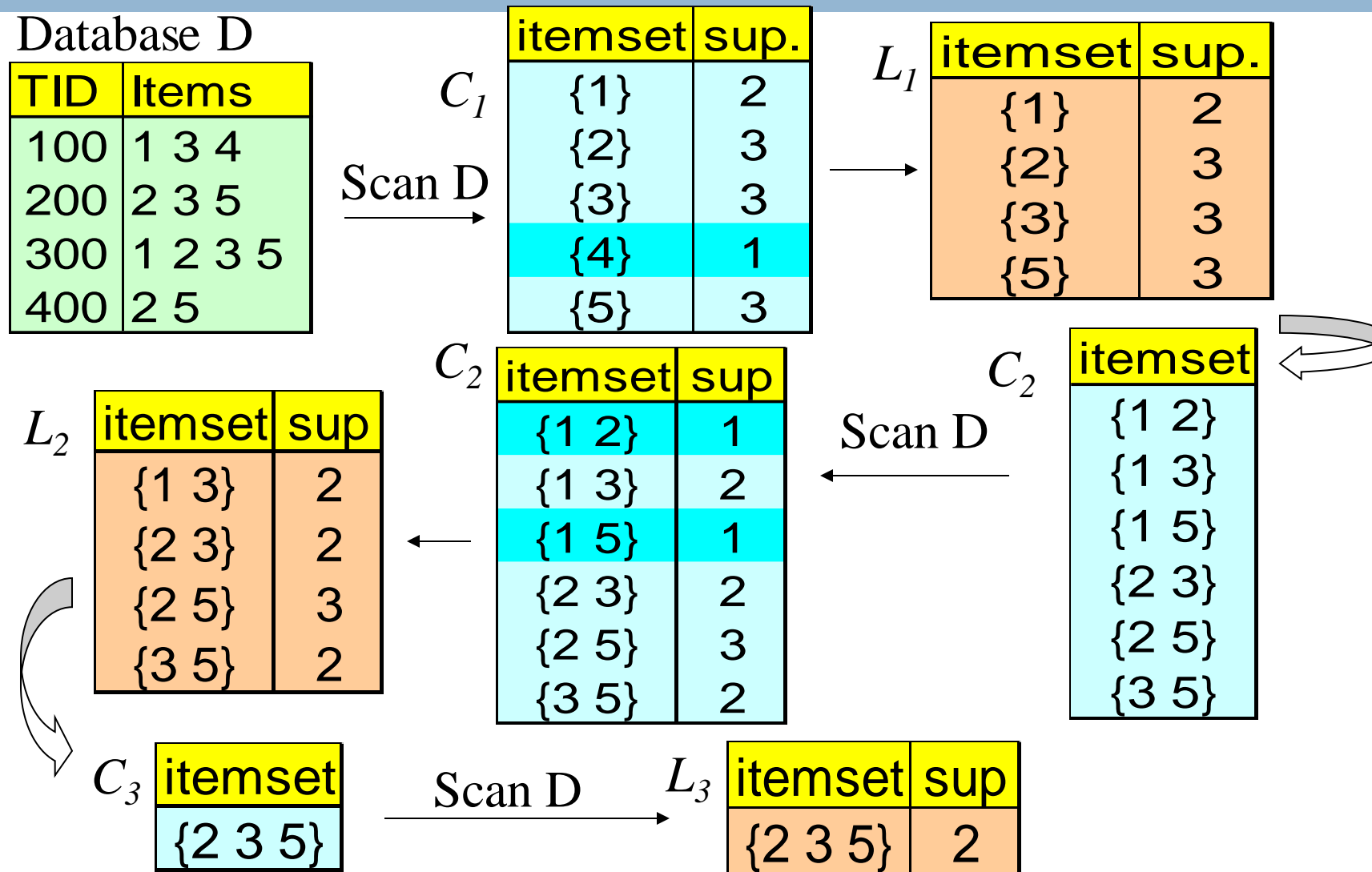
Constrained Frequent Pattern Mining: A Mining Query Optimization Problem

- Given a frequent pattern mining query with a set of constraints C , the algorithm should be
 - ▣ **sound**: it only finds frequent sets that satisfy the given constraints C
 - ▣ **complete**: all frequent sets satisfying the given constraints C are found

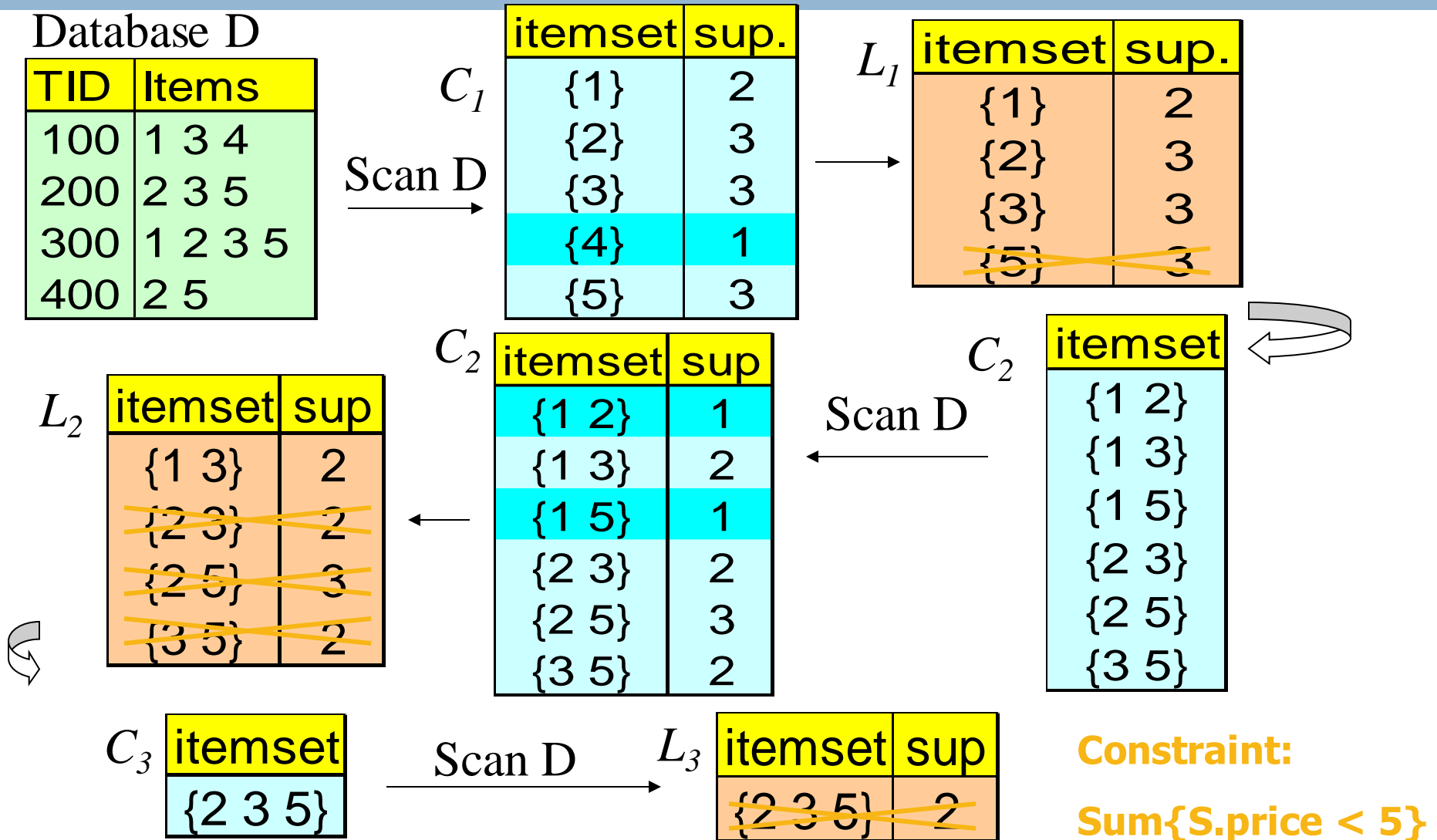
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- A naive solution
 - ▣ First find all frequent sets, and **then** test them for constraint satisfaction

The Apriori Algorithm — Example



Naïve Algorithm: Apriori + Constraint (Naïve Solution)



Constrained Frequent Pattern Mining: A Mining Query Optimization Problem

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 - **sound**: it only finds frequent sets that satisfy the given constraints C
 - **complete**: all frequent sets satisfying the given constraints C are found
- A naive solution
 - First find all frequent sets, and **then** test them for constraint satisfaction
- More efficient approaches:
 - Analyze the properties of **constraints** comprehensively
 - **Push them as deeply as possible inside** the frequent pattern computation.

Anti-Monotonicity in Constraint-Based Mining

- Anti-monotonicity
 - *When an itemset S **violates** the constraint, so does any of its superset*
 - $sum(S.Price) \leq v$ is **anti-monotone?**
 - $sum(S.Price) \geq v$ is **anti-monotone?**

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Anti-Monotonicity in Constraint-Based Mining

TDB (min_sup=2)

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 - $sum(S.Price) \leq v$ is **anti-monotone**
 - $sum(S.Price) \geq v$ is **not anti-monotone**
- Example. C: $range(S.profit) \leq 15$ is **anti-monotone**
 - Itemset ab violates C
 - So does every superset of ab
 - Define $range(S.profit) = max(S.A) - min(S.A)$

TID	Transaction
10	a, b, c, d, f
20	b, c, d, f, g, h
30	a, c, d, e, f
40	c, e, f, g

Item	Profit
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Which Constraints Are Anti-Monotone?

Constraint	Antimonotone
$v \in S$	No
$S \supseteq V$	no
$S \subseteq V$	yes
$\min(S) \leq v$	no
$\min(S) \geq v$	yes
$\max(S) \leq v$	yes
$\max(S) \geq v$	no
$\text{count}(S) \leq v$	yes
$\text{count}(S) \geq v$	no
$\text{sum}(S) \leq v (a \in S, a \geq 0)$	yes
$\text{sum}(S) \geq v (a \in S, a \geq 0)$	no
$\text{range}(S) \leq v$	yes
$\text{range}(S) \geq v$	no
$\text{avg}(S) \theta v, \theta \in \{=, \leq, \geq\}$	convertible
$\text{support}(S) \geq \xi$	yes
$\text{support}(S) \leq \xi$	no

Monotonicity in Constraint-Based Mining

- Monotonicity
 - When an itemset S **satisfies** the constraint, so does any of its superset
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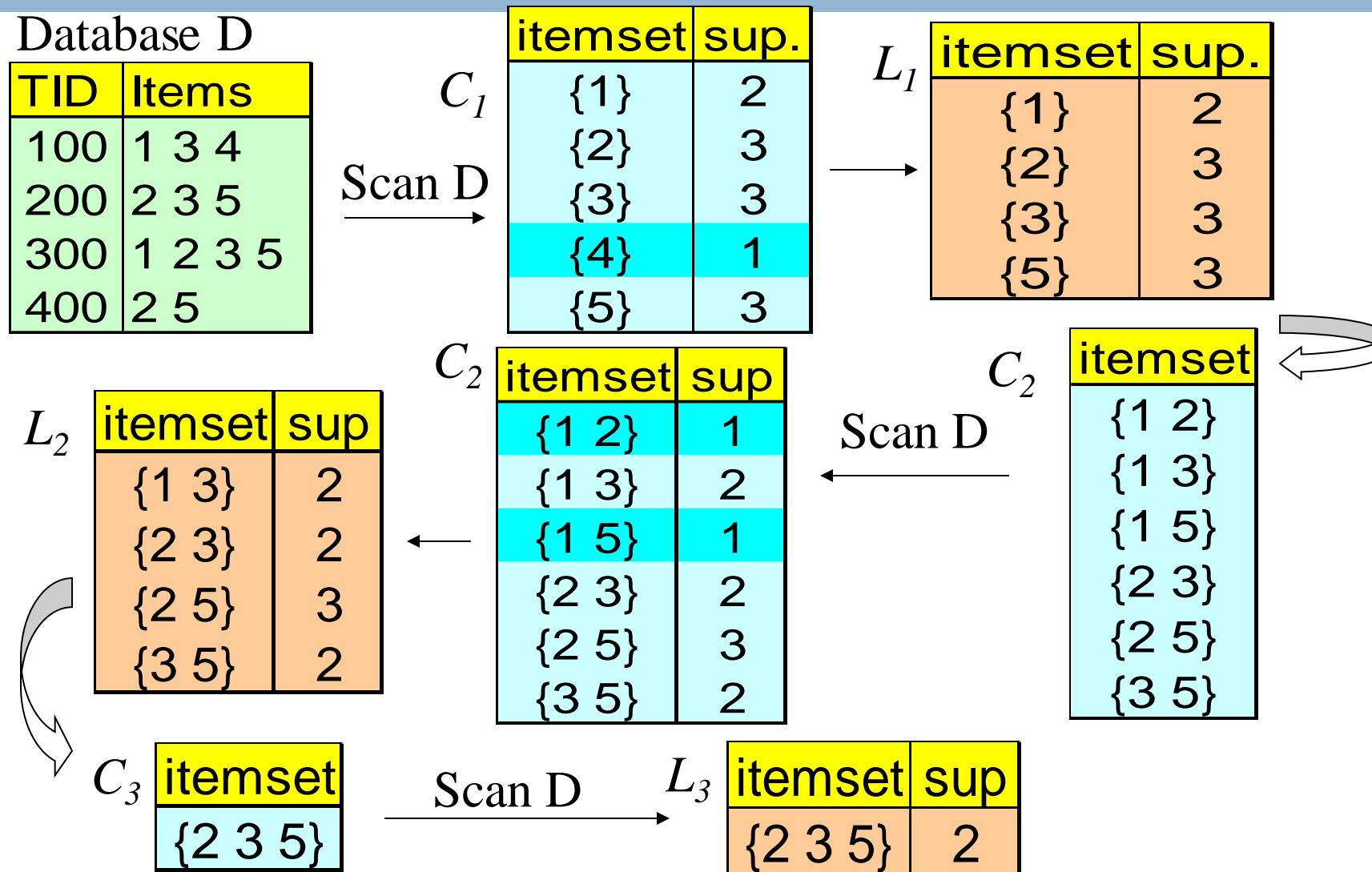
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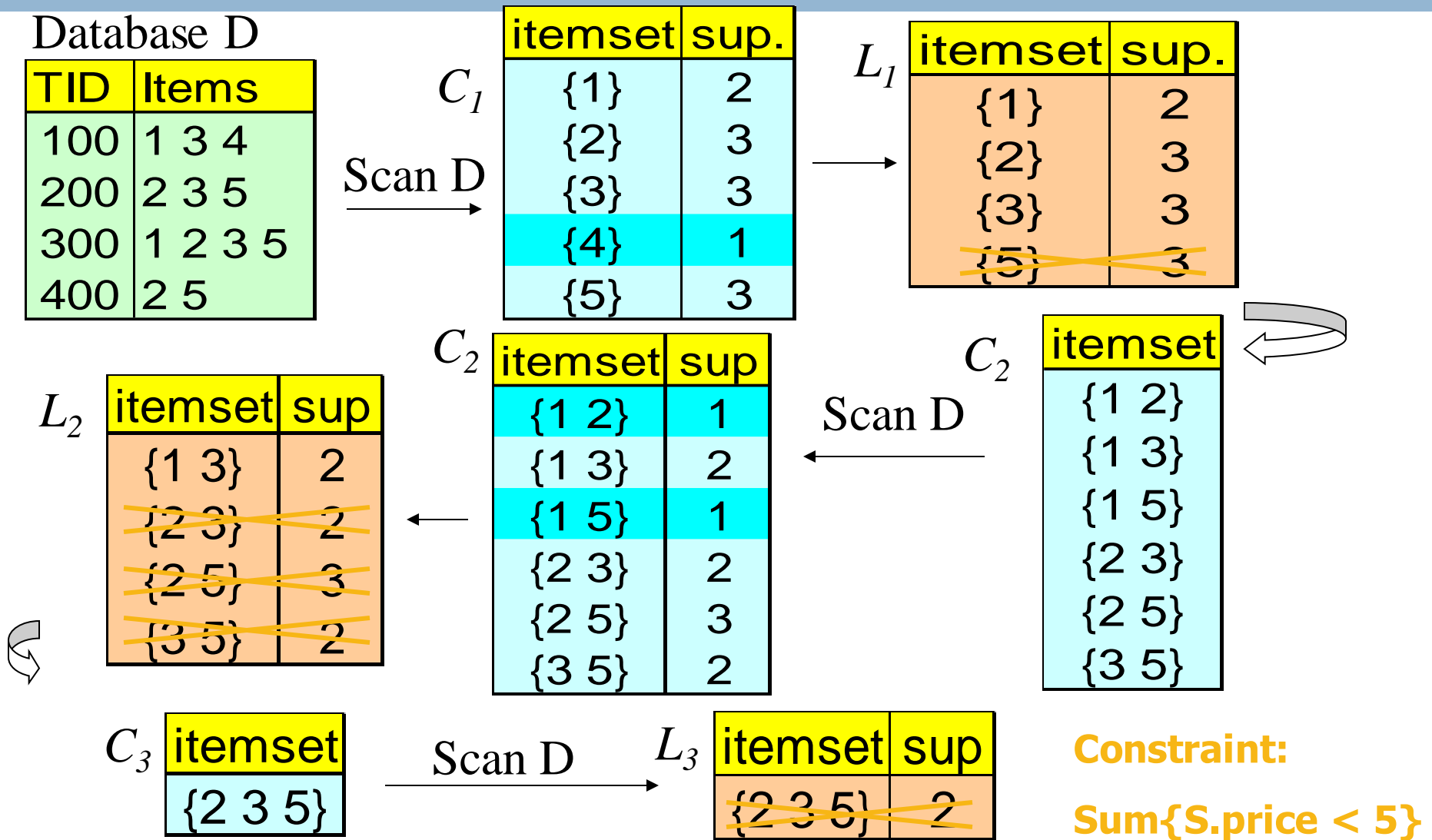
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$\text{count}(S) \geq v$	yes
$\text{sum}(S) \leq v (a \in S, a \geq 0)$	no
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$\text{avg}(S) \theta v, \theta \in \{=, \leq, \geq\}$	convertible
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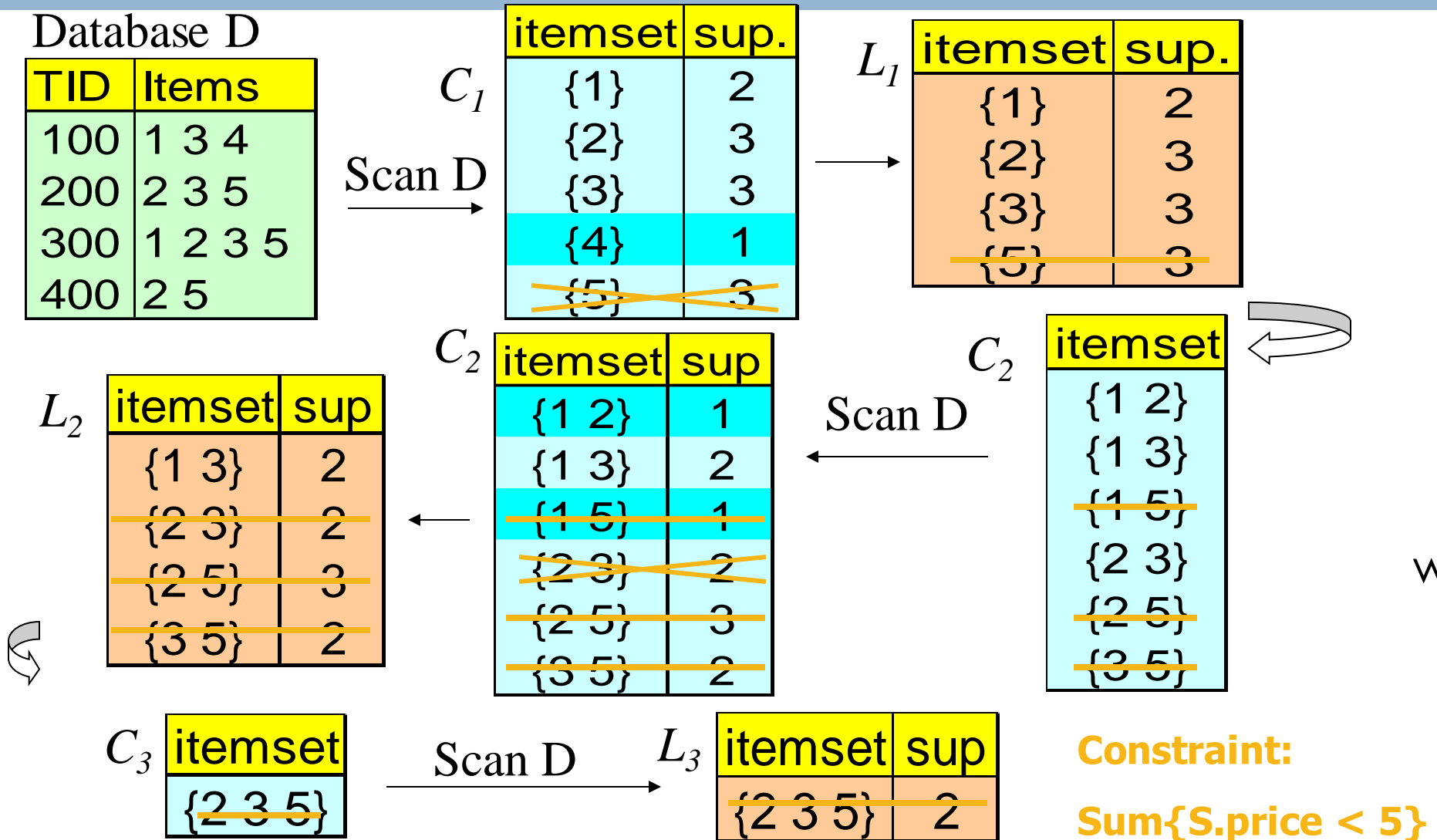
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Naïve Algorithm: Apriori + Constraint



Pushing the constraint deep into the process



Converting “Tough” Constraints

- Convert tough constraints into anti-monotone or monotone by properly ordering items

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- Convert tough constraints into anti-monotone or monotone by properly ordering items
- Examine C: $\text{avg}(S.\text{profit}) \geq 25$
 - ▣ Order items in value-descending order
 - $\langle a, f, g, d, b, h, c, e \rangle$
 - ▣ If an itemset afb violates C
 - So does $afbh, afb^*$
 - It becomes **anti-monotone!**

Converting “Tough” Constraints

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TDB (min_sup=2)

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c	-20
d	10
e	-30
f	30
g	20
h	-10

Convertible Constraints

- Let R be an order of items
- Convertible anti-monotone
 - If an itemset S violates a constraint C , so does every itemset having S as a prefix w.r.t. R
 - Ex. $\text{avg}(S) \leq v$ w.r.t. item value ascending order

Why?

Convertible Constraints

- Let R be an order of items
- Convertible anti-monotone
 - ▣ If an itemset S violates a constraint C , so does every itemset having S as a prefix w.r.t. R
 - ▣ Ex. $\text{avg}(S) \leq v$ w.r.t. item value ascending order
- Convertible monotone
 - ▣ If an itemset S satisfies constraint C , so does every itemset having S as a prefix w.r.t. R
 - ▣ Ex. $\text{avg}(S) \geq v$ w.r.t. item value ascending order

Strongly Convertible Constraints

- $\text{avg}(X) \geq 25$ is convertible anti-monotone w.r.t. item **value descending** order $R: \langle a, f, g, d, b, h, c, e \rangle$
 - If an itemset af violates a constraint C , so does every itemset with af as prefix, such as afd
- $\text{avg}(X) \geq 25$ is convertible monotone w.r.t. item **value ascending** order $R^{-1}: \langle e, c, h, b, d, g, f, a \rangle$
 - If an itemset d satisfies a constraint C , so does itemsets df and dfa , which having d as a prefix
- Thus, $\text{avg}(X) \geq 25$ is **strongly convertible**

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d	10
e	-30
f	30
g	20
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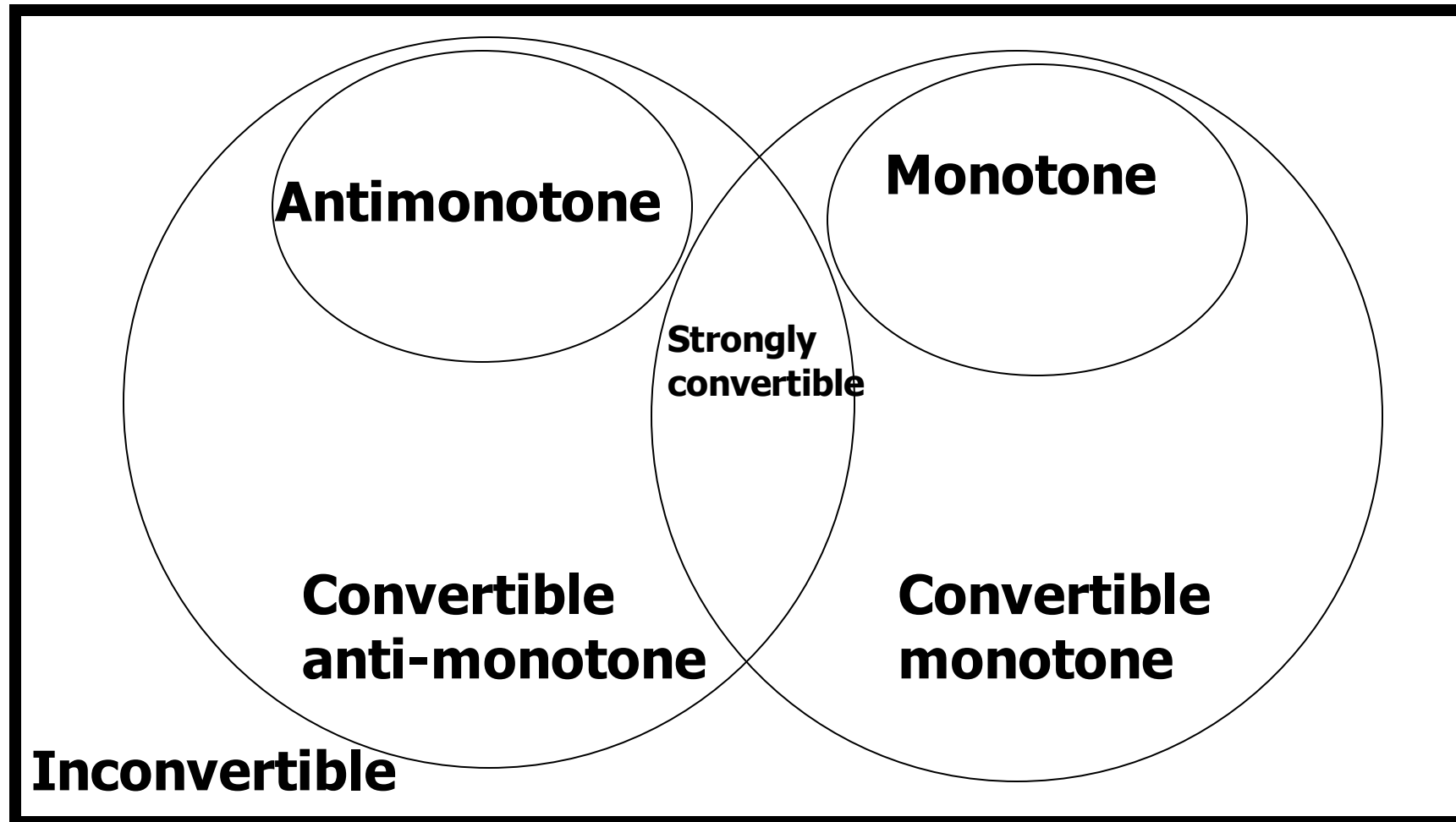
What Constraints Are Convertible?

Constraint	Convertible anti-monotone	Convertible monotone	Strongly convertible
$\text{avg}(S) \leq, \geq v$	Yes	Yes	Yes
$\text{median}(S) \leq, \geq v$	Yes	Yes	Yes
$\text{sum}(S) \leq v$ (items could be of any value, $v \geq 0$)	Yes	No	No
$\text{sum}(S) \leq v$ (items could be of any value, $v \leq 0$)	No	Yes	No
$\text{sum}(S) \geq v$ (items could be of any value, $v \geq 0$)	No	Yes	No
$\text{sum}(S) \geq v$ (items could be of any value, $v \leq 0$)	Yes	No	No
.....			

Combing Them Together—A General Picture

Constraint	Antimonotone	Monotone
$v \in S$	no	yes
$S \supseteq V$	no	yes
$S \subseteq V$	yes	no
$\min(S) \leq v$	no	yes
$\min(S) \geq v$	yes	no
$\max(S) \leq v$	yes	no
$\max(S) \geq v$	no	yes
$\text{count}(S) \leq v$	yes	no
$\text{count}(S) \geq v$	no	yes
$\text{sum}(S) \leq v (a \in S, a \geq 0)$	yes	no
$\text{sum}(S) \geq v (a \in S, a \geq 0)$	no	yes
$\text{range}(S) \leq v$	yes	no
$\text{range}(S) \geq v$	no	yes
$\text{avg}(S) \theta v, \theta \in \{=, \leq, \geq\}$	convertible	convertible
$\text{support}(S) \geq \xi$	yes	no
$\text{support}(S) \leq \xi$	no	yes

Classification of Constraints



Mining With Convertible Constraints

- C: $\text{avg}(S.\text{profit}) \geq 25$

- Scan transaction DB once
 - remove infrequent items
 - Item *h* in transaction 40 is dropped

 - Itemsets *a* and *f* are good

TDB (min_sup=2)

TID	Transaction
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Can Apriori Handle Convertible Constraint?

- A convertible, not monotone nor anti-monotone cannot be pushed deep into the an Apriori mining algorithm
 - Within the level wise framework, no direct pruning based on the constraint can be made
 - Itemset df violates constraint $C: \text{avg}(X) \geq 25$
 - **Can we prune df afterwards?**

Item	Value
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Can Apriori Handle Convertible Constraint?

- A convertible, not monotone nor anti-monotone cannot be pushed deep into the an Apriori mining algorithm
 - Within the level wise framework, no direct pruning based on the constraint can be made
 - Itemset df violates constraint C: $\text{avg}(X) \geq 25$
 - **Since adf satisfies C, Apriori needs df to assemble adf, df cannot be pruned**
- But it can be pushed into frequent-pattern growth framework!

Item	Value
a	40
b	0
c	-20
d	10
e	-30
f	30
g	20
h	-10

Mining With Convertible Constraints in FP-Growth Framework

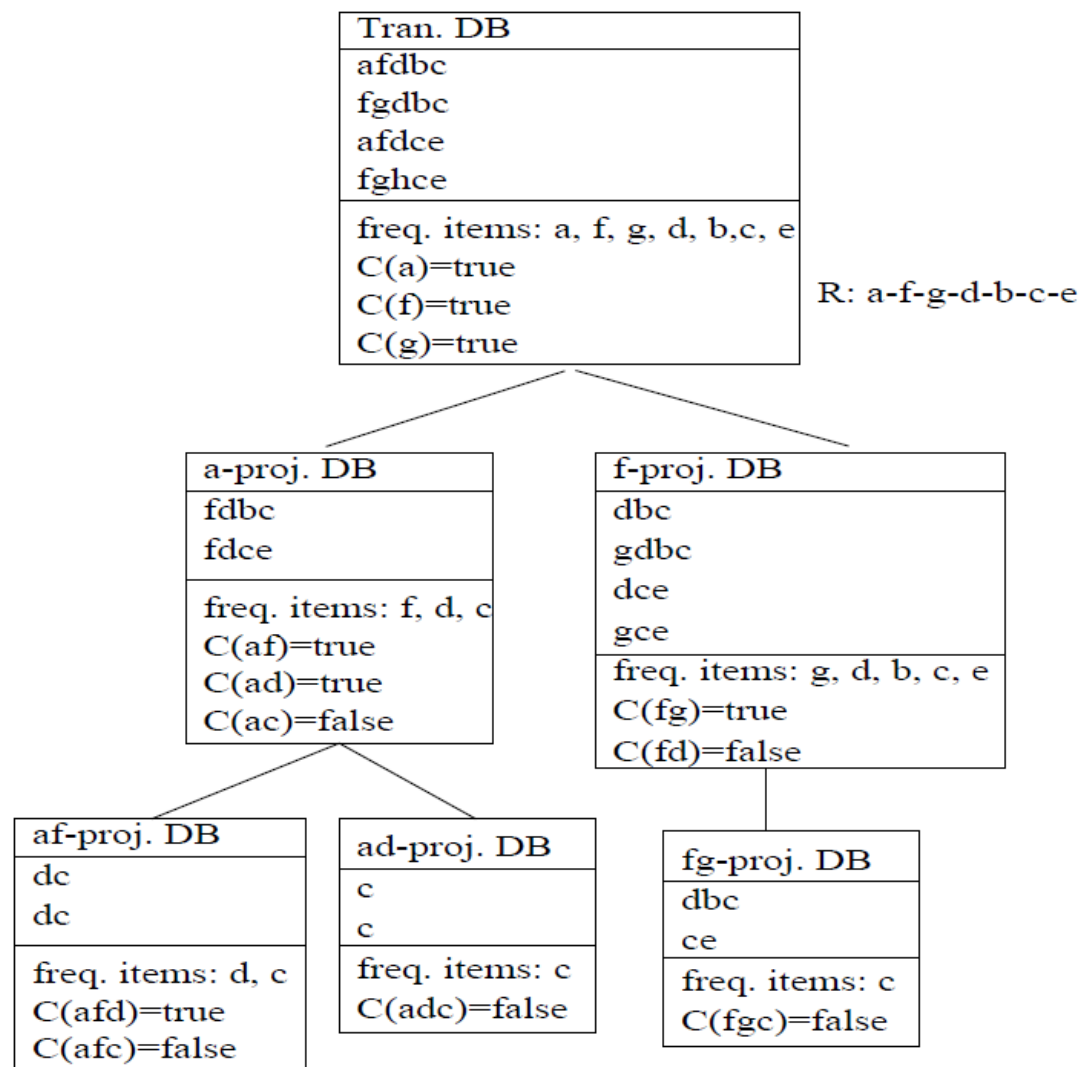
- C: $\text{avg}(X) \geq 25$, $\text{min_sup} = 2$
- List items in every transaction in value descending order R: $\langle a, f, g, d, b, h, c, e \rangle$
 - ▣ C is convertible anti-monotone w.r.t. R
- Scan TDB once
 - ▣ remove infrequent items
 - Item h is dropped
 - ▣ Itemsets a and f are good, ...
- Projection-based mining
 - ▣ Imposing an appropriate order on item projection
 - ▣ Many tough constraints can be converted into (anti)-monotone

Item	Value
a	40
f	30
g	20
d	10
b	0
h	-10
c	-20
e	-30

TDB ($\text{min_sup} = 2$)

TID	Transaction
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Mining With Convertible Constraints in FP-Growth Framework



Constrained Frequent Pattern Mining: A Pattern-Growth View


Jian Pei, Jiawei Han, SIGKDD 2002

Figure 1: Mining frequent itemsets satisfying constraint $avg(S) \geq 25$.

Handling Multiple Constraints

- Different constraints may require different or even conflicting item-ordering
- If there exists an order R s.t. both C_1 and C_2 are convertible w.r.t. R , then there is no conflict between the two convertible constraints
- If there exists conflict on order of items
 - ▣ Try to satisfy one constraint first
 - ▣ Then using the order for the other constraint to mine frequent itemsets in the corresponding projected database

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Sequence Databases & Sequential Patterns

- Sequential pattern mining has broad applications
 - Customer shopping sequences
 - Purchase a laptop first, then a digital camera, and then a smartphone, within 6 months
 - Medical treatments, natural disasters (e.g., earthquakes), science & engineering processes, stocks and markets, ...
 - Weblog click streams, calling patterns, ...
 - Software engineering: Program execution sequences, ...
 - Biological sequences: DNA, protein, ...
- Transaction DB, sequence DB vs. time-series DB
- Gapped vs. non-gapped sequential patterns
 - Shopping sequences, clicking streams vs. biological sequences

Sequence Mining: Description

□ Input

▣ A database ***D*** of sequences called *data-sequences*, in which:

▣ $I = \{i_1, i_2, \dots, i_n\}$ is the set of items

▣ each sequence is a list of transactions ordered by transaction-time

▣ each transaction consists of fields: sequence-id, transaction-id, transaction-time and a set of items.

□ Problem

▣ To discover **all the sequential patterns** with a user-specified minimum support

Input Database: example

Database \mathcal{D}

Sequence-Id	Transaction Time	Items
C1	1	Ringworld
C1	2	Foundation
C1	15	Ringworld Engineers, Second Foundation
C2	1	Foundation, Ringworld
C2	20	Foundation and Empire
C2	50	Ringworld Engineers

45% of customers who bought **Foundation** will buy **Foundation and Empire** within the next month.

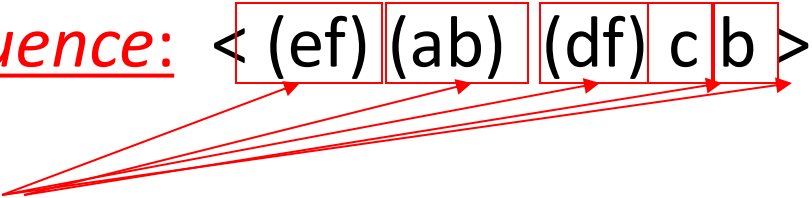
Sequential Pattern and Sequential Pattern Mining

- Sequential pattern mining: Given a set of sequences, find the **complete set of frequent subsequences** (i.e., satisfying the min_sup threshold)

A sequence database

SID	Sequence
10	<a(<u>abc</u>)(a <u>c</u>)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(<u>ab</u>)(df) <u>c</u> b>
40	<eg(af)cbc>

A sequence: < (ef) (ab) (df) c b >



- An element may contain a set of *items* (also called *events*)
- Items within an element are unordered and we list them alphabetically

<a(bc)dc> is a subsequence of <a(abc)(ac)d(cf)>

- Given support threshold $min_sup = 2$, <(ab)c> is a sequential pattern

A Basic Property of Sequential Patterns: Apriori

- A basic property: Apriori (Agrawal & Sirkant'94)
 - ▣ If a sequence S is not frequent
 - ▣ Then none of the super-sequences of S is frequent
 - ▣ E.g, $\langle hb \rangle$ is infrequent \rightarrow so do $\langle hab \rangle$ and $\langle (ah)b \rangle$

Seq. ID	Sequence
10	$\langle (bd)cb(ac) \rangle$
20	$\langle (bf)(ce)b(fg) \rangle$
30	$\langle (ah)(bf)abf \rangle$
40	$\langle (be)(ce)d \rangle$
50	$\langle a(bd)bcb(ade) \rangle$

Given *support threshold*
 $min_sup = 2$

GSP: Apriori-Based Sequential Pattern Mining

- Initial candidates: All 8-singleton sequences
 - $\langle a \rangle, \langle b \rangle, \langle c \rangle, \langle d \rangle, \langle e \rangle, \langle f \rangle, \langle g \rangle, \langle h \rangle$
- Scan DB once, count support for each candidate
- Generate length-2 candidate sequences

$min_sup = 2$

Cand.	sup
$\langle a \rangle$	3
$\langle b \rangle$	5
$\langle c \rangle$	4
$\langle d \rangle$	3
$\langle e \rangle$	3
$\langle f \rangle$	2
$\langle g \rangle$	1
$\langle h \rangle$	1

	$\langle a \rangle$	$\langle b \rangle$	$\langle c \rangle$	$\langle d \rangle$	$\langle e \rangle$	$\langle f \rangle$
$\langle a \rangle$	$\langle aa \rangle$	$\langle ab \rangle$	$\langle ac \rangle$	$\langle ad \rangle$	$\langle ae \rangle$	$\langle af \rangle$
$\langle b \rangle$	$\langle ba \rangle$	$\langle bb \rangle$	$\langle bc \rangle$	$\langle bd \rangle$	$\langle be \rangle$	$\langle bf \rangle$
$\langle c \rangle$	$\langle ca \rangle$	$\langle cb \rangle$	$\langle cc \rangle$	$\langle cd \rangle$	$\langle ce \rangle$	$\langle cf \rangle$
$\langle d \rangle$	$\langle da \rangle$	$\langle db \rangle$	$\langle dc \rangle$	$\langle dd \rangle$	$\langle de \rangle$	$\langle df \rangle$
$\langle e \rangle$	$\langle ea \rangle$	$\langle eb \rangle$	$\langle ec \rangle$	$\langle ed \rangle$	$\langle ee \rangle$	$\langle ef \rangle$
$\langle f \rangle$	$\langle fa \rangle$	$\langle fb \rangle$	$\langle fc \rangle$	$\langle fd \rangle$	$\langle fe \rangle$	$\langle ff \rangle$

	$\langle a \rangle$	$\langle b \rangle$	$\langle c \rangle$	$\langle d \rangle$	$\langle e \rangle$	$\langle f \rangle$
$\langle a \rangle$		$\langle (ab) \rangle$	$\langle (ac) \rangle$	$\langle (ad) \rangle$	$\langle (ae) \rangle$	$\langle (af) \rangle$
$\langle b \rangle$			$\langle (bc) \rangle$	$\langle (bd) \rangle$	$\langle (be) \rangle$	$\langle (bf) \rangle$
$\langle c \rangle$				$\langle (cd) \rangle$	$\langle (ce) \rangle$	$\langle (cf) \rangle$
$\langle d \rangle$					$\langle (de) \rangle$	$\langle (df) \rangle$
$\langle e \rangle$						$\langle (ef) \rangle$
$\langle f \rangle$						

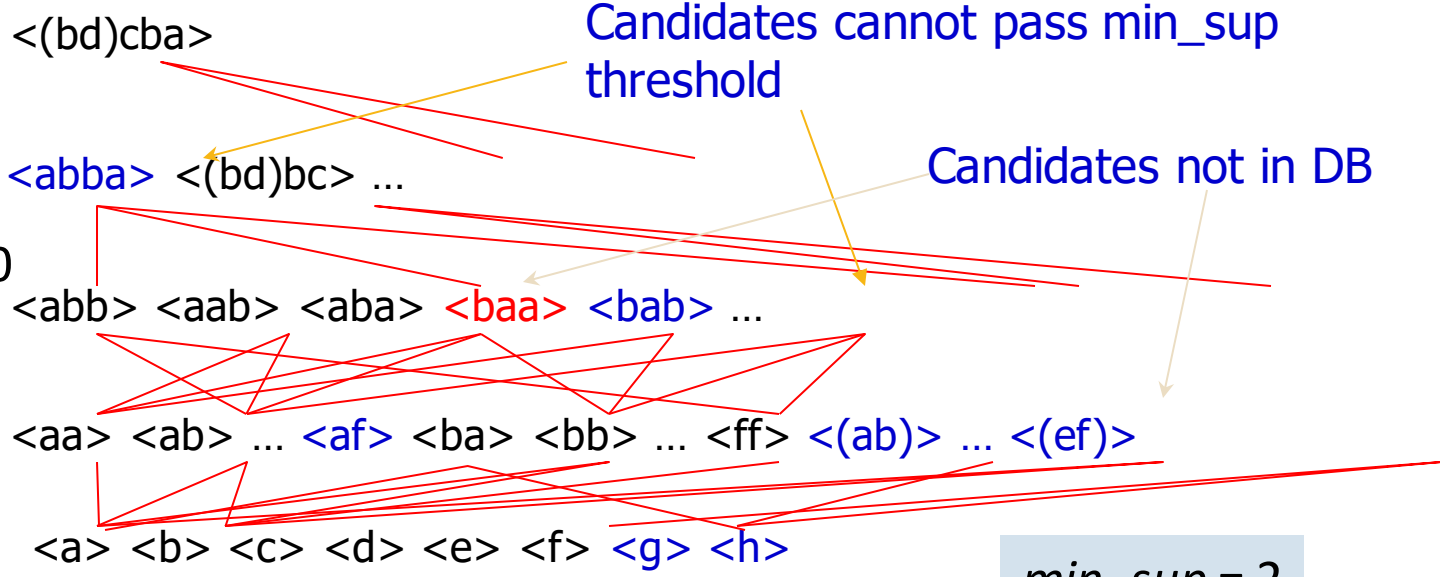
SID	Sequence
10	$\langle (bd)cb(ac) \rangle$
20	$\langle (bf)(ce)b(fg) \rangle$
30	$\langle (ah)(bf)abf \rangle$
40	$\langle (be)(ce)d \rangle$
50	$\langle a(bd)bcb(ade) \rangle$

- Without Apriori pruning:
 (8 singletons) $8*8 + 8*7/2 = 92$
 length-2 candidates
- With pruning, length-2
 candidates: $36 + 15 = 51$

GSP (Generalized Sequential Patterns): Srikant & Agrawal @ EDBT'96)

GSP Mining and Pruning

5th scan: 1 cand. 1 length-5 seq. pat.
 4th scan: 8 cand. 7 length-4 seq. pat.
 3rd scan: 46 cand. 20 length-3 seq. pat. 20 cand. not in DB at all
 2nd scan: 51 cand. 19 length-2 seq. pat. 10 cand. not in DB at all
 1st scan: 8 cand. 6 length-1 seq. pat.



min_sup = 2

- Repeat (for each level (i.e., length-k))
 - Scan DB to find length-k frequent sequences
 - Generate length-(k+1) candidate sequences from length-k frequent sequences using Apriori
 - set k = k+1
- Until no frequent sequence or no candidate can be found

SID	Sequence
10	<(bd)cb(ac)>
20	<(bf)(ce)b(fg)>
30	<(ah)(bf)abf>
40	<(be)(ce)d>
50	<a(bd)bcb(ade)>

GSP: Algorithm

□ Phase 1:

- Scan over the database to identify all the frequent items, i.e., 1 - element sequences

□ Phase 2:

- Iteratively scan over the database to discover all frequent sequences. Each iteration discovers all the sequences with the same length.
- In the iteration to generate all k -sequences
 - Generate the set of all candidate k -sequences, C_k , by joining two $(k-1)$ -sequences if only their first and last items are different
 - Prune the candidate sequence if any of its $k-1$ contiguous subsequence is not frequent
 - Scan over the database to determine the support of the remaining candidate sequences
- Terminate when no more frequent sequences can be found

GSP: Candidate Generation

Frequent 3-Sequences	Candidate 4-Sequences	
	after join	after pruning
$\langle (1, 2) (3) \rangle$	$\langle (1, 2) (3, 4) \rangle$	$\langle (1, 2) (3, 4) \rangle$
$\langle (1, 2) (4) \rangle$	$\langle (1, 2) (3) (5) \rangle$	
$\langle (1) (3, 4) \rangle$		
$\langle (1, 3) (5) \rangle$		
$\langle (2) (3, 4) \rangle$		
$\langle (2) (3) (5) \rangle$		

Figure 3: Candidate Generation: Example

The sequence $\langle (1,2) (3) (5) \rangle$ is dropped in the pruning phase, since its contiguous subsequence $\langle (1) (3) (5) \rangle$ is not frequent.

GSP: Optimization Techniques

- Applied to phase 2: computation-intensive
- Technique 1: the hash-tree data structure
 - Used for counting candidates to reduce the number of candidates that need to be checked
 - Leaf: a list of sequences
 - Interior node: a hash table
- Technique 2: data-representation transformation
 - From horizontal format to vertical format

Transaction-Time	Items
10	1, 2
25	4, 6
45	3
50	1, 2
65	3
90	2, 4
95	6



Item	Times
1	→ 10 → 50 → NULL
2	→ 10 → 50 → 90 → NULL
3	→ 45 → 65 → NULL
4	→ 25 → 90 → NULL
5	→ NULL
6	→ 25 → 95 → NULL
7	→ NULL

67

Backup slides

Sequential Pattern Mining in Vertical Data Format: The SPADE Algorithm

- A sequence database is mapped to: <SID, EID>
- Grow the subsequences (patterns) one item at a time by Apriori candidate generation

SID	Sequence
1	<a(abc)(ac)d(cf)>
2	<(ad)c(bc)(ae)>
3	<(ef)(ab)(df)cb>
4	<eg(af)cbc>

$min_sup = 2$

Ref: SPADE (Sequential Pattern Discovery using Equivalent Class) [M. Zaki 2001]

SID	EID	Items
1	1	a
1	2	abc
1	3	ac
1	4	d
1	5	cf
2	1	ad
2	2	c
2	3	bc
2	4	ae
3	1	ef
3	2	ab
3	3	df
3	4	c
3	5	b
4	1	e
4	2	g
4	3	af
4	4	c
4	5	b
4	6	c

a		b		...
SID	EID	SID	EID	...
1	1	1	2	
1	2	2	3	
1	3	3	2	
2	1	3	5	
2	4	4	5	
3	2			
4	3			

ab			ba			...
SID	EID (a)	EID(b)	SID	EID (b)	EID(a)	...
1	1	2	1	2	3	
2	1	3	2	3	4	
3	2	5				
4	3	5				

aba				...
SID	EID (a)	EID(b)	EID(a)	...
1	1	2	3	
2	1	3	4	

PrefixSpan: A Pattern-Growth Approach

$min_sup = 2$

SID	Sequence	Prefix	Suffix (Projection)
10	<a(abc)(ac)d(cf)>	<a>	<(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>	<aa>	<(_bc)(ac)d(cf)>
30	<(ef)(ab)(df)cb>	<ab>	<(_c)(ac)d(cf)>
40	<eg(af)cbc>		

Prefix and suffix

Given <a(abc)(ac)d(cf)>

Prefixes: <a>, <aa>, <a(ab)>, <a(abc)>, ...

Suffix: Prefixes-based projection

PrefixSpan Mining: Prefix Projections

Step 1: Find length-1 sequential patterns

<a>, , <c>, <d>, <e>, <f>

Step 2: Divide search space and mine each projected DB

<a>-projected DB,

-projected DB,

...

<f>-projected DB, ...

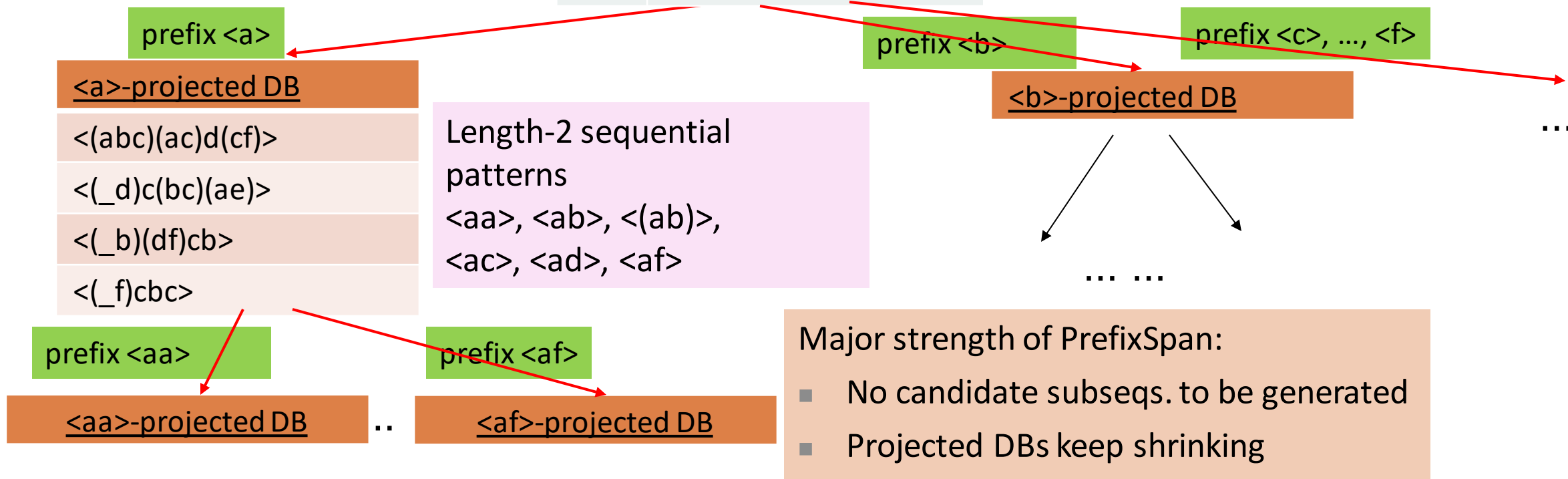
PrefixSpan (Prefix-projected Sequential pattern mining)
Pei, et al. @TKDE'04

PrefixSpan: Mining Prefix-Projected DBs

SID	Sequence
10	<a(abc)(ac)d(cf)>
20	<(ad)c(bc)(ae)>
30	<(ef)(ab)(df)cb>
40	<eg(af)cbc>

$min_sup = 2$

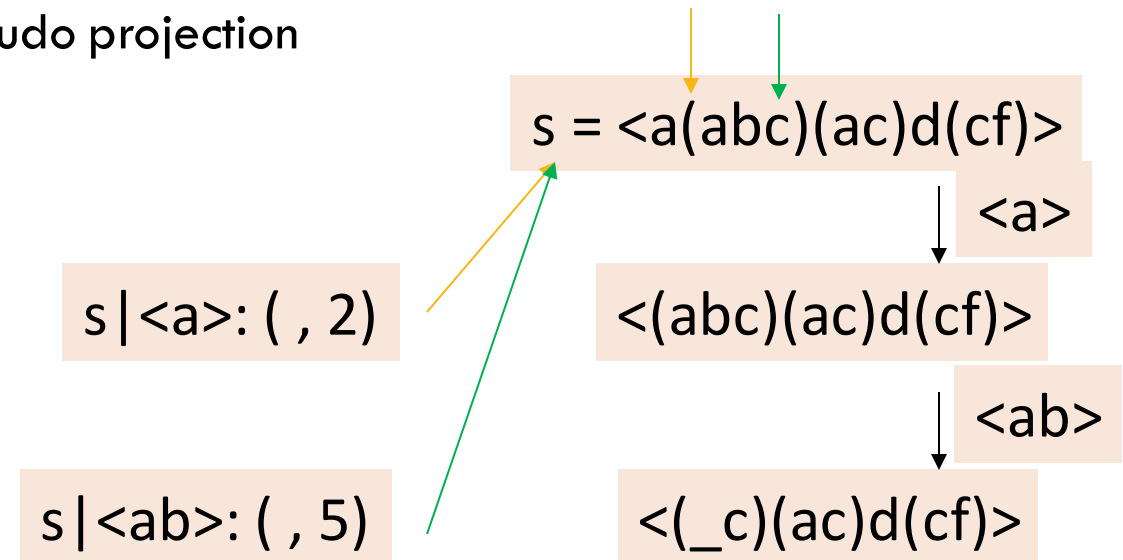
Length-1 sequential patterns
<a>, , <c>, <d>, <e>, <f>



Consideration:

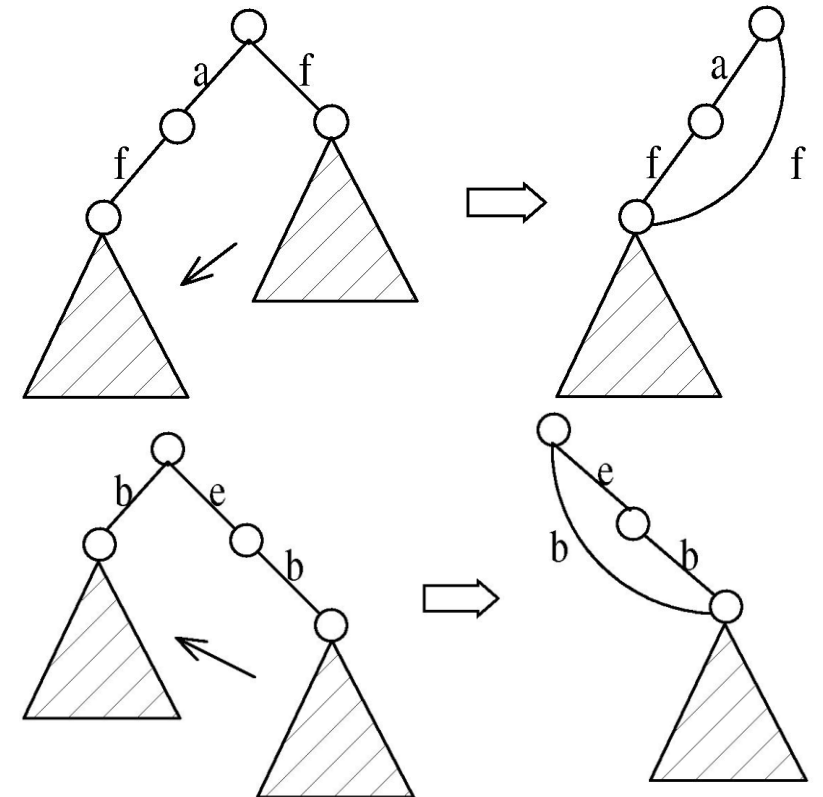
Pseudo-Projection vs. Physical Implementation

- Major cost of PrefixSpan: Constructing projected DBs
 - ▣ Suffixes largely repeating in recursive projected DBs
- When DB can be held in main memory, use pseudo projection
 - ▣ No physically copying suffixes
 - ▣ **Pointer to the sequence**
 - ▣ **Offset of the suffix**
- ▣ But if it does not fit in memory
 - ▣ Physical projection
- ▣ Suggested approach:
 - ▣ Integration of physical and pseudo-projection
 - ▣ Swapping to pseudo-projection when the data fits in memory



CloSpan: Mining Closed Sequential Patterns

- A **closed sequential pattern** s : There exists no superpattern s' such that $s' \supset s$, and s' and s have the same support
- Which ones are closed? $\langle abc \rangle$: 20, $\langle abcd \rangle$: 20, $\langle abcde \rangle$: 15
- Why directly mine closed sequential patterns?
 - Reduce # of (redundant) patterns
 - Attain the same expressive power
- Property P_1 : If $s \supset s_1$, s is closed iff two project DBs have the same size
- Explore *Backward Subpattern* and *Backward Superpattern* pruning to prune redundant search space
- Greatly enhances efficiency (Yan, et al., SDM'03)

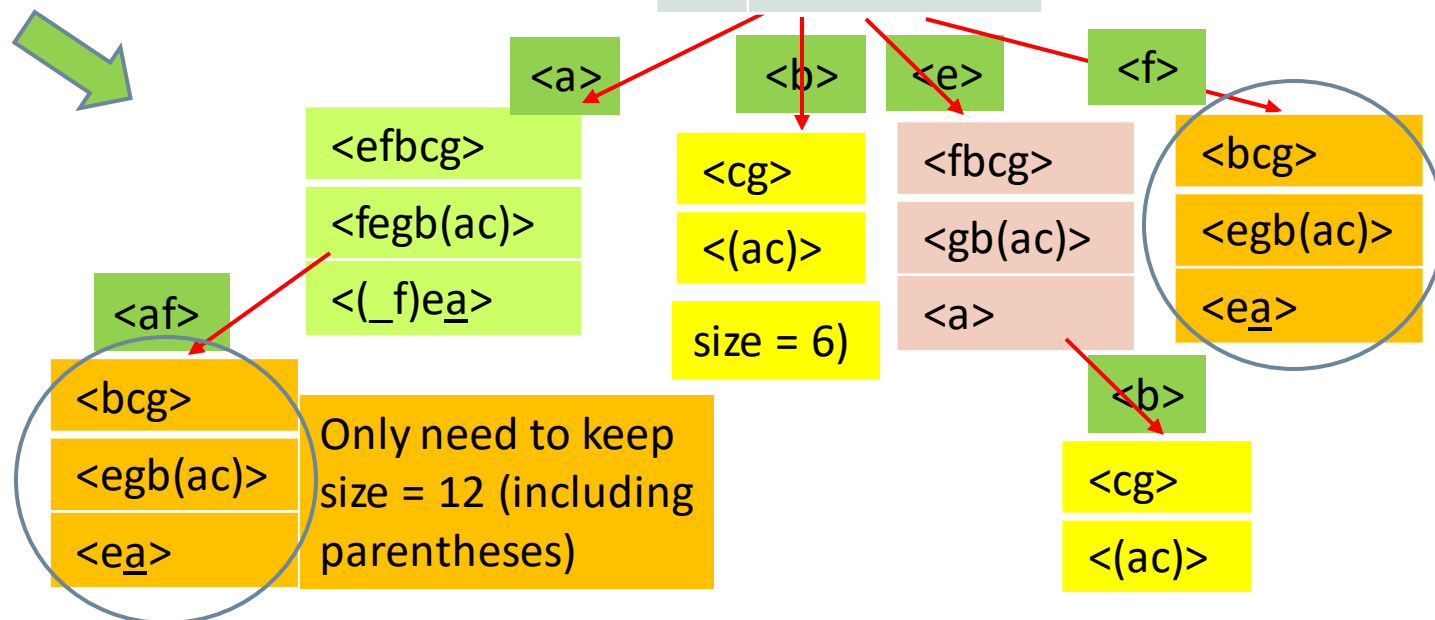
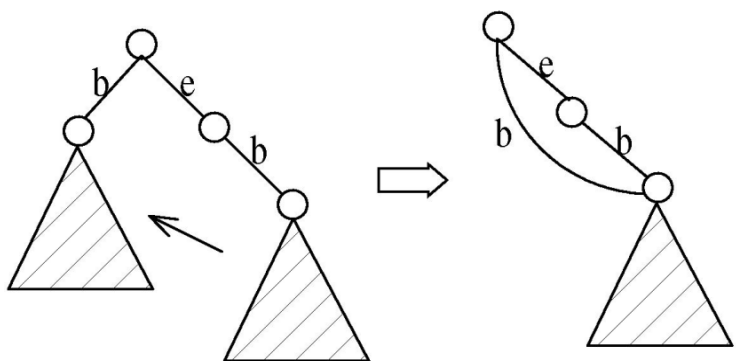
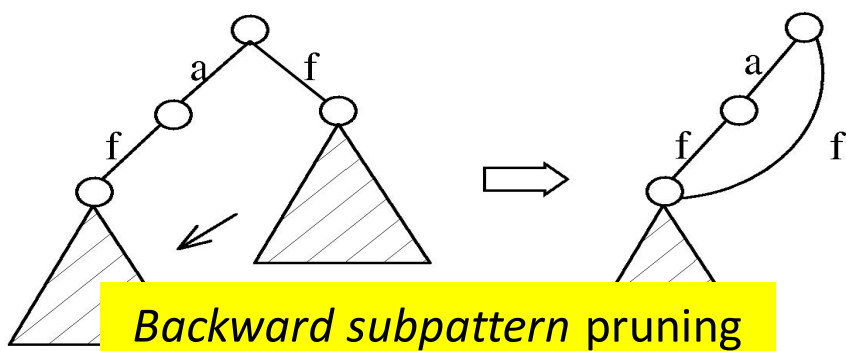


CloSpan: When Two Projected DBs Have the Same Size


- If $s \supset s_1$, s is closed iff two project DBs have the same size
 - When two projected sequence DBs have the same size?
 - Here is one example:

ID	Sequence
1	<aefbcg>
2	<afegb(ac)>
3	<(af)ea>

$min_sup = 2$



Chapter 7 : Advanced Frequent Pattern Mining

- Mining Diverse Patterns
- Sequential Pattern Mining
- Constraint-Based Frequent Pattern Mining 
- Graph Pattern Mining
- Pattern Mining Application: Mining Software Copy-and-Paste Bugs
- Summary


Constraint-Based Pattern Mining

- Why Constraint-Based Mining?
- Different Kinds of Constraints: Different Pruning Strategies
- Constrained Mining with Pattern Anti-Monotonicity
- Constrained Mining with Pattern Monotonicity
- Constrained Mining with Data Anti-Monotonicity
- Constrained Mining with Succinct Constraints
- Constrained Mining with Convertible Constraints
- Handling Multiple Constraints
- Constraint-Based Sequential-Pattern Mining

Why Constraint-Based Mining?

- Finding **all** the patterns in a dataset **autonomously**?—unrealistic!
 - ▣ Too many patterns but not necessarily user-interested!
- Pattern mining in practice: Often a user-guided, **interactive** process
 - ▣ User directs what to be mined using a **data mining query language** (or a graphical user interface), **specifying various kinds of constraints**
- What is constraint-based mining?
 - ▣ Mine together with user-provided constraints
- Why constraint-based mining?
 - ▣ User flexibility: User provides **constraints** on what to be mined
 - ▣ Optimization: System explores such constraints for mining efficiency
 - E.g., Push constraints deeply into the mining process

Various Kinds of User-Specified Constraints in Data Mining

- ❑ **Knowledge type constraint**—Specifying what kinds of knowledge to mine
 - ❑ Ex.: Classification, association, clustering, outlier finding, ...
- ❑ **Data constraint**—using SQL-like queries
 - ❑ Ex.: Find products sold together in NY stores this year
- ❑ **Dimension/level constraint**—similar to projection in relational database
 - ❑ Ex.: In relevance to region, price, brand, customer category
- ❑ **Interestingness constraint**—various kinds of thresholds
 - ❑ Ex.: Strong rules: $\text{min_sup} \geq 0.02$, $\text{min_conf} \geq 0.6$, $\text{min_correlation} \geq 0.7$
- ❑ Rule (or pattern) constraint  **The focus of this study**
 - ❑ Ex.: Small sales (price < \$10) triggers big sales (sum > \$200)

Pattern Space Pruning with Pattern Anti-Monotonicity

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
a	100	40
b	40	0
c	150	-20
d	35	-15
e	55	-30
f	45	-10
g	80	20
h	10	5

- A constraint c is **anti-monotone**
 - If an itemset S **violates** constraint c , so does any of its superset
 - That is, mining on itemset S can be terminated
- Ex. 1: $c_1: \text{sum}(S.\text{price}) \leq v$ is **anti-monotone**
- Ex. 2: $c_2: \text{range}(S.\text{profit}) \leq 15$ is **anti-monotone**
 - Itemset ab violates c_2 ($\text{range}(ab) = 40$)
 - So does every superset of ab
- Ex. 3. $c_3: \text{sum}(S.\text{Price}) \geq v$ is **not anti-monotone**
- Ex. 4. Is $c_4: \text{support}(S) \geq \sigma$ anti-monotone?
 - Yes! Apriori pruning is essentially pruning with an anti-monotonic constraint!

Note: item.price > 0
Profit can be negative

Pattern Monotonicity and Its Roles

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
a	100	40
b	40	0
c	150	-20
d	35	-15
e	55	-30
f	45	-10
g	80	20
h	10	5

- A constraint c is *monotone*: If an itemset S **satisfies** the constraint c , so does any of its superset
 - That is, we do not need to check c in subsequent mining
- Ex. 1: $c_1: \text{sum}(S.\text{Price}) \geq v$ is **monotone**
- Ex. 2: $c_2: \text{min}(S.\text{Price}) \leq v$ is **monotone**
- Ex. 3: $c_3: \text{range}(S.\text{profit}) \geq 15$ is **monotone**
 - Itemset ab satisfies c_3
 - So does every superset of ab

Note: item.price > 0
Profit can be negative

Data Space Pruning with Data Anti-Monotonicity

TID	Transaction
10	a, b, c, d, f, h
20	b, c, d, f, g, h
30	b, c, d, f, g
40	a, c, e, f, g

min_sup = 2

Item	Price	Profit
a	100	40
b	40	0
c	150	-20
d	35	-15
e	55	-30
f	45	-10
g	80	20
h	10	5

- A constraint c is **data anti-monotone**: In the mining process, if a data entry t cannot satisfy a pattern p under c , t cannot satisfy p 's superset either
 - ▣ Data space pruning: Data entry t can be pruned
- Ex. 1: $c_1: \text{sum}(S.\text{Profit}) \geq v$ is **data anti-monotone**
 - ▣ Let constraint c_1 be: $\text{sum}(S.\text{Profit}) \geq 25$
 - $T_{30}: \{b, c, d, f, g\}$ can be removed since none of their combinations can make an S whose sum of the profit is ≥ 25
- Ex. 2: $c_2: \text{min}(S.\text{Price}) \leq v$ is **data anti-monotone**
 - Consider $v = 5$ but every item in a transaction, say T_{50} , has a price higher than 10
- Ex. 3: $c_3: \text{range}(S.\text{Profit}) > 25$ is **data anti-monotone**

Note: item.price > 0
Profit can be negative

Expressing Patterns in Compressed Form: Closed Patterns

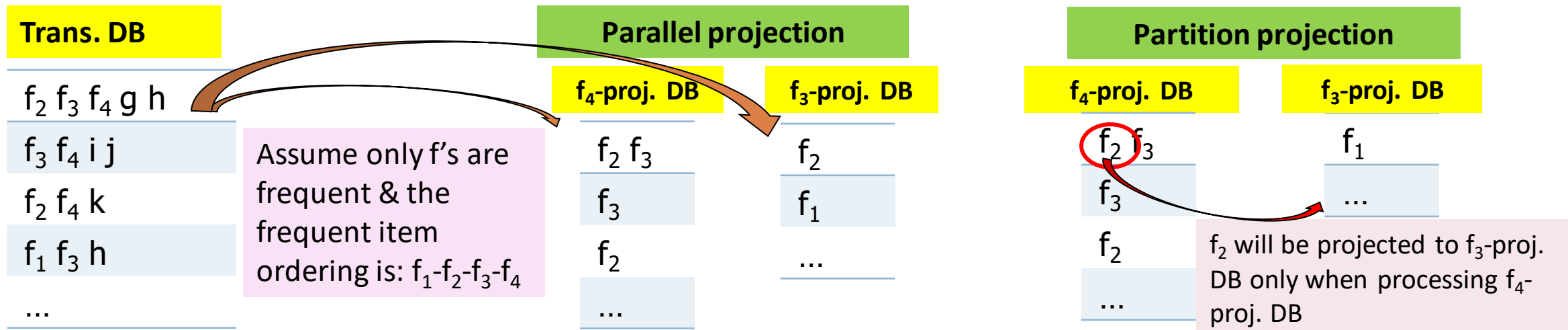
- How to handle such a challenge?
- Solution 1: **Closed patterns:** A pattern (itemset) X is closed if X is frequent, and there exists no super-pattern $Y \supset X$, with the same support as X
 - ▣ Let Transaction DB TDB_1 : $T_1: \{a_1, \dots, a_{50}\}; T_2: \{a_1, \dots, a_{100}\}$
 - ▣ Suppose $minsup = 1$. How many closed patterns does TDB_1 contain?
 - Two: $P_1: \{a_1, \dots, a_{50}\}: 2$; $P_2: \{a_1, \dots, a_{100}\}: 1$
- Closed pattern is a lossless compression of frequent patterns
 - ▣ Reduces the # of patterns but does not lose the support information!
 - ▣ You will still be able to say: $\{a_2, \dots, a_{40}\}: 2$, $\{a_5, a_{51}\}: 1$

Expressing Patterns in Compressed Form: Max-Patterns

- **Solution 2: Max-patterns:** A pattern X is a maximal frequent pattern or max-pattern if X is frequent and there exists no frequent super-pattern $Y \supset X$
- Difference from close-patterns?
 - ▣ Do not care the real support of the sub-patterns of a max-pattern
 - ▣ Let Transaction DB TDB_1 : $T_1: \{a_1, \dots, a_{50}\}$; $T_2: \{a_1, \dots, a_{100}\}$
 - ▣ Suppose $minsup = 1$. How many max-patterns does TDB_1 contain?
 - One: $P: \{\{a_1, \dots, a_{100}\}: 1\}$
- Max-pattern is a lossy compression!
 - ▣ We only know $\{a_1, \dots, a_{40}\}$ is frequent
 - ▣ But we do not know the real support of $\{a_1, \dots, a_{40}\}$, ..., any more!
 - ▣ Thus in many applications, close-patterns are more desirable than max-patterns

Scaling FP-growth by Item-Based Data Projection

- What if FP-tree cannot fit in memory?—Do not construct FP-tree
 - ▣ “Project” the database based on frequent single items
 - ▣ Construct & mine FP-tree for each projected DB
- **Parallel projection** vs. **partition projection**
 - ▣ Parallel projection: Project the DB on each frequent item
 - Space costly, all partitions can be processed in parallel
 - ▣ Partition projection: Partition the DB in order
 - Passing the unprocessed parts to subsequent partitions



Analysis of DBLP Coauthor Relationships

- DBLP: Computer science research publication bibliographic database
 - > 3.8 million entries on authors, paper, venue, year, and other information

ID	Author <i>A</i>	Author <i>B</i>	$s(A \cup B)$	$s(A)$	$s(B)$	Jaccard	<i>Cosine</i>	<i>Kulc</i>
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
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8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Advisor-advisee relation: Kulc: high, Jaccard: low, cosine: middle

- Which pairs of authors are strongly related?
 - Use Kulc to find Advisor-advisee, close collaborators

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What Measures to Choose for Effective Pattern Evaluation?

- Null value cases are predominant in many large datasets
 - ▣ Neither milk nor coffee is in most of the baskets; neither Mike nor Jim is an author in most of the papers;
- *Null-invariance* is an important property
- Lift, χ^2 and cosine are good measures if null transactions are not predominant
 - ▣ Otherwise, *Kulczynski + Imbalance Ratio* should be used to judge the interestingness of a pattern
- Exercise: Mining research collaborations from research bibliographic data
 - ▣ Find a group of frequent collaborators from research bibliographic data (e.g., DBLP)
 - ▣ Can you find the likely advisor-advisee relationship and during which years such a relationship happened?
 - ▣ Ref.: C. Wang, J. Han, Y. Jia, J. Tang, D. Zhang, Y. Yu, and J. Guo, "Mining Advisor-Advisee Relationships from Research Publication Networks", KDD'10

Mining Compressed Patterns

Pat-ID	Item-Sets	Support
P1	{38,16,18,12}	205227
P2	{38,16,18,12,17}	205211
P3	{39,38,16,18,12,17}	101758
P4	{39,16,18,12,17}	161563
P5	{39,16,18,12}	161576

- ❑ Closed patterns
 - ❑ P1, P2, P3, P4, P5
 - ❑ Emphasizes too much on support
 - ❑ There is no compression
- ❑ Max-patterns
 - ❑ P3: information loss
- ❑ Desired output (a good balance):
 - ❑ **P2, P3, P4**

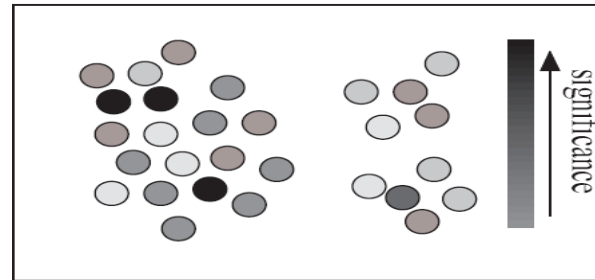
- ❑ Why mining compressed patterns?
 - ❑ Too many scattered patterns but not so meaningful
- ❑ Pattern distance measure

$$Dist(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

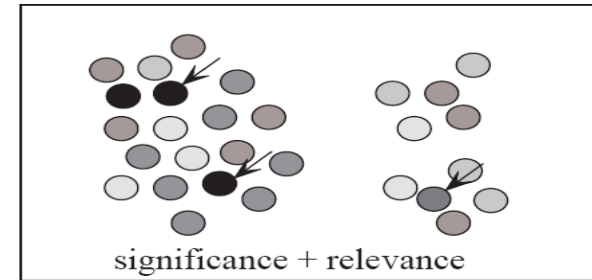
- ❑ δ -clustering: For each pattern P, find all patterns which can be expressed by P and whose distance to P is within δ (δ -cover)
- ❑ All patterns in the cluster can be represented by P
- ❑ Method for efficient, direct mining of compressed frequent patterns (e.g., D. Xin, J. Han, X. Yan, H. Cheng, "On Compressing Frequent Patterns", Knowledge and Data Engineering, 60:5-29, 2007)

Redundancy-Aware Top-k Patterns

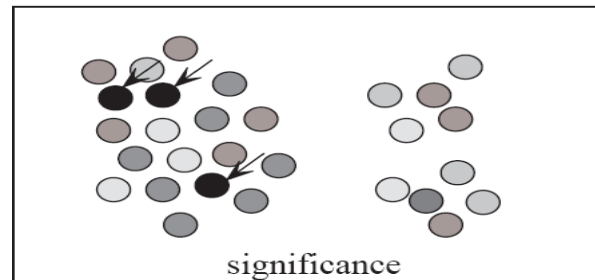
- Desired patterns: high significance & low redundancy



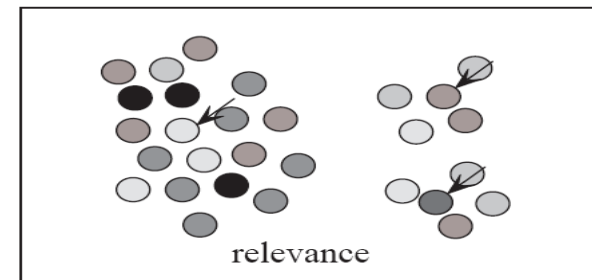
(a) a set of patterns



(b) redundancy-aware top- k



(c) traditional top- k



(d) summarization

- Method: Use MMS (Maximal Marginal Significance) for measuring the combined significance of a pattern set
- Xin et al., Extracting Redundancy-Aware Top-K Patterns, KDD'06

Redundancy Filtering at Mining Multi-Level Associations

- Multi-level association mining may generate many redundant rules
- Redundancy filtering: Some rules may be redundant due to “ancestor” relationships between items
 - ▣ milk \Rightarrow wheat bread [support = 8%, confidence = 70%] (1)
 - ▣ 2% milk \Rightarrow wheat bread [support = 2%, confidence = 72%] (2)
 - Suppose the “2% milk” sold is about “1/4” of milk sold
 - Does (2) provide any novel information?
- A rule is *redundant* if its support is close to the “expected” value, according to its “ancestor” rule, and it has a similar confidence as its “ancestor”
 - ▣ Rule (1) is an ancestor of rule (2), which one to prune?

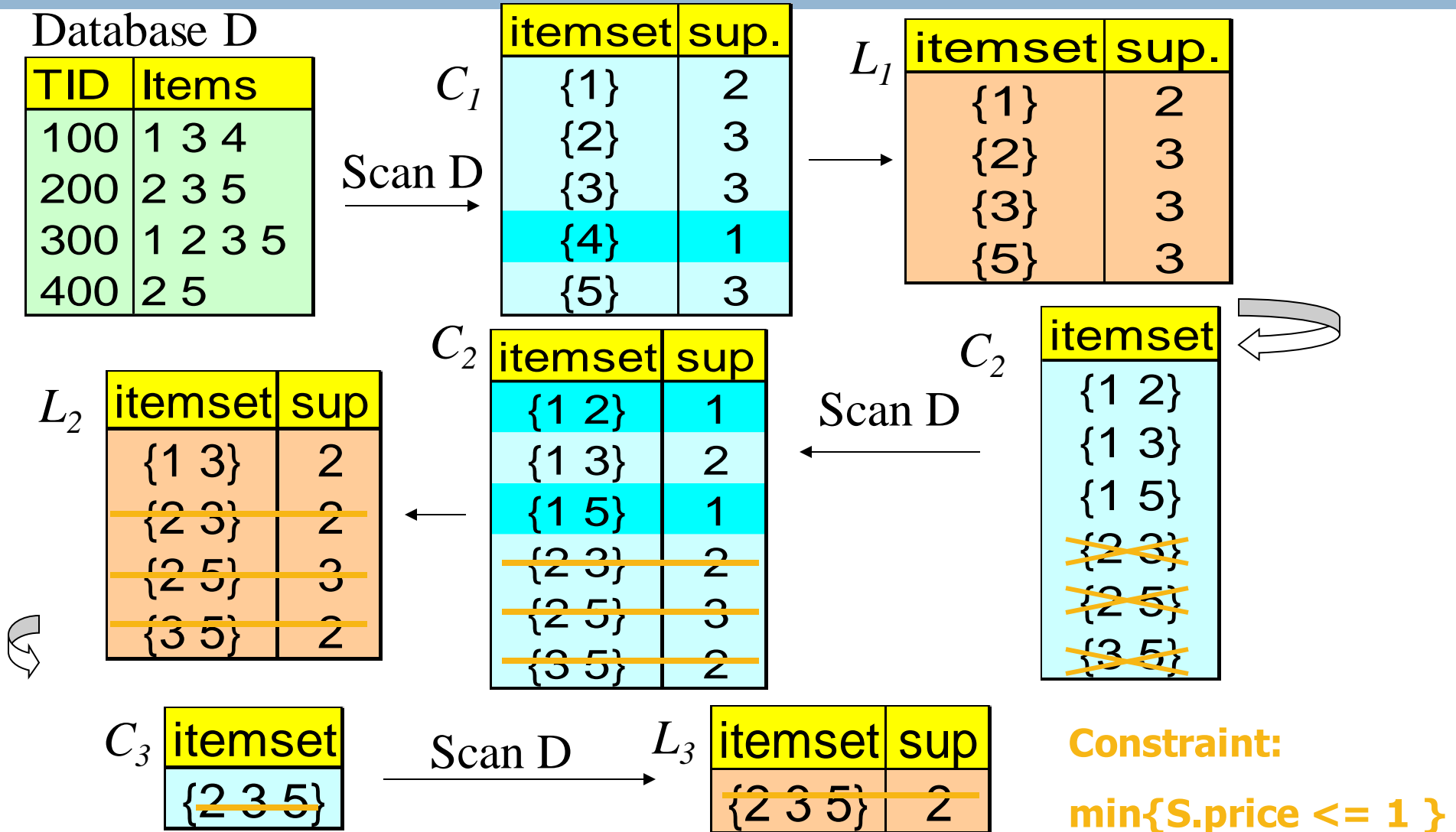
Succinctness

- Succinctness:
 - ▣ Given A_1 , the set of items satisfying a succinctness constraint C , then any set S satisfying C is based on A_1 , i.e., S contains a subset belonging to A_1
 - ▣ Idea: Without looking at the transaction database, whether an itemset S satisfies constraint C can be determined based on the selection of items
 - ▣ $\min(S.Price) \leq v$ is succinct
 - ▣ $\sum(S.Price) \geq v$ is not succinct
- Optimization: If C is succinct, C is pre-counting pushable

Which Constraints Are Succinct?

Constraint	Succinct
$v \in S$	yes
$S \supseteq V$	yes
$S \subseteq V$	yes
$\min(S) \leq v$	yes
$\min(S) \geq v$	yes
$\max(S) \leq v$	yes
$\max(S) \geq v$	yes
$\text{sum}(S) \leq v \ (a \in S, a \geq 0)$	no
$\text{sum}(S) \geq v \ (a \in S, a \geq 0)$	no
$\text{range}(S) \leq v$	no
$\text{range}(S) \geq v$	no
$\text{avg}(S) \theta v, \theta \in \{=, \leq, \geq\}$	no
$\text{support}(S) \geq \xi$	no
$\text{support}(S) \leq \xi$	no

Push a Succinct Constraint Deep



Sequential Pattern Mining

- Sequential Pattern and Sequential Pattern Mining
- GSP: Apriori-Based Sequential Pattern Mining
- SPADE: Sequential Pattern Mining in Vertical Data Format
- PrefixSpan: Sequential Pattern Mining by Pattern-Growth
- CloSpan: Mining Closed Sequential Patterns