CSE 5243 INTRO. TO DATA MINING

Data & Data Preprocessing

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Chapter 3: Data Preprocessing

- Data Preprocessing: An Overview
- Data Cleaning
- Data Integration
- Data Reduction and Transformation
- Dimensionality Reduction
- Summary

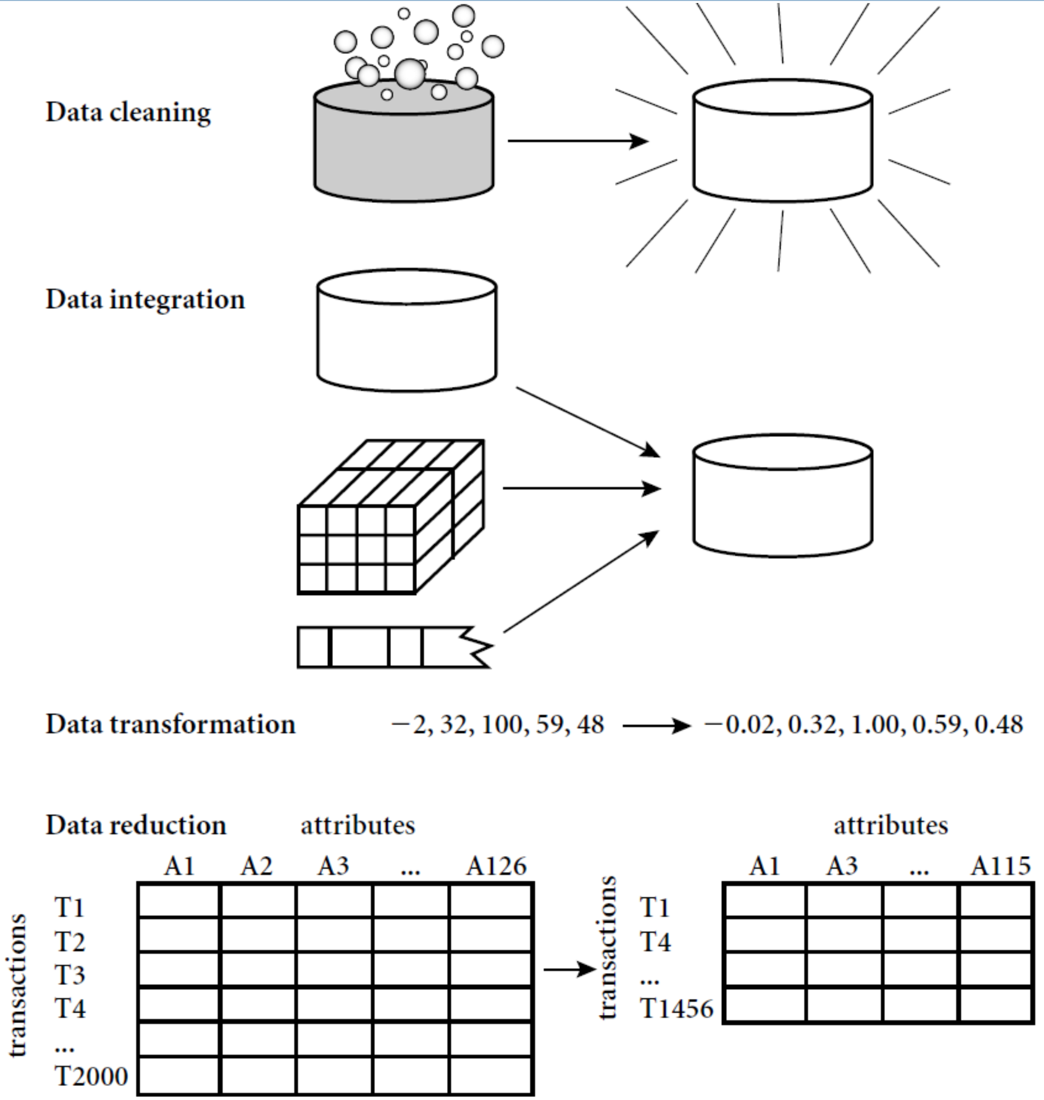
Major tasks

Why Preprocess the Data? — Data Quality Issues

- Measures for data quality: A multidimensional view
  - Accuracy: correct or wrong, accurate or not
  - Completeness: not recorded, unavailable, …
  - Consistency: some modified but some not, dangling, …
  - Timeliness: timely update?
  - Believability: how trustable the data are correct?
  - Interpretability: how easily the data can be understood?

What is Data Preprocessing? — Major Tasks

- **Data cleaning**
  - Handle missing data, smooth noisy data, identify or remove outliers, and resolve inconsistencies
- **Data integration**
  - Integration of multiple databases, data cubes, or files
- **Data reduction**
  - Dimensionality reduction
  - Numerosity reduction
  - Data compression
- **Data transformation and data discretization**
  - Normalization
  - Concept hierarchy generation



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Incomplete (Missing) Data

- Data is not always available
  - E.g., many tuples have no recorded value for several attributes, such as customer income in sales data
- Various reasons for missing:
  - Equipment malfunction
  - Inconsistent with other recorded data and thus deleted
  - Data were not entered due to misunderstanding
  - Certain data may not be considered important at the time of entry
  - Did not register history or changes of the data
- Missing data may need to be inferred

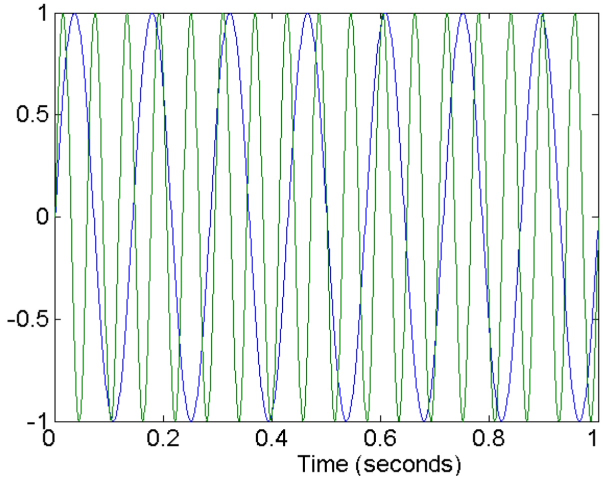
How to Handle Missing Data?

- Ignore the tuple: usually done when class label is missing (when doing classification)—not effective when the % of missing values per attribute varies considerably
- Fill in the missing value manually: tedious + infeasible?
- Fill in it automatically with
  - a global constant : e.g., "unknown", a new class?!
  - the attribute mean
  - the attribute mean for all samples belonging to the same class: smarter
  - the most probable value: inference-based such as Bayesian formula or decision tree

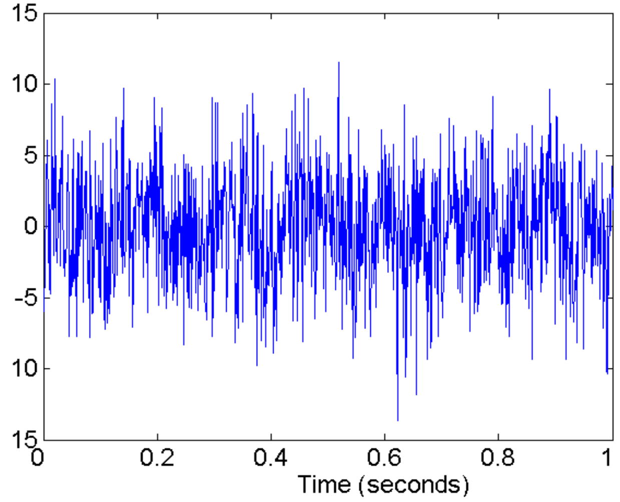
Noisy Data

- Noise: random error or variance in a measured variable
- Incorrect attribute values may be due to
  - Faulty data collection instruments
  - Data entry problems
  - Data transmission problems
  - Technology limitation
  - Inconsistency in naming convention
- Other data problems
  - Duplicate records
  - Inconsistent data

Two Sine Waves



Two Sine Waves + Noise



How to Handle Noisy Data?

- Binning
  - First sort data and partition into (equal-frequency) bins
  - Then one can smooth by bin means, smooth by bin median, smooth by bin boundaries, etc.
- Regression
  - Smooth by fitting the data into regression functions
- Clustering
  - Detect and remove outliers
- Semi-supervised: Combined computer and human inspection
  - Detect suspicious values and check by human (e.g., deal with possible outliers)

Data Cleaning as a Process

- **Data discrepancy detection**
  - Use metadata (e.g., domain, range, dependency, distribution)
  - Check field overloading
  - Check based on rules: uniqueness rule, consecutive rule and null rule
  - Use commercial tools
    - Data scrubbing: use simple domain knowledge (e.g., postal code, spell-check) to detect errors and make corrections
    - Data auditing: by analyzing data to discover rules and relationship to detect violators (e.g., correlation and clustering to find outliers)

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- **Data migration and integration**
  - Data migration tools: allow transformations to be specified
  - ETL (Extraction/Transformation/Loading) tools: allow users to specify transformations through a graphical user interface
- **Integration of the two processes**
  - Iterative and interactive (e.g., Potter's Wheels, a publicly available data cleaning tool)

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Data Integration

- Data integration
  - Combining data from multiple sources into a coherent store
- Schema integration: e.g., A.cust-id ≡ B.cust-#
  - Integrate metadata from different sources
- Entity identification:
  - Identify real world entities from multiple data sources, e.g., Bill Clinton = William Clinton
- Detecting and resolving data value conflicts
  - For the same real world entity, attribute values from different sources are different
  - Possible reasons: different representations, different scales, e.g., metric vs. British units

Handling Redundancy in Data Integration

- Redundant data occur often when integration of multiple databases
  - *Object identification*: The same attribute or object may have different names in different databases
  - *Derivable data:* One attribute may be a "derived" attribute in another table, e.g., annual revenue

Handling Redundancy in Data Integration

- Redundant data occur often when integration of multiple databases
  - *Object identification*: The same attribute or object may have different names in different databases
  - *Derivable data:* One attribute may be a "derived" attribute in another table, e.g., annual revenue
- Redundant attributes may be able to be detected by ***correlation analysis* and *covariance analysis***
- Careful integration of the data from multiple sources may help reduce/avoid redundancies and inconsistencies and improve mining speed and quality

Correlation Analysis (for Categorical Data)

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  - Contingency table: How many times the joint event (A_i, B_j), "attribute A takes on values a_i and attribute B takes on value b_j", happens based on the observed data tuples.

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$$\chi^2 = \sum_{i=1}^{c} \sum_{j=1}^{r} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$$

Where o_{ij} is the observed frequency (or, actual count) of the joint event (A_i, B_j), and e_{ij} is the expected frequency:

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  - The larger the X^2 value, the more likely the variables are related

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- Null hypothesis: The two variables are independent
- The cells that contribute the most to the X^2 value are those whose actual count is very different from the expected count
  - The larger the X^2 value, the more likely the variables are related
- Note: Correlation does not imply causality
  - # of hospitals and # of car-theft in a city are correlated
  - Both are causally linked to the third variable: population

Chi-Square Calculation: An Example

	Play chess	Not play chess	Sum (row)
Like science fiction	250 (90)	200 (360)	450
Not like science fiction	50 (210)	1000 (840)	1050
Sum (col.)	300	1200	1500

Contingency Table

Numbers outside bracket mean the observed frequencies of a joint event, and numbers inside bracket mean the expected frequencies.

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(450* 300)/1500 = 90

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- X^2 (chi-square) calculation

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

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450/1500 * 300 = 90

- X^2 (chi-square) calculation (numbers in parenthesis are expected counts calculated based on the data distribution in the two categories)

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Given a threshold 10.828

- It shows that *like_science_fiction* and *play_chess* are correlated in the group

Review: Variance for Single Variable (Numerical Data)

- The variance of a random variable X provides a measure of how much the value of X deviates from the mean or expected value of X:

$$\sigma^2 = \mathrm{var}(X) = E[(X-\mu)^2] = \begin{cases} \sum_x (x-\mu)^2 f(x) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx & \text{if } X \text{ is continuous} \end{cases}$$

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  - It can also be written as:

$$\boxed{\sigma^2 = \mathrm{var}(X) = E[(\mathrm{X}-\mu)^2] = E[\mathrm{X}^2] - \mu^2 = E[\mathrm{X}^2] - [E(x)]^2}$$

Covariance for Two Variables

- Covariance between two variables X_1 and X_2

$$\sigma_{12} = E[(X_1 - \mu_1)(X_2 - \mu_2)] = E[X_1 X_2] - \mu_1\mu_2 = E[X_1 X_2] - E[X_1]E[X_2]$$

where $\mu_1 = E[X_1]$ is the mean or **expected value** of X_1; similarly for μ_2

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- Sample covariance between X_1 and X_2: $\hat{\sigma}_{12} = \frac{1}{n}\sum_{i=1}^{n}(x_{i1} - \hat{\mu}_1)(x_{i2} - \hat{\mu}_2)$

- Sample covariance is a generalization of the sample variance:

$$\hat{\sigma}_{11} = \frac{1}{n}\sum_{i=1}^{n}(x_{i1} - \hat{\mu}_1)(x_{i1} - \hat{\mu}_1) = \frac{1}{n}\sum_{i=1}^{n}(x_{i1} - \hat{\mu}_1)^2 = \hat{\sigma}_1^{\,2}$$

For unbiased estimator, n => n-1

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- **Positive covariance:** If $\sigma_{12} > 0$

- **Negative covariance:** If $\sigma_{12} < 0$

- **Independence:** If X_1 and X_2 are independent, $\sigma_{12} = 0$, but the reverse is not true
  - Some pairs of random variables may have a covariance 0 but are not independent
  - Only under some additional assumptions (e.g., the data follow multivariate normal distributions) does a covariance of 0 imply independence

Example: Calculation of Covariance

- Suppose two stocks X_1 and X_2 have the following values in one week:
  - Day 1: $(X_1, X_2) = (2, 5)$,
  - Day 2: $(X_1, X_2) = (3, 8)$,
  - Day 3: $(X_1, X_2) = (5, 10)$,
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  - $\sigma_{12} = \underbrace{(2\times5 + 3\times8 + 5\times10 + 4\times11 + 6\times14)/5}_{E[X_1X_2]} - 4 \times 9.6 = 4$

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  - $\sigma_{12} = (2\times5 + 3\times8 + 5\times10 + 4\times11 + 6\times14)/5 - 4 \times 9.6 = 4$
- Thus, X_1 and X_2 rise together since $\sigma_{12} > 0$

Correlation Coefficient between Two Numerical Variables

- **Correlation** between two variables X_1 and X_2 is the standard covariance, obtained by normalizing the covariance with the standard deviation of each variable

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{\sigma_{12}}{\sqrt{\sigma_1^{\,2} \sigma_2^{\,2}}}$$

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where n is the number of tuples, μ_1 and μ_2 are the respective means of X_1 and X_2 , σ_1 and σ_2 are the respective standard deviation of X_1 and X_2

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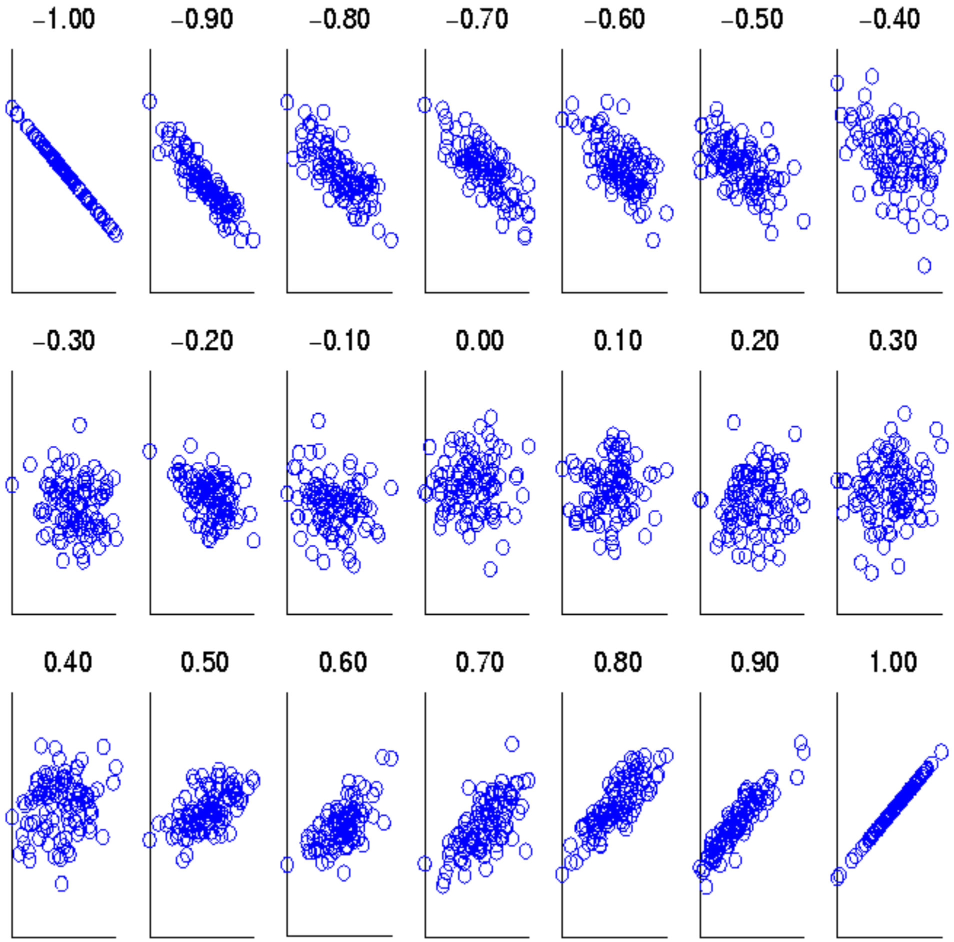
- If $\rho_{12} > 0$: A and B are positively correlated (X_1's values increase as X_2's)
  - The higher, the stronger correlation
- If $\rho_{12} = 0$: independent (under the same assumption as discussed in co-variance)
- If $\rho_{12} < 0$: negatively correlated

Visualizing Changes of Correlation Coefficient



- Correlation coefficient value range: [–1, 1]
- A set of scatter plots shows sets of points and their correlation coefficients changing from –1 to 1

Covariance Matrix

- The variance and covariance information for the two variables X_1 and X_2 can be summarized as 2 X 2 covariance matrix as

$$\Sigma = E[(\mathbf{X}-\mu)(\mathbf{X}-\mu)^T] = E[\begin{pmatrix} X_1-\mu_1 \\ X_2-\mu_2 \end{pmatrix}(X_1-\mu_1 \quad X_2-\mu_2)] = \begin{pmatrix} E[(X_1-\mu_1)(X_1-\mu_1)] & E[(X_1-\mu_1)(X_2-\mu_2)] \\ E[(X_2-\mu_2)(X_1-\mu_1)] & E[(X_2-\mu_2)(X_2-\mu_2)] \end{pmatrix}$$

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- Generalizing it to *d* dimensions, we have,

$$D = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1d} \\ x_{21} & x_{22} & \cdots & x_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ x_{d1} & x_{d2} & \cdots & x_{dd} \end{pmatrix} \qquad \boxed{\mathbf{\Sigma} = E[(\mathbf{X}-\mu)(\mathbf{X}-\mu)^T] = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1d} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2d} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{d1} & \sigma_{d2} & \cdots & \sigma_d^2 \end{pmatrix}}$$

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Data Reduction

- **Data reduction:**
  - Obtain a reduced representation of the data set
    - much smaller in volume but yet produces *almost* the same analytical results
- Why data reduction?—A database/data warehouse may store terabytes of data
  - Complex analysis may take a very long time to run on the complete data set

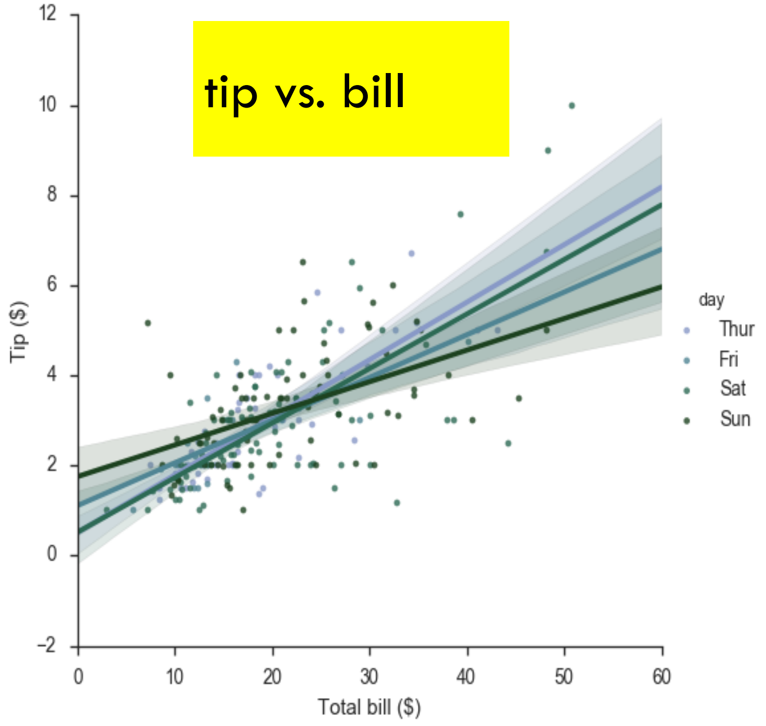
Data Reduction

- **Data reduction:**
  - Obtain a reduced representation of the data set
    - much smaller in volume but yet produces *almost* the same analytical results
- Why data reduction?—A database/data warehouse may store terabytes of data
  - Complex analysis may take a very long time to run on the complete data set

- **Methods for data reduction** (also *data size reduction* or *numerosity reduction*)
  - Regression and Log-Linear Models
  - Histograms, clustering, sampling
  - Data cube aggregation
  - Data compression

Data Reduction: **Parametric vs. Non-Parametric Methods**

- Reduce data volume by choosing alternative, *smaller forms* of data representation
- **Parametric methods** (e.g., regression)
  - Assume the data fits some model, estimate model parameters, store only the parameters, and discard the data (except possible outliers)



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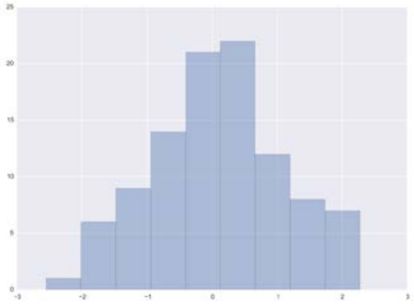
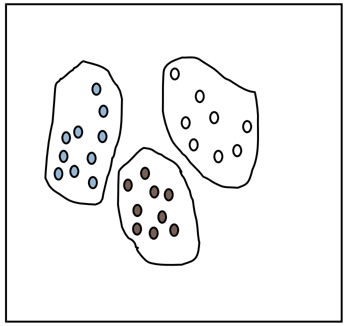
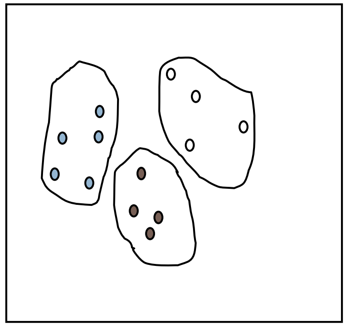
  - Assume the data fits some model, estimate model parameters, store only the parameters, and discard the data (except possible outliers)
  - Ex.: Log-linear models—obtain value at a point in *m*-D space as the product on appropriate marginal subspaces
- **Non-parametric** methods
  - Do not assume models
  - Major families: histograms, clustering, sampling, …



Histogram



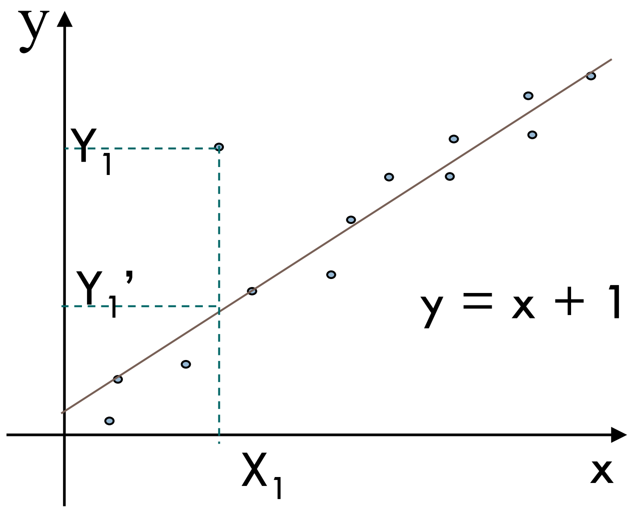
Clustering on the Raw Data



Stratified Sampling

Parametric Data Reduction: Regression Analysis

- Regression analysis: A collective name for techniques for the modeling and analysis of numerical data consisting of values of a ***dependent variable*** (also called ***response variable*** or *measurement*) and of one or more *independent variables* (also known as ***explanatory variables*** or ***predictors***)



- Used for prediction (including forecasting of time-series data), inference, hypothesis testing, and modeling of causal relationships

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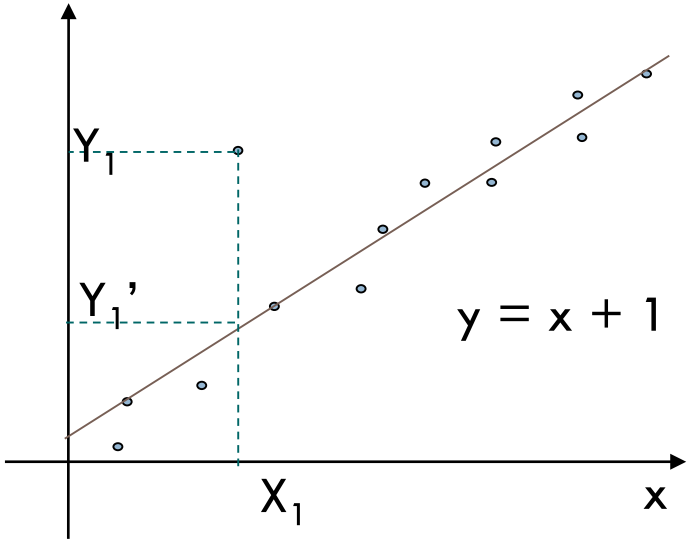

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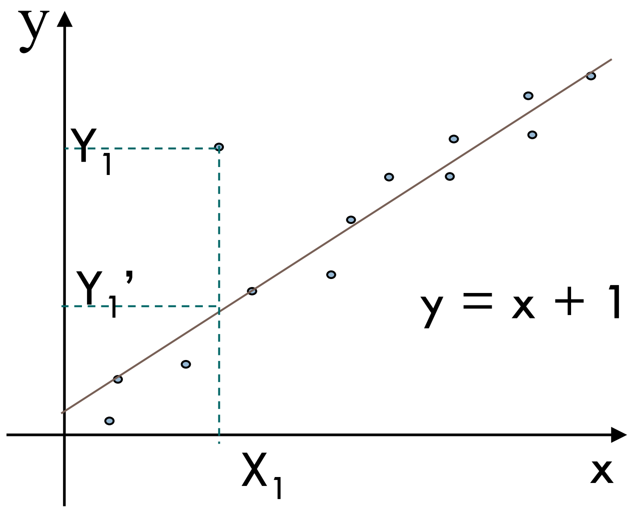
- Most commonly the best fit is evaluated by using the ***least squares method***, but other criteria have also been used



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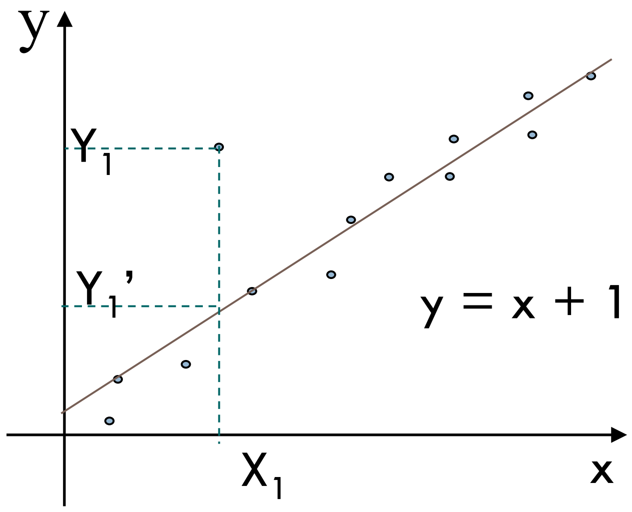

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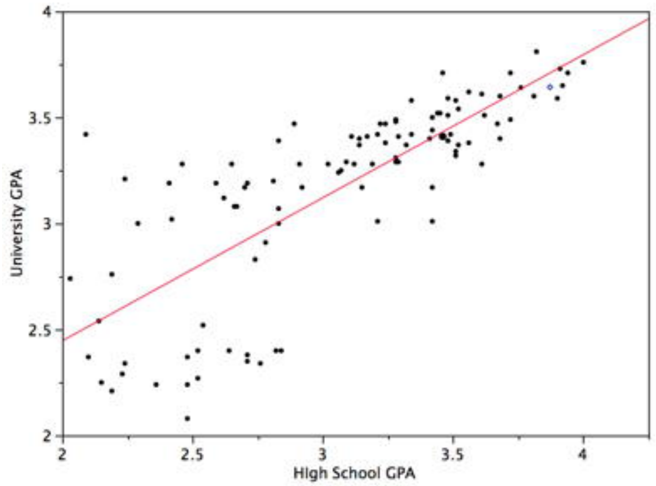
Linear and Multiple Regression

- <u>Linear regression</u>: $Y = w\,X + b$
  - Data modeled to fit a straight line
  - Often uses the least-square method to fit the line
  - Two regression coefficients, *w* and *b*, specify the line and are to be estimated by using the data at hand
  - Using the least squares criterion to the known values of $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$



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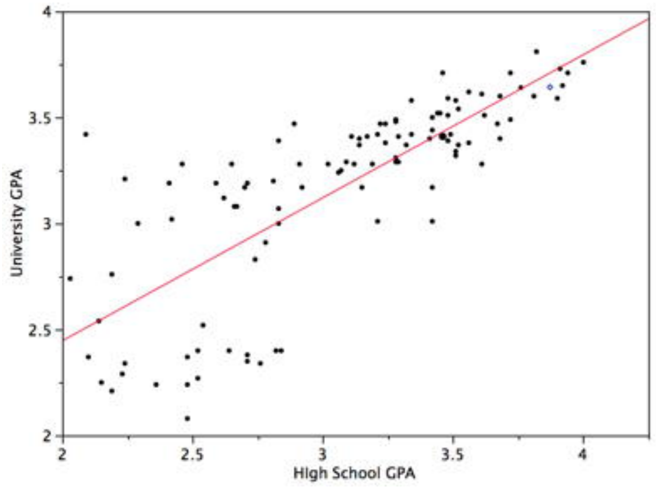
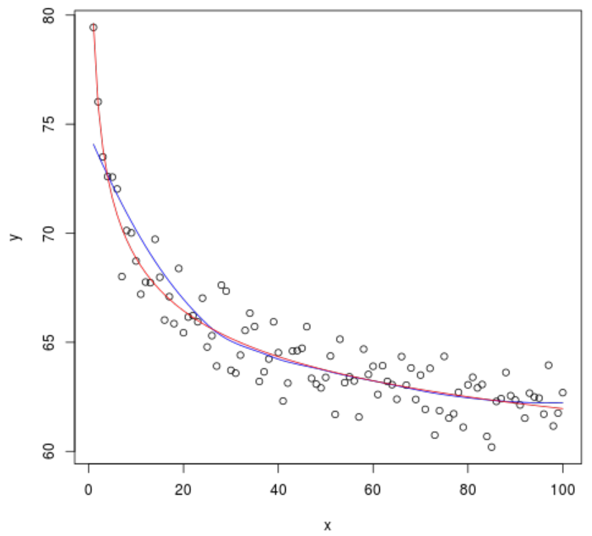
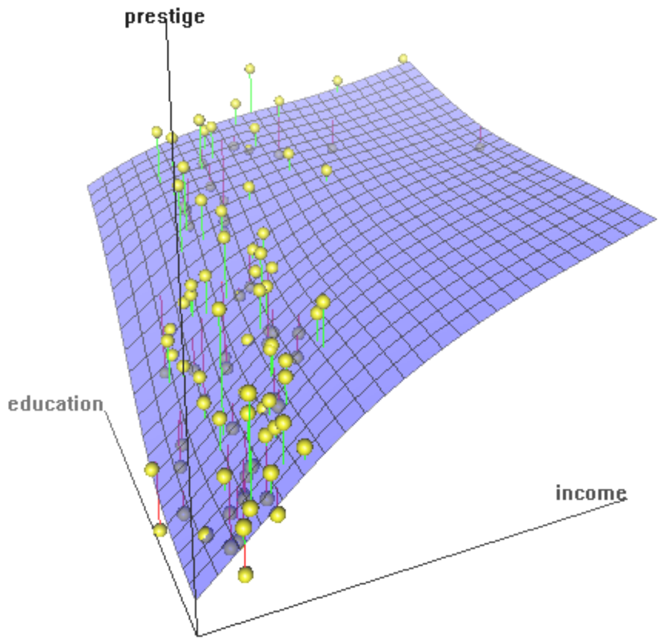


- <u>Nonlinear regression</u>:
  - Data are modeled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables
  - The data are fitted by a method of successive approximations



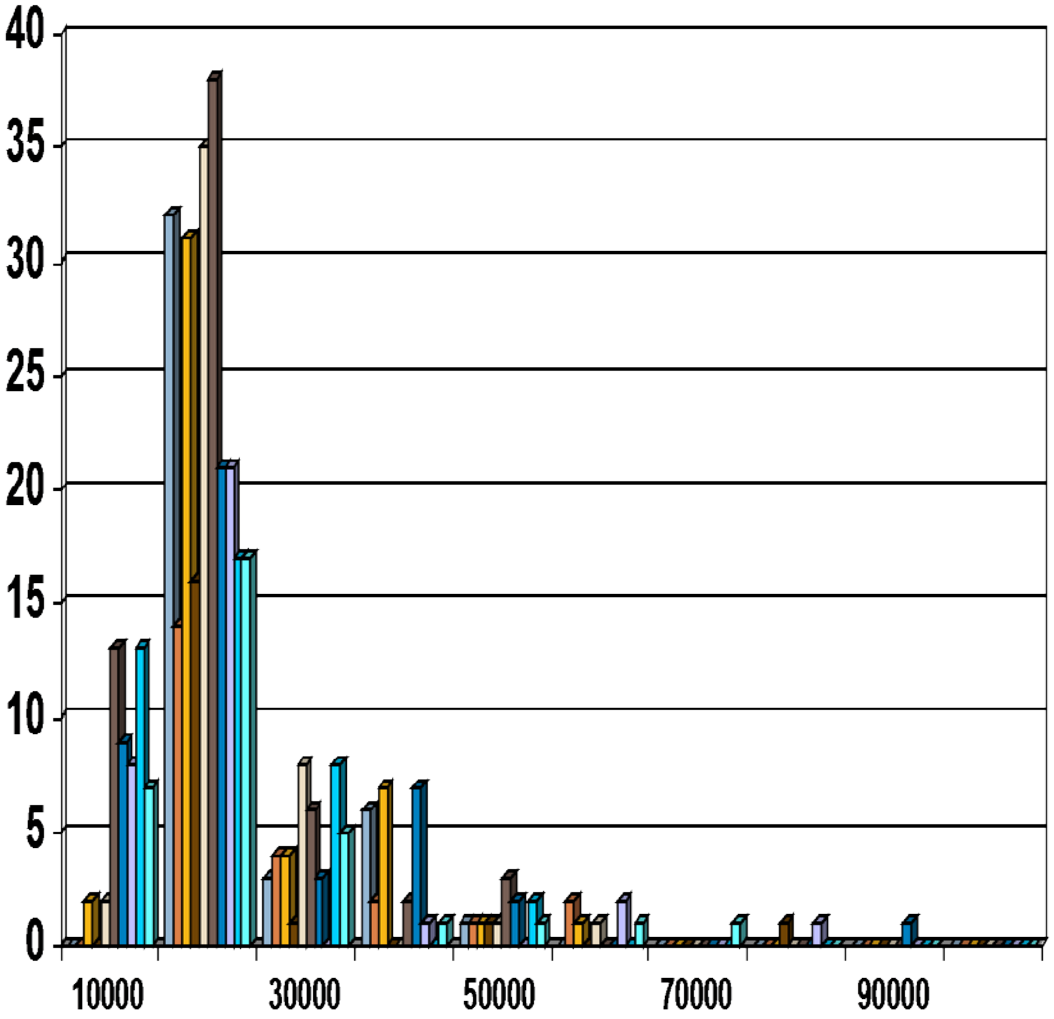
Multiple Regression

- <u>Multiple regression</u>: $Y = b_0 + b_1 X_1 + b_2 X_2$
  - Allows a response variable Y to be modeled as a linear function of multidimensional feature vector
  - Many nonlinear functions can be transformed into the above



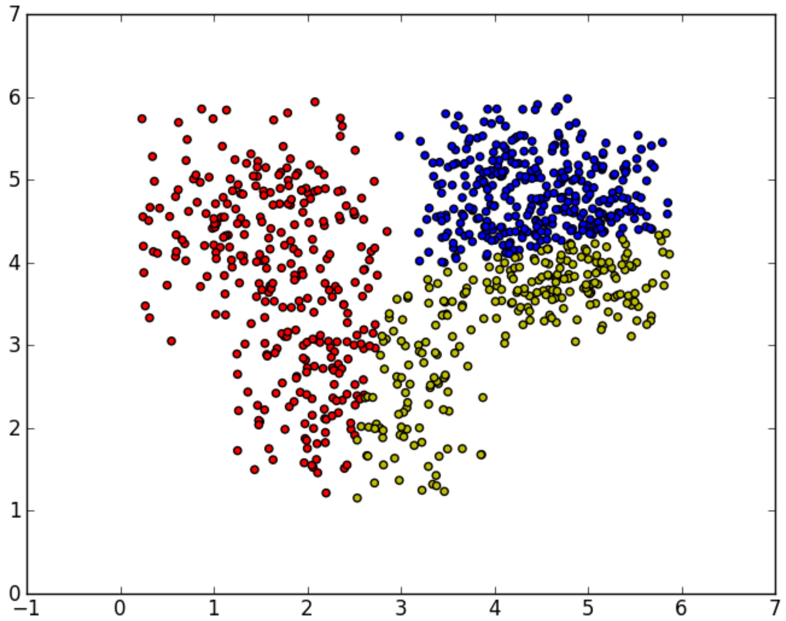
Histogram Analysis

- Divide data into buckets and store average (sum) for each bucket
- Partitioning rules:
  - Equal-width: equal bucket range
  - Equal-frequency (or equal-depth)



Clustering

- Partition data set into clusters based on similarity, and store cluster representation (e.g., centroid and diameter) only
- Can be very effective if data is clustered but not if data is "smeared"
- Can have hierarchical clustering and be stored in multi-dimensional index tree structures
- There are many choices of clustering definitions and clustering algorithms
- Cluster analysis will be studied in later this semester

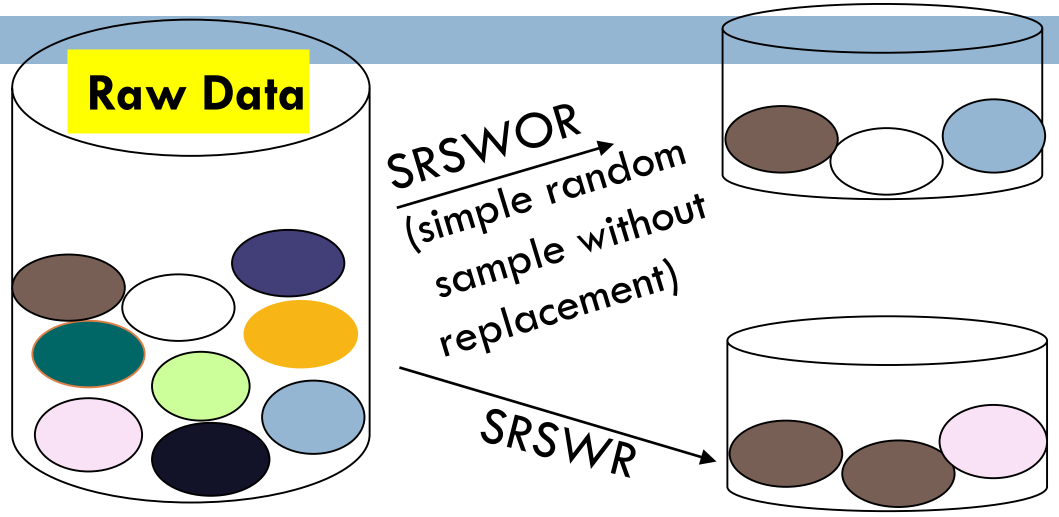


Sampling

- Sampling: obtaining a small sample *s* to represent the whole data set *N*
- Allow a mining algorithm to run in complexity that is potentially sub-linear to the size of the data
- Key principle: **Choose a representative subset of the data**
  - Simple random sampling may have very poor performance in the presence of skew
  - Develop adaptive sampling methods, e.g., stratified sampling:

Types of Sampling

- **Simple random sampling:** equal probability of selecting any particular item
- **Sampling without replacement**
  - Once an object is selected, it is removed from the population
- **Sampling with replacement**
  - A selected object is not removed from the population



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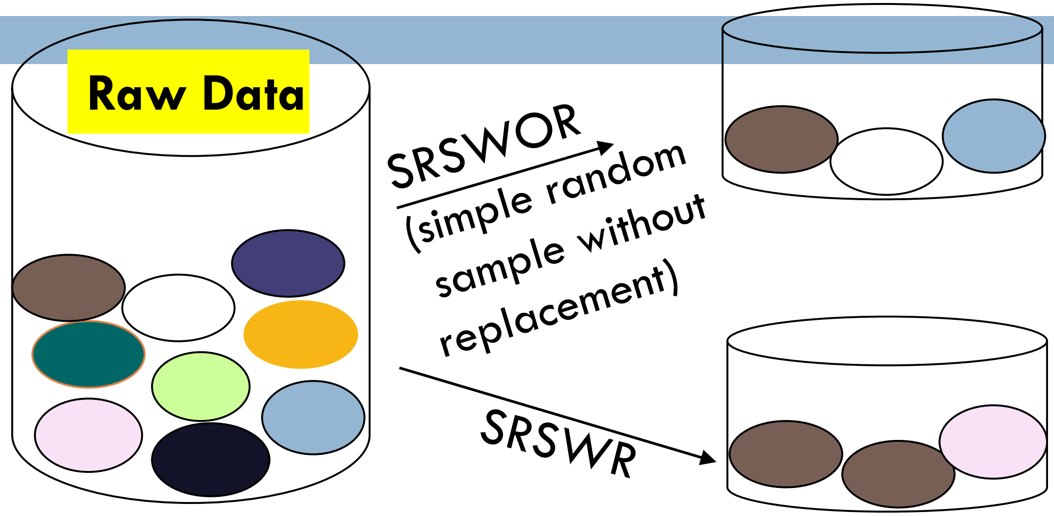
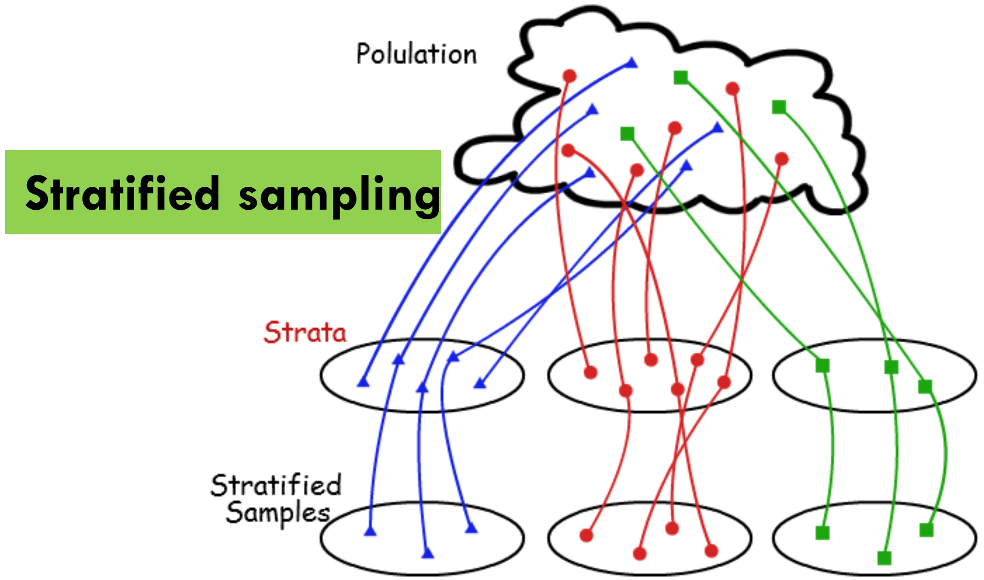
  - Once an object is selected, it is removed from the population
- **Sampling with replacement**
  - A selected object is not removed from the population
- **Stratified sampling**
  - Partition (or cluster) the data set, and draw samples from each partition (proportionally, i.e., approximately the same percentage of the data)





Chapter 3: Data Preprocessing

- Data Preprocessing: An Overview
- Data Cleaning
- Data Integration
- Data Reduction and Transformation
- Dimensionality Reduction
- Summary

Data Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values, s.t. each old value can be identified with one of the new values

Data Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values s.t. each old value can be identified with one of the new values
- Methods
  - Smoothing: Remove noise from data
  - Attribute/feature construction
    - New attributes constructed from the given ones
  - Aggregation: Summarization, data cube construction
  - Normalization: Scaled to fall within a smaller, specified range
    - min-max normalization; z-score normalization; normalization by decimal scaling
  - Discretization: Concept hierarchy climbing

Normalization

- **Min-max normalization:** to [new_min$_A$, new_max$_A$]

$$\boxed{v' = \frac{v - min_A}{max_A - min_A}(new\_max_A - new\_min_A) + new\_min_A}$$

Ex. Let income range \$12,000 to \$98,000 normalized to [0.0, 1.0]

- Then \$73,600 is mapped to $\frac{73{,}600 - 12{,}000}{98{,}000 - 12{,}000}(1.0 - 0) + 0 = 0.716$

Normalization

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- **Z-score normalization** (μ: mean, σ: standard deviation):

$$v' = \frac{v - \mu_A}{\sigma_A}$$

Z-score: The distance between the raw score and the population mean in the unit of the standard deviation

  - Ex. Let μ = 54,000, σ = 16,000. Then,

$$\frac{73{,}600 - 54{,}000}{16{,}000} = 1.225$$

Normalization

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- **Normalization by decimal scaling**

$$v' = v / 10^j$$, Where j is the smallest integer such that Max(|v'|) < 1

Discretization

- Three types of attributes
  - Nominal—values from an unordered set, e.g., color, profession
  - Ordinal—values from an ordered set, e.g., military or academic rank
  - Numeric—real numbers, e.g., integer or real numbers

- Discretization: Divide the range of a continuous attribute into intervals
  - Interval labels can then be used to replace actual data values
  - Reduce data size by discretization
  - Supervised vs. unsupervised
  - Split (top-down) vs. merge (bottom-up)
  - Discretization can be performed recursively on an attribute
  - Prepare for further analysis, e.g., classification

Data Discretization Methods

- Binning
  - Top-down split, unsupervised
- Histogram analysis
  - Top-down split, unsupervised
- Clustering analysis
  - Unsupervised, top-down split or bottom-up merge
- Decision-tree analysis
  - Supervised, top-down split
- Correlation (e.g., χ^2) analysis
  - Unsupervised, bottom-up merge
- Note: All the methods can be applied recursively

Simple Discretization: Binning

- Equal-width (distance) partitioning
  - Divides the range into N intervals of equal size: uniform grid
  - if A and B are the lowest and highest values of the attribute, the width of intervals will be: $W = (B - A)/N$.
  - The most straightforward, but outliers may dominate presentation
  - Skewed data is not handled well

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  - The most straightforward, but outliers may dominate presentation
  - Skewed data is not handled well
- Equal-depth (frequency) partitioning
  - Divides the range into N intervals, each containing approximately same number of samples
  - Good data scaling
  - Managing categorical attributes can be tricky

Example: Binning Methods for Data Smoothing

- Sorted data for price (in dollars): 4, 8, 9, 15, 21, 21, 24, 25, 26, 28, 29, 34
- Partition into equal-frequency (**equi-width**) bins:
  - Bin 1: 4, 8, 9, 15
  - Bin 2: 21, 21, 24, 25
  - Bin 3: 26, 28, 29, 34
- Smoothing by **bin means:**
  - Bin 1: 9, 9, 9, 9
  - Bin 2: 23, 23, 23, 23
  - Bin 3: 29, 29, 29, 29
- Smoothing by **bin boundaries:**
  - Bin 1: 4, 4, 4, 15
  - Bin 2: 21, 21, 25, 25
  - Bin 3: 26, 26, 26, 34

Discretization by Classification & Correlation Analysis

- Classification (e.g., decision tree analysis)
  - Supervised: Given class labels, e.g., cancerous vs. benign
  - Using *entropy* to determine split point (discretization point)
  - Top-down, recursive split
  - Details to be covered in “Classification” sessions

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- Dimensionality Reduction
- Summary

Dimensionality Reduction

- **Curse of dimensionality**
  - When dimensionality increases, data becomes increasingly sparse
  - Density and distance between points, which is critical to clustering, outlier analysis, becomes less meaningful
  - The possible combinations of subspaces will grow exponentially

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- **Dimensionality reduction**
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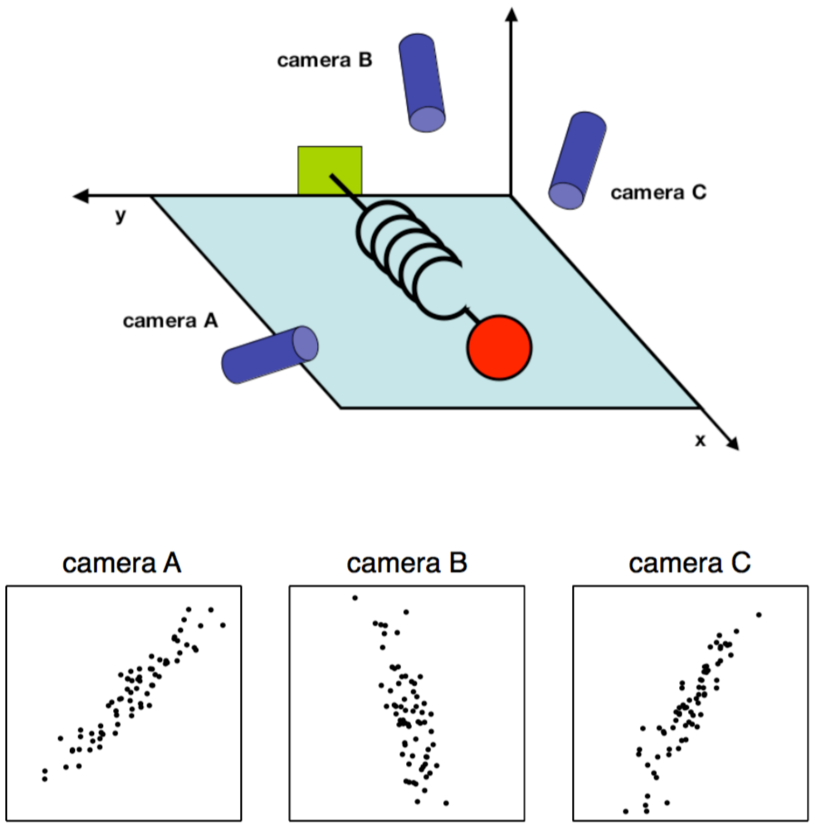
- **Advantages of dimensionality reduction**
  - Avoid the curse of dimensionality
  - Help eliminate irrelevant features and reduce noise
  - Reduce time and space required in data mining
  - Allow easier visualization

Dimensionality Reduction Techniques

- Dimensionality reduction methodologies
  - **Feature selection:** Find a subset of the original variables (or features, attributes)
  - **Feature extraction:** Transform the data in the high-dimensional space to a space of fewer dimensions
- Some typical dimensionality reduction methods
  - Principal Component Analysis
  - Supervised and nonlinear techniques
    - Feature subset selection
    - Feature creation

Principal Component Analysis (PCA)

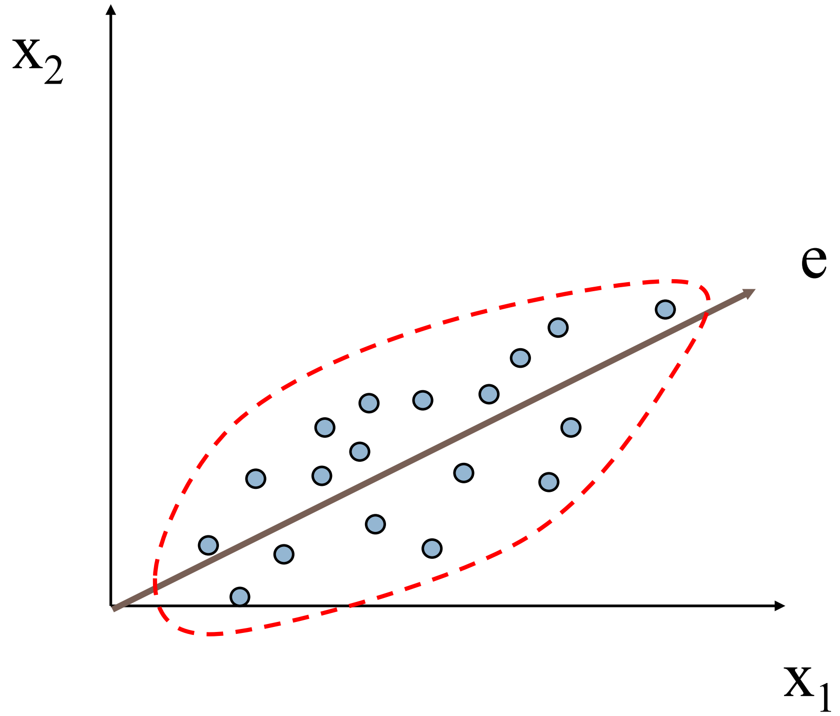
- PCA: A statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called ***principal components***
- The original data are projected onto a much smaller space, resulting in dimensionality reduction
- Method: Find the eigenvectors of the covariance matrix, and these eigenvectors define the new space



Ball travels in a straight line. Data from three cameras contain much redundancy

Principal Components Analysis: Intuition

- Goal is to find a projection that captures the largest amount of variation in data
- Find the eigenvectors of the covariance matrix
- The eigenvectors define the new space



Principal Component Analysis: Details

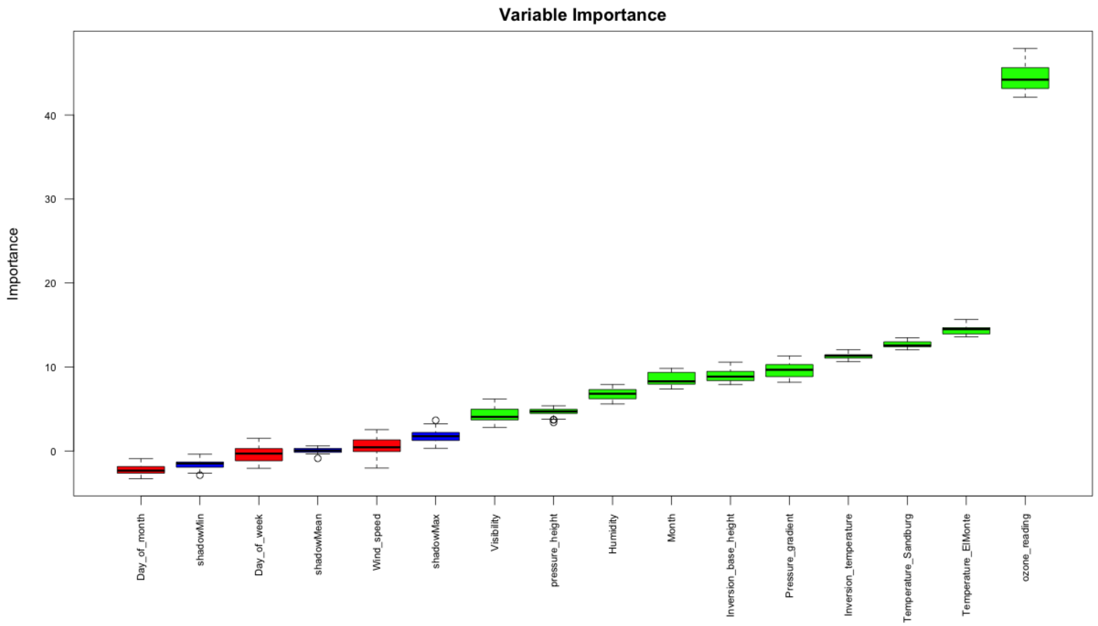
- Let A be an $n \times n$ matrix representing the correlation or covariance of the data.
  - λ is an **eigenvalue** of A if there exists a non-zero vector **v** such that:

    $$A\mathbf{v} = \lambda\,\mathbf{v} \text{ often } \textbf{rewritten as } (A - \lambda I)v = 0$$

- In this case, vector **v** is called an **eigenvector** of A corresponding to λ. For each eigenvalue λ, the set of all vectors **v** satisfying $A\mathbf{v} = \lambda\,\mathbf{v}$ is called the **eigenspace** of A corresponding to λ.

Attribute Subset Selection

- Another way to reduce dimensionality of data
- Redundant attributes
  - Duplicate much or all of the information contained in one or more other attributes
    - E.g., purchase price of a product and the amount of sales tax paid
- Irrelevant attributes
  - Contain no information that is useful for the data mining task at hand
    - Ex. A student's ID is often irrelevant to the task of predicting his/her GPA



Heuristic Search in Attribute Selection

- There are 2^d possible attribute combinations of d attributes
- Typical heuristic attribute selection methods:
  - Best single attribute under the attribute independence assumption: choose by significance tests
  - Best step-wise feature selection:
    - The best single-attribute is picked first
    - Then next best attribute condition to the first, ...
  - Step-wise attribute elimination:
    - Repeatedly eliminate the worst attribute
  - Best combined attribute selection and elimination
  - Optimal branch and bound:
    - Use attribute elimination and backtracking

Attribute Creation (Feature Generation)

- Create new attributes (features) that can capture the important information in a data set more effectively than the original ones
- Three general methodologies
  - Attribute extraction
    - Domain-specific
  - Mapping data to new space (see: data reduction)
    - E.g., Fourier transformation, wavelet transformation, manifold approaches (not covered)
  - Attribute construction
    - Combining features (see: discriminative frequent patterns in Chapter on "Advanced Classification")
    - Data discretization

Summary

- **Data quality**: accuracy, completeness, consistency, timeliness, believability, interpretability
- **Data cleaning**: e.g. missing/noisy values, outliers
- **Data integration** from multiple sources:
  - Entity identification problem; Remove redundancies; Detect inconsistencies
- **Data reduction**
  - Dimensionality reduction; Numerosity reduction; Data compression
- **Data transformation and data discretization**
  - Normalization; Concept hierarchy generation

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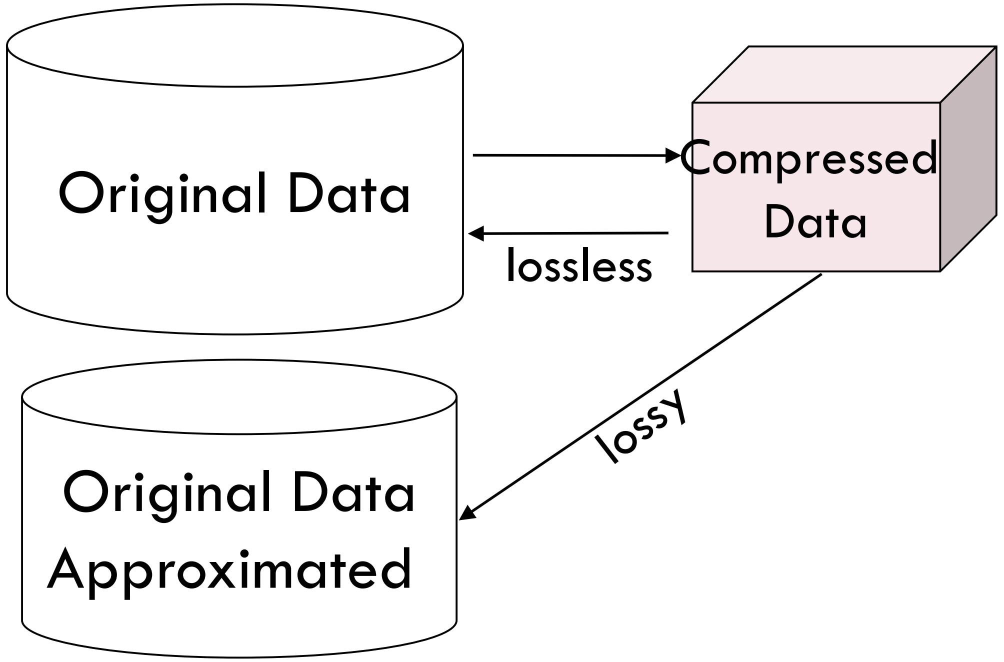
Backup Slides:

Data Compression

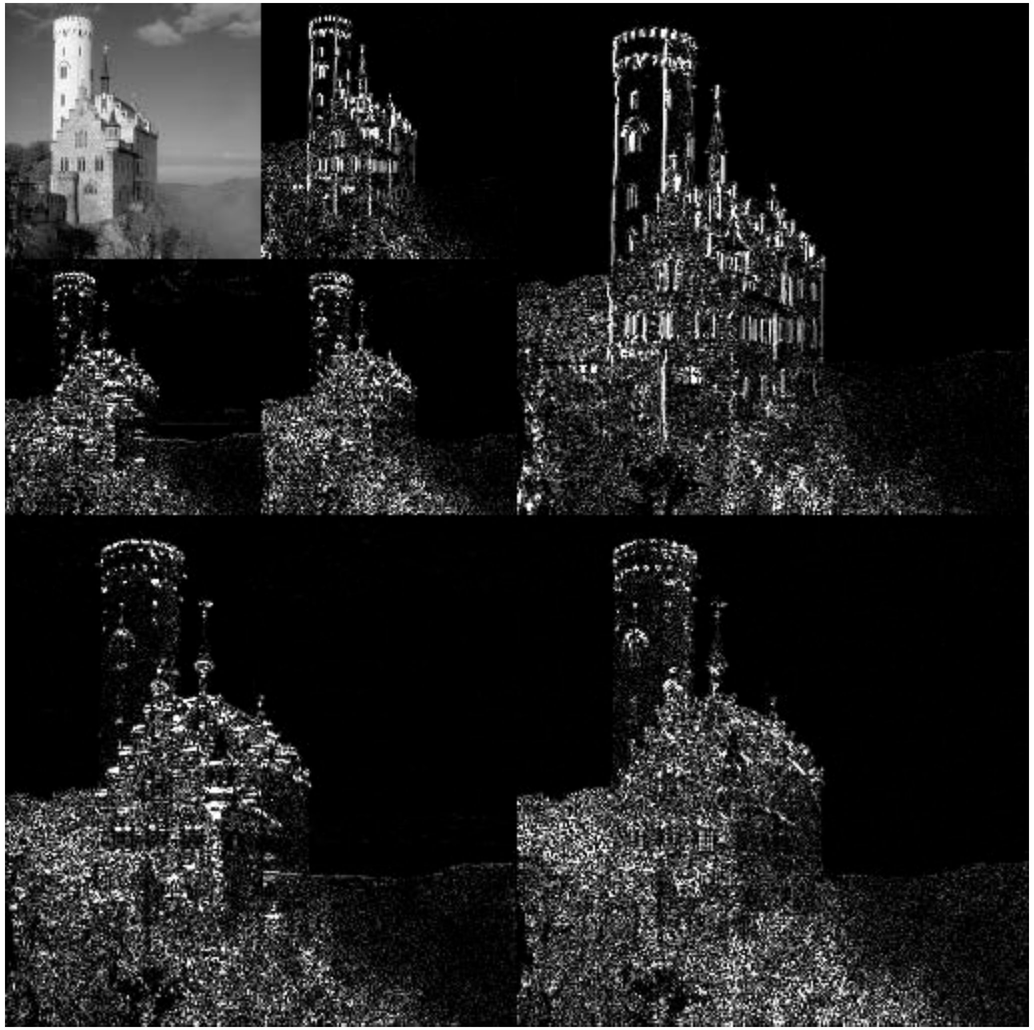
- String compression
  - There are extensive theories and well-tuned algorithms
  - Typically lossless, but only limited manipulation is possible without expansion
- Audio/video compression
  - Typically lossy compression, with progressive refinement
  - Sometimes small fragments of signal can be reconstructed without reconstructing the whole
- Time sequence is not audio
  - Typically short and vary slowly with time
- Data reduction and dimensionality reduction may also be considered as forms of data compression



Lossy vs. lossless compression

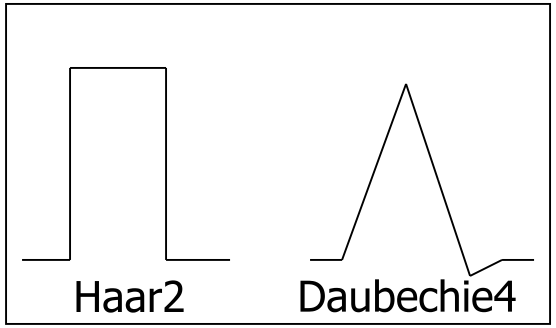
Wavelet Transform: A Data Compression Technique

- Wavelet Transform
  - Decomposes a signal into different frequency subbands
  - Applicable to n-dimensional signals
- Data are transformed to preserve relative distance between objects at different levels of resolution
- Allow natural clusters to become more distinguishable
- Used for image compression



Wavelet Transformation

- Discrete wavelet transform (DWT) for linear signal processing, multi-resolution analysis
- Compressed approximation: Store only a small fraction of the strongest of the wavelet coefficients
- Similar to discrete Fourier transform (DFT), but better lossy compression, localized in space
- Method:
  - Length, L, must be an integer power of 2 (padding with 0's, when necessary)
  - Each transform has 2 functions: smoothing, difference
  - Applies to pairs of data, resulting in two set of data of length L/2
  - Applies two functions recursively, until reaches the desired length



Wavelet Decomposition

- Wavelets: A math tool for space-efficient hierarchical decomposition of functions
- S = [2, 2, 0, 2, 3, 5, 4, 4] can be transformed to $S_\wedge$ = [$2^3/_4$, -$1^1/_4$, $^1/_2$, 0, 0, -1, -1, 0]
- Compression: many small detail coefficients can be replaced by 0's, and only the significant coefficients are retained

Resolution	Averages	Detail Coefficients
8	$[2,\ 2,\ 0,\ 2,\ 3,\ 5,\ 4,\ 4]$	
4	$[2,\ 1,\ 4,\ 4]$	$[0,\ -1,\ -1,\ 0]$
2	$[1\frac{1}{2},\ 4]$	$[\frac{1}{2},\ 0]$
1	$[2\frac{3}{4}]$	$[-1\frac{1}{4}]$

Why Wavelet Transform?

- Use hat-shape filters
  - Emphasize region where points cluster
  - Suppress weaker information in their boundaries
- Effective removal of outliers
  - Insensitive to noise, insensitive to input order
- Multi-resolution
  - Detect arbitrary shaped clusters at different scales
- Efficient
  - Complexity O(N)
- Only applicable to low dimensional data

Concept Hierarchy Generation

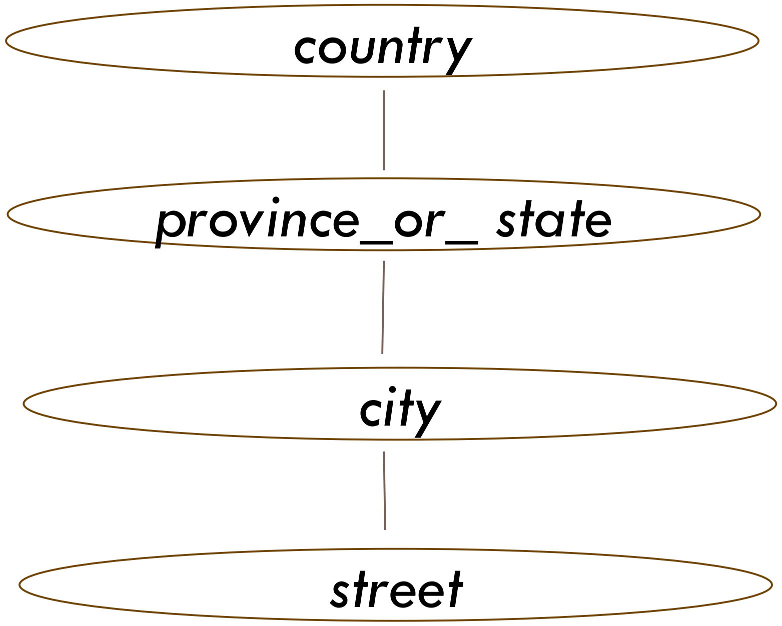
- **Concept hierarchy** organizes concepts (i.e., attribute values) hierarchically and is usually associated with each dimension in a data warehouse
- Concept hierarchies facilitate <u>drilling and rolling</u> in data warehouses to view data in multiple granularity
- Concept hierarchy formation: Recursively reduce the data by collecting and replacing low level concepts (such as numeric values for *age*) by higher level concepts (such as *youth*, *adult*, or *senior*)
- Concept hierarchies can be explicitly specified by domain experts and/or data warehouse designers
- Concept hierarchy can be automatically formed for both numeric and nominal data—For numeric data, use discretization methods shown

Concept Hierarchy Generation for Nominal Data

- Specification of a partial/total ordering of attributes explicitly at the schema level by users or experts
  - *street* < *city* < *state* < *country*
- Specification of a hierarchy for a set of values by explicit data grouping
  - {Urbana, Champaign, Chicago} < Illinois
- Specification of only a partial set of attributes
  - E.g., only *street* < *city*, not others
- Automatic generation of hierarchies (or attribute levels) by the analysis of the number of distinct values
  - E.g., for a set of attributes: {*street, city, state, country*}

Automatic Concept Hierarchy Generation

- Some hierarchies can be automatically generated based on the analysis of the number of distinct values per attribute in the data set
  - The attribute with the most distinct values is placed at the lowest level of the hierarchy
  - Exceptions, e.g., weekday, month, quarter, year



15 distinct values

365 distinct values

3567 distinct values

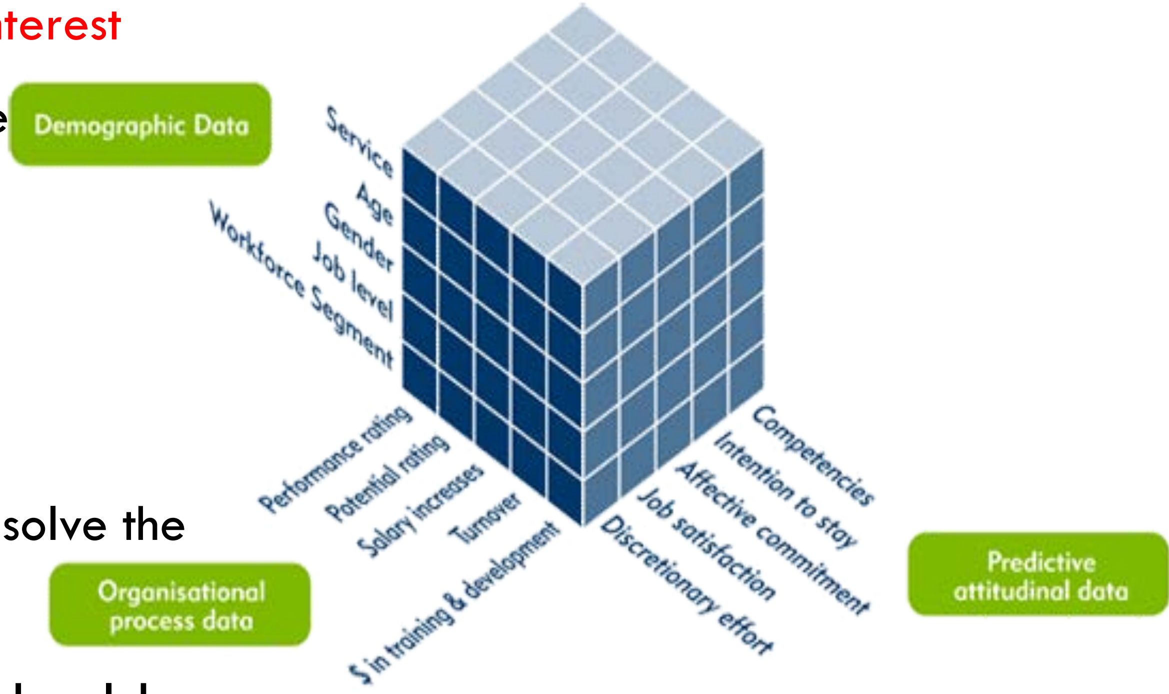
674,339 distinct values

Data Cleaning

- Data in the Real World Is Dirty: Lots of potentially incorrect data, e.g., instrument faulty, human or computer error, and transmission error
  - Incomplete: lacking attribute values, lacking certain attributes of interest, or containing only aggregate data
    - e.g., *Occupation* = " " (missing data)
  - Noisy: containing noise, errors, or outliers
    - e.g., *Salary* = "−10" (an error)
  - Inconsistent: containing discrepancies in codes or names, e.g.,
    - *Age* = "42", *Birthday* = "03/07/2010"
    - Was rating "1, 2, 3", now rating "A, B, C"
    - discrepancy between duplicate records
  - Intentional (e.g., *disguised missing* data)
    - Jan. 1 as everyone's birthday?

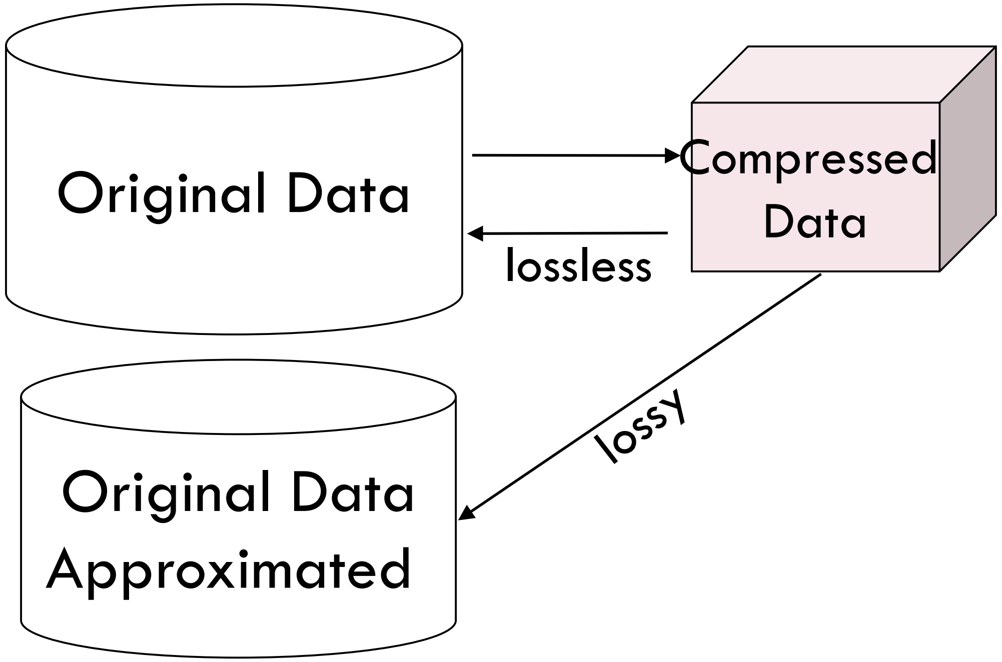
Data Cube Aggregation

- The lowest level of a data cube (base cuboid)
  - The aggregated data for an individual entity of interest
  - E.g., a customer in a phone calling data warehouse
- Multiple levels of aggregation in data cubes
  - Further reduce the size of data to deal with
- Reference appropriate levels
  - Use the smallest representation which is enough to solve the task
- Queries regarding aggregated information should be answered using data cube, when possible



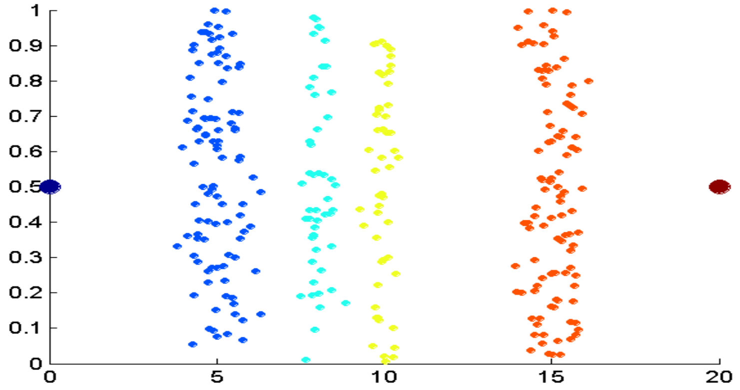
Data Compression

- String compression
  - There are extensive theories and well-tuned algorithms
  - Typically lossless, but only limited manipulation is possible without expansion
- Audio/video compression
  - Typically lossy compression, with progressive refinement
  - Sometimes small fragments of signal can be reconstructed without reconstructing the whole
- Time sequence is not audio
  - Typically short and vary slowly with time
- Data reduction and dimensionality reduction may also be considered as forms of data compression

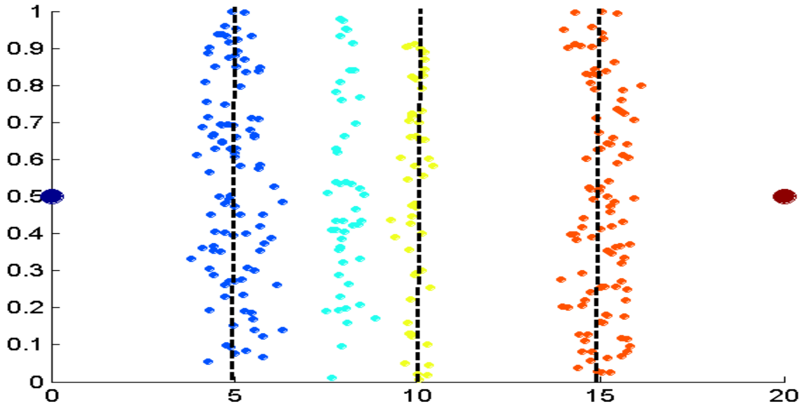


Lossy vs. lossless compression

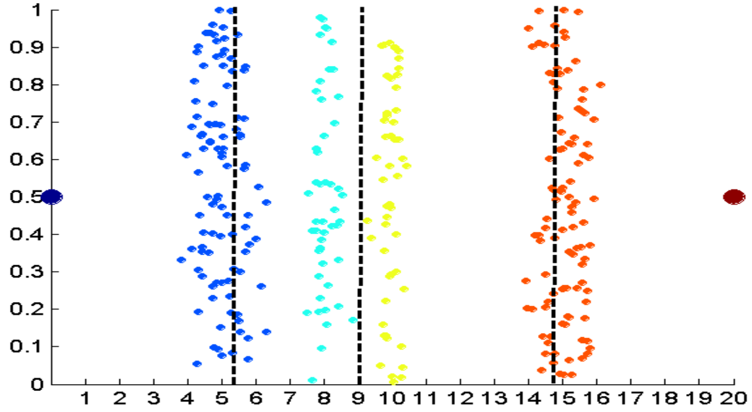
Discretization Without Supervision: Binning vs. Clustering



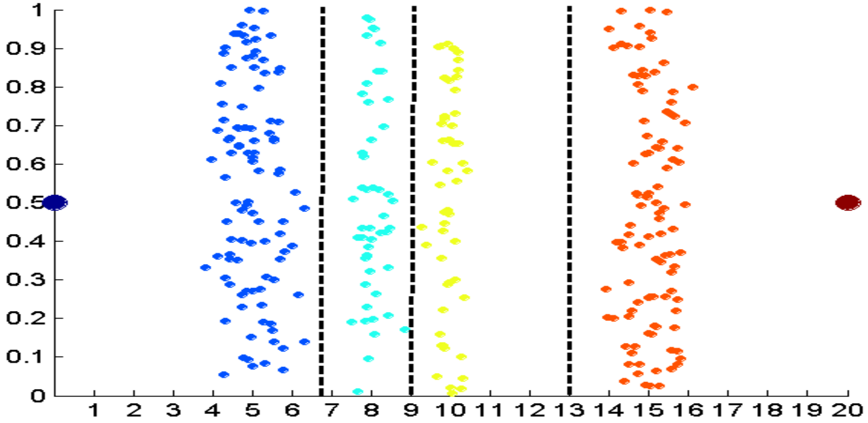
Data



Equal width (distance) binning



Equal depth (frequency) (binning)



K-means clustering leads to better results