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Center for Cognitive Science
The Ohio State University
208 Ohio Stadium East
1961 Tuttle Park Place
Columbus, OH 43210-1102, U.S.A.
PH: 614-292-8200 FX: 614-292-0321
EML: `culicover.1@osu.edu`

Image Segmentation Based on Oscillatory Correlation

DeLiang Wang[†] and David Terman[‡]

[†]Department of Computer and Information Science and Center for Cognitive Science

[‡]Department of Mathematics

The Ohio State University, Columbus, Ohio 43210, USA

Abstract

We study image segmentation on the basis of locally excitatory globally inhibitory oscillator networks (LEGION), whereby the phases of oscillators encode the binding of pixels. We introduce a potential for each oscillator so that only those oscillators with strong connections from their neighborhood can develop high potentials. Based on the concept of potential, a solution to remove noisy regions in an image is proposed for LEGION, so that it suppresses the oscillators corresponding to noisy regions, without affecting those corresponding to major regions. We show analytically that the resulting oscillator network separates an image into several major regions, plus a background consisting of all noisy regions, and illustrate network properties by computer simulation. The network exhibits a natural capacity in segmenting images. The oscillatory dynamics leads to a computer algorithm, which is applied successfully to segmenting real gray-level images. A number of issues regarding biological plausibility and perceptual organization are discussed. We argue that LEGION provides a novel and effective framework for image segmentation and figure-ground segregation.

1. Introduction

The segmentation of a visual scene (image) into a set of coherent patterns (objects) is a fundamental aspect of perception, which underlies a variety of tasks such as image processing, figure-ground segregation, and automatic target recognition. Scene segmentation plays a critical role in the understanding of natural scenes. Although humans perform it with apparent ease, the general problem of image segmentation remains unsolved in sensory information processing. As the technology of single-object recognition becomes more and more advanced in recent years, the demand for a solution to image segmentation is increasing since both natural scenes and manufacturing applications of computer vision are rarely composed of a single object.

Objects appear in a natural scene as the grouping of similar sensory features and the segregation of dissimilar ones. Sensory features are generally taken to be local, and in the simplest case may correspond to single pixels. To approach the problem of scene segmentation, three basic issues must be addressed: What are the cues that determine grouping and segregation? What is the proper representation for the result of segmentation? How are the cues used to give rise to segmentation?

Much is known about sensory cues that are important for segmentation. In particular, Gestalt psychology has uncovered a set of principles guiding the grouping process in the visual domain (Wertheimer 1923; Koffka 1935; Rock and Palmer 1990). We briefly summarize some of the most important principles (see also Rock and Palmer 1990):

- *Proximity*. The closer the features lie to each other, the easier they are to be grouped into the same segment.
- *Similarity*. Features that have similar attributes, such as grayness, color, depth, texture, etc., tend to group together.
- *Common fate*. Features that have similar temporal behavior tend to group together. For instance, a group of features that move coherently (common motion) would form a single object. Notice that common fate may be regarded as one aspect of similarity. We list it separately to emphasize the importance of time as a separate dimension.
- *Connectedness*. A uniform, connected region, such as a spot, line, or more extended area, tends to form a single segment.
- *Good continuation*. A set of features that form a smooth and continuous curve tend to group together.

The above principles concern only the qualities within the input image. Such groupings may be referred to as the bottom-up process. Another important aspect has to do with prior knowledge (memory), i.e., if a set of features belong to the same familiar pattern, they tend to group together. Prior knowledge can strongly influence the grouping process in a top-down fashion, and it is listed in the following.

- *Prior knowledge*.

All of these principles (possibly more) work together to give rise to segmentation. The integration of these differing principles by itself can pose a significant computational problem.

In computer vision algorithms for image segmentation, the result of segmentation can be represented in many ways. However, it is not a trivial task to represent the outcome of segmentation in a neural network. One proposal is naturally derived from the so-called *neuron doctrine* (Barlow 1972), where neurons at higher brain areas are assumed to become more selective and eventually a single neuron represents each single object. This representation is also called the grandmother-cell representation. Multiple objects in a visual scene would be represented by the coactivation of multiple units at some level of the nervous system. This representation faces major theoretical and neurobiological problems (von der Malsburg 1981; Abeles 1991; Singer 1993). Another proposal relies on *temporal correlation* to encode the binding (Milner 1974; von der Malsburg 1981; Abeles 1982). In particular, the correlation theory of von der Malsburg (1981)

asserts that an object is represented by the temporal correlation of the firing activities of the scattered cells that encode different features of the object. Multiple objects are represented by different correlated firing patterns that alternate in time, each corresponding to a single object.

Temporal correlation provides an elegant way to represent the result of segmentation. A special form of temporal correlation is *oscillatory correlation*, where the basic unit is a neural oscillator (see Terman and Wang 1995; Wang and Terman 1995). However, this representation does not, by itself, reveal *how* segmentation is achieved using Gestalt grouping principles. As a matter of fact, despite an extensive body of literature dealing with segmentation using temporal correlation (starting perhaps from von der Malsburg and Schneider 1986), little progress has been made in building successful neural systems for image segmentation. There are two major challenges facing the oscillatory correlation theory. The first challenge is how to achieve fast synchronization within a population of locally coupled oscillators. Most of the models proposed for achieving phase synchrony rely on all-to-all connections (see Sect. 2 for more details). However, as pointed out by Sporns et al. (Sporns *et al.* 1991) and Wang (Wang 1993a), a network with full connections indiscriminately connects all the oscillators which are activated simultaneously by different objects, because the network is dimensionless and loses critical information about geometry. The second challenge is how to achieve fast desynchronization among different groups of oscillators representing distinct objects. This is necessary in order to segment multiple objects simultaneously presented.

We have previously proposed a neural network framework to deal with the problem of image segmentation, called Locally Excitatory Globally Inhibitory Oscillator Networks (LEGION) (Wang and Terman 1995; Terman and Wang 1995). Each oscillator is modeled as a standard relaxation oscillator. Local excitation is implemented by positive coupling between neighboring oscillators and global inhibition is realized by a global inhibitor. LEGION exhibits the mechanism of *selective gating*, whereby oscillators stimulated by the same pattern tend to synchronize due to local excitation and oscillator groups stimulated by different patterns tend to desynchronize due to global inhibition (Wang and Terman 1995; Terman and Wang 1995). We have proven that, with the selective gating mechanism, LEGION rapidly achieves both synchronization within groups of oscillators that are stimulated by connected regions and desynchronization between different groups. In sum, LEGION provides an elegant solution to both challenges outlined above.

In this paper, we study LEGION for segmenting real images. Before we demonstrate image segmentation, the original version of LEGION needs to be extended to handle images with many tiny (noisy) regions. One such example is shown in Fig. 1, where three objects with a noisy background form a visual image. Without extension, LEGION would treat each region, no matter how small it is, as a separate segment. Thus, it would lead to many tiny fragments. We call this problem *fragmentation*. Because different segments alternate in time, a large number of fragments slow down the segmentation process drastically. Another and more serious problem is that it is difficult to choose parameters so that LEGION is able to achieve more than several (5 to 10) segments (Terman and Wang 1995). Noisy fragments may, therefore, compete with major image regions for becoming segments, so that it may not be possible to extract all of the major segments from an image. The problem of fragmentation is solved by introducing a concept of potential for each oscillator. The stimulated oscillators that receive significant input from their neighborhoods will develop high potentials, while other stimulated oscillators will not. Shortly after they are stimulated, low-potential oscillators that cannot be recruited by high-potential ones will cease oscillating. The extended dynamics is fully analyzed, and the resulting LEGION network is applied to gray-level images and yields successful segmentation.

In the next section we review prior work relevant to image segmentation and neural networks, particularly neural models using the representation of oscillatory correlation. In Section 3, our model is described in detail, and it is analyzed in Section 4. In Section 5, computer simulations of the extended LEGION network are presented. Section 6 presents the segmentation results on real

images. Further discussions concerning our approach are given in Section 7. Finally, Section 8 concludes our exposition.

2. Related Work

2.1 Image Segmentation Algorithms

Due to its critical importance for computer vision, image segmentation has been studied extensively. Many techniques have been invented (for reviews of the subject see Zucker 1976; Haralick 1979; Lowe 1985; Haralick and Shapiro 1985; Sarkar and Boyer 1993b; Pal and Pal 1993). Broadly speaking, there are three categories of algorithms: pixel classification, edge-based organization, or region-based segmentation. A simple classification technique is thresholding: a pixel is assigned a specific label if some measure of the pixel passes a certain threshold. This idea can be extended to a complex form including multiple thresholds which are determined by pixel histograms (Kohler 1981). This way, a region of pixels is grouped together if its pixels fall between two threshold values. Some learning techniques may be introduced to make classification more flexible (see Uchiyama and Arbib 1994, for example). Edge-based techniques generally start with an edge-detection algorithm, which is followed by grouping edge elements into rectilinear or curvilinear lines. These lines are then grouped into boundaries that can be used to segment images into various regions (see, for example, Geman *et al.* 1990; Sarkar and Boyer 1993a; Foresti *et al.* 1994). Finally, region-based techniques operate directly on regions. A classical method is region growing/splitting (or split-and-merge, see Horowitz and Pavlidis 1976; Zucker 1976; Pavlidis 1977; Adams and Bischof 1994), where iterative steps are taken to grow (split) pixels into a connected region if all the pixels in the region satisfy some conditions. One such condition is that the distance between the minimum and the maximum pixel values is within a pre-defined range. Most of the techniques use one or more Gestalt grouping principles that emphasize relationships among object components (Mohan and Nevatia 1989; Sarkar and Boyer 1993b).

One of the apparent deficits with these algorithms is their iterative (serial) nature (Liou *et al.* 1991). There are some recent algorithms which are partially parallel. In Sha'ashua and Ullman (1988), a globally consistent curve structure is detected using a locally connected network. In Liou *et al.* (1991), a parallel technique is used to search a partition space. In Mohan and Nevatia (1992), part of the segmentation process is performed by a neural network for cost optimization. A similar approach is taken by Manjunath and Chellappa (1993), who use a competitive network for reducing noise and illumination effects. A parallelizable inference network based on Bayesian statistics is used by Sarkar and Boyer for detecting a set of structures, such as ellipses, circles, and ribbons (Sarkar and Boyer 1993a).

Most of these techniques rely on domain-specific heuristics to perform segmentation, and no unified computational framework exists to explain the general phenomenon of scene segmentation (Haralick and Shapiro 1985). The problem of scene segmentation is computationally hard (Gurari and Wechsler 1982), and largely regarded unsolved.

2.2 Neural Network Efforts

Neural networks have proven to be a successful approach to pattern recognition (Schalkoff 1992; Wang 1993b). Unfortunately, little work has been devoted to scene segmentation which is generally regarded as part of preprocessing (often meaning manual segmentation). Scene segmentation is a particularly challenging task for neural networks, partly because traditional neural networks lack the representational power for encoding multiple objects simultaneously. The two most popular network architectures, namely multilayer perceptrons and associative memories, both require learning or memory of patterns first, whereas scene segmentation, except that based on prior knowledge, seems to be an innate and immediate process, thus difficult to be formulated in these architectures.

Sejnowski and Hinton proposed a scheme for separating a figure from a background using the Boltzmann machine algorithm (Sejnowski and Hinton 1987). Their method may reverse the figure and the background by shifting top-down input (they call it attention). Mozer *et al.* (1992) presented an interesting method of using multilayer perceptrons for scene segmentation, in which each unit is extended to code both intensity and phase. Grossberg and Wyse (1991) proposed a model for image segmentation, based on the contour detection model of Grossberg and Mingolla (1985). The model uses filling-in (dye-spreading) to restore regions from closed contours. Computer simulations implementing some of the ideas proposed in Grossberg and Mingolla (Grossberg and Mingolla 1985) have been presented by Gove *et al.* (in press). However, all of these methods were tested only on small synthetic images, and it is not clear how they can be extended to handle real images.

Since neural networks can be readily applied to classification tasks, they are applicable to segmentation based on pixel classification. In particular, Kohonen's self-organizing maps have been used for segmentation (Kohonen 1995). Koh *et al.* (1995) have developed a system based on self-organizing maps to segment range images. Raghu *et al.* (1995) have studied texture classification using a self-organizing map followed by a multilayer perceptron. A primary drawback of these methods is that the number of segments (objects) is assumed to be known *a priori*.

Because temporal (oscillatory) correlation offers an elegant way of representing multiple objects in neural networks (von der Malsburg and Schneider 1986), most of the neural network efforts on image segmentation have centered around this theme. In particular, the discovery of synchronous oscillations in the visual cortex has triggered much interest in exploring oscillatory correlation to solve the problems of segmentation and figure-ground segregation (Wang *et al.* 1990; Baldi and Meir 1990; Sompolinsky *et al.* 1991; Sporns *et al.* 1991; von der Malsburg and Buhmann 1992; Hummel and Biederman 1992; Murata and Shimizu 1993; Schillen and König 1994). One type of model uses all-to-all connections to reach synchronization (Wang *et al.* 1990; Baldi and Meir 1990; Sompolinsky *et al.* 1991; von der Malsburg and Buhmann 1992). As explained in Sect. 1, these models cannot extend very far in solving the segmentation problem because fundamental information concerning the geometry among sensory features is lost. Another type of model uses lateral connections to reach synchrony (Sporns *et al.* 1991; Hummel and Biederman 1992; Murata and Shimizu 1993; Schillen and König 1994). Unfortunately, it is unclear to what extent these oscillator networks can synchronize on the basis of local connectivity since no analysis is given and only simulation results on small networks are provided. Moreover, recent insights into the contrasting behavior between sinusoidal and relaxation oscillators makes clear that sinusoid-typed oscillators, which encompass most of the oscillator models used, have severe limitations to support fast synchronization (Wang 1995; Terman and Wang 1995; Somers and Kopell in press). In fact, in all of the above models, nothing close to a real image has ever been used for testing these models.

3. Model Description

In this section, we provide the precise definition of our model. In Sect. 3.1 we describe the model in detail, which will be studied in this paper. In Sect. 3.2 we give some alternative definitions that can be used without altering model dynamics.

3.1 Model Definition

The building block of LEGION, a single oscillator i , is defined as a feedback loop between an excitatory unit x_i and an inhibitory unit y_i :

$$x'_i = 3x_i - x_i^3 + 2 - y_i + I_i H(p_i + \exp(-\alpha t) - \theta) + S_i + \rho \quad (1a)$$

$$y'_i = \varepsilon (\gamma(1 + \tanh(x_i/\beta)) - y_i) \quad (1b)$$

Here H stands for the Heaviside step function, which is defined as $H(v) = 1$ if $v \geq 0$ and $H(v) = 0$ if $v < 0$. I_i represent external stimulation which is assumed to be applied at time 0, and S_i denotes the coupling from other oscillators in the network. ρ denotes the amplitude of Gaussian noise, the mean of which is set to $-\rho$. The negative mean is used to reduce the chance of self-generating oscillations, which will become clear in the next paragraph. The noise term is introduced for two purposes. The first one is obvious: to test the robustness of the system. The second one, perhaps more important, is to play an active role in separating different input patterns (for more discussions see Sect. 4 and Terman and Wang 1995).

The parameter ε is a small positive number. Hence (1), without any coupling or noise and with constant stimulation, corresponds to a standard relaxation oscillator. The x -nullcline of (1) is a cubic curve, while the y -nullcline is a sigmoid function. If $I > 0$ and $H = 1$, these curves intersect along the middle branch of the cubic when β is small. In this case, we call the oscillator *enabled* (see Fig. 2A). It produces a stable periodic orbit for all sufficiently small values of ε . The periodic solution alternates between *silent* and *active* phases of near steady-state behavior. As shown in Fig. 2A, the silent and the active phases correspond to the left L and the right R branches of the cubic, respectively. The transitions between the two phases occur rapidly (thus referred to as *jumping*). The parameter γ is used to control the ratio of the times that the solution spends in these two phases. For a larger value of γ , the solution spends a shorter time in the active phase. If $I \leq 0$ and $H = 1$, the nullclines of (1) intersect at a stable fixed point along the left branch of the cubic (see Fig. 2B). In this case (1) produces no periodic orbit, and the oscillator is referred to as *excitable*, indicating the oscillator has not yet been but can be excited by stimulation. An excitable oscillator may be oscillatory if it receives, through the term S , large enough coupling from other oscillators. Because of this dependency on external stimulation, the oscillations are stimulus-dependent. We say that the oscillator is *stimulated* if $I > 0$, and *unstimulated* if $I \leq 0$.

The parameter β specifies the steepness of the sigmoid function, and is chosen to be small. The oscillator model (1) may be interpreted as a model for the spiking behavior of a single neuron, the envelope of a bursting neuron, or a mean field approximation to a network of excitatory and inhibitory binary neurons (Buhmann 1989; Sporns *et al.* 1989). The primary difference between (1) and the model in Terman and Wang (1995) is the introduction of the Heaviside function in which $\alpha > 0$ and $0 < \theta < 1$. The parameter α is chosen to

be $O(\varepsilon)$ so that the exponential function in (1a) decays on a slow time scale. It is the Heaviside term which allows the network to distinguish between major blocks and noisy fragments. The basic idea is that a major block must contain at least one oscillator, denoted as a *leader*, which lies in the center of a large homogeneous region. This oscillator will be able to receive large lateral excitation from its neighborhood. A noisy fragment does not contain such an oscillator. The variable p_i in (1a) determines whether or not an oscillator is a leader. It is referred to as the *potential* of the oscillator i . We assume that p_i satisfies the differential equation:

$$p'_i = \lambda (1 - p_i) H\left[\sum_{k \in N_1(i)} T_{ik} H(x_k - \theta_x) - \theta_p\right] - \mu p_i \quad (2)$$

Here $\lambda > 0$, T_{ik} is the *permanent* connection weight (explained later) from oscillator k to i , and $N_1(i)$ is some neighborhood of i , called the *potential neighborhood*. Note that the outer Heaviside function in (2) equals 1 if the weighted sum oscillator i receives from $N_1(i)$ exceeds the threshold

θ_p . Hence, if this weighted sum exceeds the threshold θ_p , p_i approaches 1. If this weighted sum is below θ_p , p_i relaxes to 0 on a time scale determined by μ , which is chosen to be $O(\varepsilon)$ resulting in a slow time scale. It follows that p_i can only exceed the threshold θ in (1a) if i is able to receive a large enough lateral excitation from its potential neighborhood.

In order to develop a high potential, it is not sufficient that a large number of neighbors of i are oscillatory. They must also have a certain degree of synchrony in their oscillations. In particular, they must all exceed the threshold θ_x at the same time in their oscillations. Although (2) appears complicated, it should be clear by now that the equation and the idea behind it are quite straightforward.

The purpose of introducing the potential is that an oscillator with a high potential can *lead* the activation of an oscillator block corresponding to an object. Though it is not needed that a high-potential oscillator be stimulated, it must be stimulated in order to play the role of leading an oscillator block; otherwise, the oscillator will stay excitable according to (1). Thus, we require that a leader be always stimulated. More formally, an oscillator i is defined as a leader if $p_i \geq \theta$ and i is stimulated. The potential of every oscillator is initialized to zero.

The network we study for image segmentation is two dimensional. Each oscillator has a permanent connection with the oscillators in its potential neighborhood. Figure 3 shows the simplest case of permanent connectivity, where an oscillator is connected only with its four immediate neighbors except on the boundaries where no wrap-around is used. Such connectivity forms a 2-D grid. In general, however, $N_1(i)$ should be larger, and the permanent connection weights should take on the form of a Gaussian distribution, i.e., the weight between two oscillators should fall off exponentially.

The coupling term S_i in (1) is given by

$$S_i = \sum_{k \in N_2(i)} W_{ik} H(x_k - \theta_x) - W_z H(z - \theta_{xz}) \quad (3)$$

where W_{ik} is the *dynamic* connection weight from k to i , and $N_2(i)$ is another set of neighboring oscillators of i , called the *coupling neighborhood*. Unlike the potential neighborhood, the coupling neighborhood affects the activity of an oscillator directly.

Why do we need two neighborhoods? $N_1(i)$ determines whether i can become a leader, while $N_2(i)$ determines which oscillators can affect the activity of i . In general, we want $N_1(i)$ to be relatively large, so that only a sizable, homogeneous region can produce leaders. Figure 4 illustrates this point. With the size of $N_1(i)$ in Fig. 4, it is easy to select θ_p so that only the three major regions can produce leaders. Unlike the oscillators lying in the three major regions, the oscillator whose $N_1(i)$ lies in the upper-left part of the figure, for example, has much fewer stimulated oscillators within $N_1(i)$. Such a choice of $N_1(i)$ is desirable since noisy regions are not supposed to produce leaders. On the other hand, if $N_2(i)$ is the same as $N_1(i)$, then it is difficult to distinguish, for example, the two oscillators in Fig. 4, one lying inside the tree-like object but near the border and another one lying outside of the object but also near the border, in terms of which one should join the tree-like segment. This troubling situation can be resolved by using a smaller $N_2(i)$. As illustrated in the figure, the choice of a small $N_2(i)$ makes it clear which of the two oscillators lies inside the tree-like object. This analysis leads to our following assumption

$$N_2(i) \subseteq N_1(i)$$

Now let us explain the use of permanent and dynamic connection weights. To facilitate synchrony and desynchrony (more discussions later), we assume that there are two kinds of synaptic weight (link) between two oscillators. The permanent weight, or T_{ik} , embodies the hardwired structure of a network, and does not change¹. On the other hand, the dynamic weight, or W_{ik} , rapidly changes. W_{ik} is formed on the basis of T_{ik} according to the mechanism of dynamic normalization (Wang 1995). Dynamic normalization was previously defined as a two-step procedure: First update dynamic links and then normalization (Wang 1995; Terman and Wang 1995). There are different ways to realize such normalization. In the following, we give one way to implement dynamic normalization in differential equations,

$$u_i' = \eta (1 - u_i) I_i - \nu u_i \quad (4a)$$

$$W_{ik}' = W_T T_{ik} u_i u_k - W_{ik} \sum_{j \in N_2(i)} T_{ij} u_i u_j \quad (4b)$$

The function u_i measures whether oscillator i is stimulated, and it is initialized to 0. The parameter η determines the rate of updating u_i , and it is chosen so that η is $O(1)$ with respect to ε . When $I_i > 0$, $u_i \rightarrow 1$ quickly; otherwise when $I_i = 0$, $u_i = 0$. For this equation we assume $I_i = 0$ if oscillator i is unstimulated (otherwise it is easy to enforce this by applying a step function on I_i). The parameter ν is chosen to be $O(\varepsilon)$, so that u_i slowly relaxes back to 0 after the external stimulus is withdrawn.

We assume that W_{ik} are initialized to 0 for all i and k . It is easy to see that if oscillator i is unstimulated, W_{ik} remains to be 0 for all k , and if oscillator k is unstimulated $W_{ik} = 0$ for all i . Otherwise, if $u_i = 1$ and $u_k = 1$ for at least one $k \in N_2(i)$, then at equilibrium,

$$W_{ik} = \frac{W_T T_{ik} u_i u_k}{\sum_{j \in N_2(i)} T_{ij} u_i u_j} \quad \text{and} \quad \sum_{k \in N_2(i)} W_{ik} = W_T$$

Thus the total dynamic weights converging to a single oscillator equals W_T , which gives the desired normalization. Notice that dynamic weights, not permanent weights, participate in determining S_i (see (3)). Weight normalization of this type is commonly used in competitive learning networks (von der Malsburg 1973; Goodhill and Barrow 1994). Notice that W_{ik} can be properly set up in one step at the beginning based on external stimulation, which should be useful for engineering applications.

It should be mentioned that weight normalization is not a necessary condition for the selective gating mechanism to work. This conclusion has been established previously (Terman and Wang 1995). With normalized weights, however, the quality of synchronization within each oscillator block is better (Terman and Wang 1995). In Sect. 3.2, we give an alternative system definition

¹ To be more precise, a permanent weight may reflect the traces of long-term memory and thus may change on a very slow time scale compared to a dynamic link.

without weight normalization. Indeed, in an algorithmic version of LEGION to be introduced in Sect. 6 for handling gray-level images, no weight normalization is employed.

In (3), W_z is the weight of inhibition from the global inhibitor z , whose activity, also denoted by z , is defined as

$$z' = \phi(\sigma_\infty - z) \quad (5)$$

where $\sigma_\infty = 1$ if $x_i \geq \theta_{zx}$ for at least one oscillator i , and $\sigma_\infty = 0$ otherwise. Hence θ_{zx} represents another threshold, and it is chosen so that only an oscillator jumping to the active phase can trigger the global inhibitor. If σ_∞ equals 1, $z \rightarrow 1$. The parameter ϕ represents the rate at which the inhibitor reacts to the stimulation from the oscillator network.

The introduction of a potential provides a solution to the problem of fragmentation. There is an initial period when the term $\exp(\alpha t)$ exceeds the threshold θ . During this period, every stimulated oscillator is enabled. This allows the leaders to receive sufficient lateral excitation so that they can achieve a high potential. After this initial period, the only oscillators which can jump up without stimulation from other oscillators are the leaders. When a leader jumps up, it spreads its activity to other oscillators within its own block, so they can also jump up. These oscillators are referred to as *followers*. Oscillators not in this block are prevented from jumping up, because of the global inhibitor. The oscillators which belong to the noisy fragments will not be able to jump up beyond the initial period, because these oscillators will not be able to develop a sufficiently high potential by themselves and they cannot be recruited by leaders. These oscillators are referred to as *loners*.

In order to be oscillatory beyond the initial time period, an oscillator must either be a leader or a follower. This indicates that the oscillator is not part of a noisy fragment, because noisy fragments in an image tend to be small and isolated (see Fig. 4). The collection of all noisy regions whose corresponding oscillators are loners is called the *background*, which is not a uniform region and generally discontinuous.

The global inhibitor plays the same role of desynchronization as it did before (Terman and Wang 1995; Wang and Terman 1995). Thus, this extended LEGION network can still be characterized as local cooperation via excitatory coupling among neighboring oscillators and global competition via the global inhibitor. See Terman and Wang (1995) and Wang and Terman (1995) for more discussions on other aspects of this model.

3.2 Alternative Definitions

There are alternative ways of defining the model without affecting its essential dynamics. Here we give some alternatives. First, the product of external stimulation and the Heaviside function in (1a) can be replaced by a sum,

$$x_i' = 3x_i - x_i^3 + 1 - y_i + I_i + H(p_i + \exp(-\alpha t) - \theta) + S_i + \rho \quad (3.1)$$

In this case, we require that an oscillator be oscillatory if and only if two of the three terms: I , H , and S , are positive. This requirement can be implemented by changing the constant 2 in (1a) to 1 in the above equation. The same analysis as in Sect. 4 will apply to the resulting model.

Another possible change to (1a) is more substantial. Again, we change (1a) to the following,

$$x'_i = 3x_i - x_i^3 + 2 - y_i + I_i + W_p H(p_i + \exp(-\alpha t) - \theta) + W_T H(S_i - \theta_S) - W_z H(z - \theta_{xz}) + \rho \quad (3.2)$$

Here, W_p and θ_S are new parameters, and W_T and W_z are as defined in (4b) and (3), respectively. In addition, the z term in (3) is removed from the definition of S_i . More importantly, dynamic normalization of connection strengths in (4) is not needed in this modified model, because the second Heaviside function in (3.2) plays the role of normalizing the overall coupling from other oscillators. Because of this, the analysis in Sect. 4 will apply to the resulting model in the same manner.

We choose not to include (3.2) in the main model described in Sect. 3.1 because the quality of synchrony within each block and the flexibility for choosing parameters seem somewhat lessened. However, due to its relative simplicity, (3.2) may be more desirable for engineering applications. More discussions will be given in Sect. 5, along with some simulation results.

In (1a), there is an explicit use of time t in the exponential function. Explicit time makes it difficult to segment a sequence of images, because every time a new image is presented, t has to be reset to 0. The ability to deal with time-varying images is necessary for perceiving motion patterns (see "common fate" in Sect. 1), and it is crucial for real-time segmentation of nonstationary scenes. We can, however, eliminate explicit time by the following modification to (1a)²

$$x'_i = 3x_i - x_i^3 + 2 - y_i + I_i H(p_i + q'_i - \theta) + S_i + \rho \quad (3.3)$$

Where q_i is defined as

$$q'_i = -\alpha q_i + I_i(t)$$

Here α is a decay parameter. We assume for simplicity that $I_i(t)$ is a step function that suddenly turns on when external input stimulates oscillator i . With q_i initialized to 0, q'_i equals 1 when $I_i(t)$ turns on, and q'_i decays to 0 exponentially when $I_i(t)$ turns off, thus playing the same role as the exponential function in (1a).

As will be clear in Section 4, the precise nonlinear forms of the x -nullcline and the y -nullcline in (1) are not important for the system to work. The specific cubic and sigmoid functions (see Fig. 2) are used in the model because of their simplicity.

4. Analysis

4.1 Preliminaries

Before stating the main result, it will be necessary to make some definitions and introduce some notation. Let e_i be any oscillator. A *block* is defined to be a maximally connected set of stimulated oscillators. This definition of a block implies that $N_1(i)$ is assumed to contain only immediate neighbors of i . Suppose that the oscillator e_i belongs to the block B . According to (2), e_i is a leader if $\sum_{k \in N_1(i) \cap B} T_{ik} > \theta_p$. That is, the maximum stimulus that e_i can receive from

² We thank P. Linsay for this suggestion.

permanent connections with oscillators in $N_1(i) \cap B$ is above the threshold θ_p . A *major block* is any block that contains at least one leader, and a *minor block* is a block with no leader.

The analysis will take extensive advantage of the singular nature of solutions to the model. We dissect the solutions into fast and slow components using the small parameter ε . This allows us to construct singular solutions of (1)-(5) in which ε is formally set to zero. Here we briefly review the singular perturbation construction of a single oscillator. This will help motivate the notation and more complicated constructions which follow.

Consider the relaxation oscillator

$$\begin{aligned} x' &= f(x, y) + I \\ y' &= \varepsilon g(x, y) \end{aligned} \tag{4.1}$$

where $f(x, y) \equiv 3x - x^3 + 2 - y$ and $g(x, y) \equiv \gamma(1 + \tanh(x/\beta)) - y$. If $I > 0$ and β is sufficiently small, then the x -nullcline $C \equiv \{(x, y): f(x, y) + I = 0\}$ intersects the y -nullcline $H \equiv \{(x, y): g(x, y) = 0\}$ at a unique point which lies on the middle branch of C , and (4.1) gives rise to a stable periodic solution for all $\varepsilon > 0$. The singular solution of (4.1) is shown in Fig. 2A. It consists of four pieces. The pieces which lie on the left branch L and the right branch R of C correspond to the silent and active phases, respectively. The other two pieces connect L with R . The piece which connects the left knee LK of L with R corresponds to when the singular solution jumps up. The piece which connects the right knee RK of R with L corresponds to when the singular solution jumps down.

If $I \leq 0$, then the nullclines intersect at a point which lies on the left branch of C . This is a stable fixed point of (4.1), and the system is excitable. This situation is illustrated in Fig. 2B.

In the analysis that follows, we construct singular solutions for more general networks. Each oscillator will lie on the left or the right branch of some cubic, or will be in the process of jumping up or jumping down between such branches. The relevant cubic is determined by the level of stimulus which the oscillator receives. It will be convenient to introduce the following notations. Referring to (1a) and (3), we set

$$S_I^i = I_i H(p_i + \exp(-\alpha t) - \theta); \quad S_E^i = \sum_{k \in N_2(i)} W_{ik} H(x_k - \theta_x), \quad S_z = W_z H(z - \theta_{xz})$$

Let $C(S_I, S_E, S_Z) \equiv \{(x, y): f(x, y) + S_I + S_E - S_Z = 0\}$ and denote the left and right branches of this cubic by $L(S_I, S_E, S_Z)$ and $R(S_I, S_E, S_Z)$, respectively. Denote the left and the right knees of this cubic by $LK(S_I, S_E, S_Z)$ and $RK(S_I, S_E, S_Z)$, respectively.

We will assume that the parameter I_i takes one of two values. In other words, the input image is assumed to be a binary one. This assumption simplifies the following analysis, but, as discussed in Sect. 6, the analysis can be extended to gray-level images. If the oscillator e_i is stimulated, then $I_i = I^+ > 0$, while if e_i is unstimulated, then $I_i = I^- < 0$.

Assume that the left branch of $L \equiv L(I^+, 0, 0)$ is given by $L \equiv \{(x, y): x = h(y)\}$, and consider the scalar equation

$$y' = g(h(y), y) \tag{4.2}$$

This equation determines the evolution of singular solutions during the silent phase. For $(x, y) \in \mathbb{L}$, let $\psi(y, t)$ be the solution of (4.2) such that $\psi(y, 0) = y$. For $y_1 < y_2$, define $\tau(y_1, y_2)$ by $\psi(y_2, \tau(y_1, y_2)) = y_1$. That is, $\tau(y_1, y_2)$ is the time of excursion from y_2 to y_1 .

4.2 Main Result

We now formally state our main result. It basically states that it is possible to choose the parameters so that the network quickly evolves to a state in which only the oscillators belonging to a major block oscillate. All of the oscillators within a major block oscillate in synchrony, while distinct major blocks oscillate out of phase from each other. The unstimulated oscillators never oscillate, and after an initial transient period, loners do not oscillate either.

The following theorem is concerned with singular solutions of (1)-(5) in which ε is formally set to zero. The precise definition of a singular solution is very similar to that given in Terman and Wang (1995). In the theorem t is considered to be the slow time. Using analysis similar to that given in Terman and Wang (1995, Section 4.5), one can show that the following theorem extends to solutions of (1)-(5) if $\varepsilon > 0$ is sufficiently small. We will ignore the role of noise in the proof of the theorem. While this simplifies the proof, a small amount of Gaussian noise not only does not disrupt oscillatory dynamics but also aids the process of separating oscillator blocks, as illustrated in the numerical simulations of Section 5.

Theorem: Fix $M > 0$. Assume that each oscillator begins, when $t = 0$, on \mathbb{L} . Moreover, if (x_1, y_1) and (x_2, y_2) are distinct oscillators then $y_1 \neq y_2$, and if $y_1 < y_2$, then $\tau(y_1, y_2) < M$. There exists $T > 0$ such that if the parameters in (1)-(5) are chosen appropriately, then the following hold:

- A) Let B be any major block. For each $t > 0$, either every oscillator in B is in a silent phase, in an active phase, or every oscillator in B is jumping up or jumping down.
- B) If $t > T$, then at most one block can be in the active phase.
- C) Every loner is in the silent phase for every $t > T$.
- D) Every unstimulated oscillator is in the silent phase for every $t > 0$.

Remarks: a) The proof demonstrates that it is possible to choose the parameters so that the time T corresponds to no more than $N_M + 1$ cycles where N_M is equal to the number of major blocks.

b) We assume that every oscillator begins in the silent phase. The choice of parameters depends on the constant M which measures how close the oscillators begin to each other. Our numerical simulations indicate that these restrictions are not necessary. For the numerical simulations, we use random initial conditions.

c) The proof is very constructive, and gives rather precise estimates on how the parameters need to be chosen. We comment on this later after the analysis.

The proof of the Theorem is carried out in a number of steps. We first prove a weaker result in which T may correspond to more than $N_M + 1$ cycles. For this weaker result, we make very few restrictions on the parameters. We then prove the stronger result by choosing the parameters more

carefully. The approach we take is very similar to the proofs given in Terman and Wang (1995). We will not give all of the details of the analysis here whenever an argument is identical to that given in Terman and Wang (1995).

4.3 Unstimulated Oscillators

We require that unstimulated oscillators never jump up to the active phase. This will be the case if certain conditions on the parameters are satisfied. Recall that if e_i is an unstimulated oscillator, then $I_i = I < 0$. The maximal input that e_i can receive due to coupling from its neighbors is W_T . Then we require that

$$I + W_T < 0 \tag{4.3}$$

From the discussion in the preceding section, this implies that the cubic corresponding to e_i always has a fixed point on its left branch. This cubic may change, because the input which e_i receives may change. However, e_i will always tend towards the fixed point on the left branch which it lies on.

4.4 Synchronization of Oscillators within a Major Block

Let B be any major block. We demonstrate that if all of the oscillators in B begin sufficiently close to each other, then they will synchronize in the following sense: at any given (slow) time, all of the oscillators in B are either in their silent phase, in their active phase, jumping up, or jumping down. Moreover, the maximum distance between any two leaders in B or any two followers in B approaches zero at an exponential rate as time approaches infinity. This will only be true for appropriate choices of the parameters, and the analysis will clarify how to choose the parameters. For this analysis, there may or may not exist other oscillators besides those in B .

The oscillators in B begin in the silent phase on the left branch of $L(I^+, 0, 0)$. Here we need the term $\exp(-\alpha t)$ in (1a). This guarantees that every stimulated oscillator is initially enabled. If α is sufficiently small, then the oscillators in B remain enabled for at least the first cycle. Hence these oscillators must eventually jump up, and there are two ways in which this can happen. Here we briefly describe both mechanisms. For this, it will be necessary to make the following definition. Let e_i and e_j be any stimulated oscillators in B . We say that e_j is a *coupling neighbor* of e_i if $j \in N_2(i) \cap B$. Referring to (3), this implies that the coupling neighbors of e_i are those oscillators in the same block as e_i which receive input from e_i whenever e_i is active.

The first mechanism by which the oscillators in B may jump up is called fast threshold modulation (Somers and Kopell 1993). One oscillator, say e_i , in B reaches the left knee of $L(I^+, 0, 0)$ and then jumps up to the active phase. When x_i crosses θ_x , e_i 's coupling neighbors feel excitation. This has the effect of raising the cubics corresponding to these neighbors. If, at this time, the neighbors lie below the left knee of their new cubic, then they must jump up to the active phase. In Terman and Wang (1995), we prove that this will be the case if the parameters are chosen appropriately. We must be careful in choosing the parameters because e_i also turns on inhibition; this has the effect of lowering the cubics. We assume that

$$\min_{j \in N_1(i) \cap B} \{W_{ji}\} > W_Z \tag{4.4}$$

so that the net effect of both excitation and inhibition is to raise an oscillator's cubic. Once e_i 's coupling neighbors jump up, then they will cause their coupling neighbors to jump up by the same mechanism. Continuing in this way, every oscillator in B will jump up at the same (slow) time.

The second mechanism by which that the oscillators in B may jump up is often referred to as *rebound* (Perkel and Mulloney 1974). Suppose that the oscillators in B are in their silent phase and some other oscillators are active. Then the oscillators in B lie on $L(I^+, 0, W_Z)$. When the active oscillators jump down, they turn off inhibition so that the oscillators in B jump towards the cubic $L(I^+, 0, 0)$. If, at this time, an oscillator in B lies below $LK(I^+, 0, 0)$, then that oscillator must jump up to the active phase. If this is the case, then all of the oscillators in B will jump up because of fast threshold modulation.

We now consider what happens when all of the oscillators in B lie in their active phase. We still assume that this is the first cycle so that every oscillator in B is still enabled. Our assumption of normalized weights implies that each oscillator in B receives the same input W_T from its coupling neighbors. This is because the equations (4a) and (4b) respond on the fast time scale. Hence, during the active phase each oscillator in B lies on $R(I^+, W_T, W_Z)$. This active phase continues until one of the oscillators, say e_k , in B reaches its right knee and then jumps down to the silent phase. We claim that because of fast threshold modulation, this causes the other oscillators in B to jump down. When x_k crosses θ_x , the excitation to e_k 's coupling neighbors is reduced. This has the effect of lowering the cubics corresponding to e_k 's coupling neighbors. The analysis in Terman and Wang (1995) demonstrates that if the parameters are chosen appropriately, then these neighbors must lie above the right knees of their new cubics. Hence, these neighbors must jump down. In a similar way, the coupling neighbors of e_k 's coupling neighbors must jump down and so on until all of the oscillators in B jump down to the silent phase.

We have shown that during the first cycle, all of the oscillators in B jump up and jump down together. We now show why this is true for every cycle. The leaders will, in fact, continue to behave in this manner. This is because each time the oscillators in B jump up, the leaders in B receive enough input from their neighbors in $N_1 \cap B$ so that they achieve a high potential. Each leader, therefore, remains enabled for at least the next cycle. It follows that every leader lies on either $L(I^+, 0, 0)$ or $L(I^+, 0, W_Z)$ during the silent phase and on $R(I^+, W_T, W_Z)$ during the active phase.

The followers in B , however, must become excitable. This is because the followers must eventually have low potential so that the term $p_i + \exp(-\alpha t)$ in (1a) must fall below the threshold θ . This is not completely obvious since while the followers are in the active phase, there may be other oscillators besides those in B which lie in the active phase. Hence, the followers in B may achieve a high potential by receiving input from other oscillators not in B . In the next subsection, however, we will show that eventually no other oscillator besides those in B will jump up while the oscillators in B are active. Since the followers in B can then only receive input from other oscillators in B , their potential must decay to zero at a rate determined by μ .

The followers become excitable when their potential falls below θ . They then lie on either $L(0, 0, 0)$ or $L(0, 0, W_Z)$ during the silent phase and on $R(0, W_T, W_Z)$ during the active phase. Since

the y -nullcline intersects $L(0, 0, 0)$ along its left branch, the followers are never able to reach their left knee. Hence, one difference between the first and later cycles is that after the followers become excitable, the silent phase cannot terminate by a follower reaching a left knee and jumping up. The silent phase can only terminate if a leader in B jumps up either because of its reaching a left knee or because of rebound. All of the oscillators in B still jump up together, however, because of fast threshold modulation. Another difference is that after the followers become excitable, they lie on the right branch of a different cubic than the leaders during the active phase. The same considerations as before, however, show that all of the oscillators in B jump down at the same (slow) time because of fast threshold modulation.

It is not hard to show that if two oscillators lie on the same branch of the same cubic, then the distance between them must decrease at an exponential rate. Since the followers or leaders always lie on the same left or right branch, the maximal distance between the followers or leaders must decrease at an exponential rate. We note that there is also a compression of the "time metric" (LoFaro 1994) between oscillators when they jump up from the silent to the active phase. This is because the rate at which the oscillators move along the left and right branches is determined by the nonlinear function $g(x, y)$. If the oscillators lie near the left knee, and this knee is close to the y -nullcline, then the g values of the oscillators must be small. In particular, even though the Euclidean distance between two oscillators near the left knee may appear to be small, the actual time it takes for one oscillator to reach the other may be large. Immediately after the oscillators jump up to the right branch, the Euclidean distance between them does not change. However, the rate at which they evolve along the right branch is now determined by the values of $g(x, y)$ along the right branch. The oscillators are now much further from the y -nullcline than they were before the jump. Hence, the time it takes the trailing oscillator to reach the leading one is much shorter after the jump. This compression in the time metric can be used to prove that both the followers and leaders must synchronize at an exponential rate. Note that while the oscillators evolve along the same branch, the time metric remains invariant, although the Euclidean metric must decrease.

4.5 Separation of Blocks

In the preceding section, we demonstrated that the oscillators within each block remain synchronized in the sense that they jump up and jump down together. This analysis applies to all blocks, including minor ones, as long as they continue to oscillate. Here, we show that even if the blocks are initially close to each other, they eventually separate. By this we mean that after some transient period there can be at most one major block in the active phase at any particular time, and these blocks take turns to jump up to the active phase. This is what is meant by *desynchronization* of the blocks. The analysis is very similar to that in Terman and Wang (1995), and we will only sketch the proof of this result.

Recall that there are two mechanisms by which a block B can jump up to the active phase. The first is if one of the oscillators in B reaches its left knee, jumps up, and then the remaining oscillators in B jump up because of fast threshold modulation. We claim that this mechanism will cause B to separate from the remaining blocks. That is, no other oscillator besides those in B will be able to jump up during this active phase or any other later time during which B is in the active phase. The reason why this is true is because once one oscillator in B crosses θ_{zx} , the global inhibitor is turned on. This lowers the cubic corresponding to an oscillator not in B to either $C(I^+, 0, W_z)$, if the oscillator is enabled, or to $C(0, 0, W_z)$, if the oscillator is excitable. We assume that

$$I^+ - W_z < 0 \tag{4.5}$$

This implies that the y -nullcline intersects both of these cubics at a point on the left branch of the cubic. The oscillators not in B must then tend towards one of these fixed points while B is in the active phase and will not be able to jump up until B returns to the silent phase. This process of separation of the blocks is referred to as selective gating in Terman and Wang (1995).

The second mechanism by which the oscillators in B can jump up is through rebound. That is, oscillators in B jump up when other oscillators not in B jump down and release the global inhibition. It is perfectly possible that other oscillators, besides those in B , will jump up at this time. This is the major difficulty in proving that the blocks must eventually separate from each other.

In Terman and Wang (1995), we prove that during each cycle at least one new block jumps up because some oscillator in that block reaches its left knee, i.e. not by rebound. From the previous discussion, this implies that until the system reaches full separation of blocks, at least one new block separates itself from the remaining blocks during each cycle. Hence, if N_B is the total number of blocks (major or minor), then all of the blocks are separated from each other after N_B cycles. Here we include the minor blocks, because unless further restrictions are placed on the parameters, they may be oscillatory for up to N_B cycles before they are no longer able to jump up.

For the proof of this result, we require that the time each oscillator spends in the active phase is small compared to the time it spends in the silent phase. More precisely, let τ_S and τ_A equal to the times that the singular solution of (4.1) spends in the silent and active phases, respectively. We require that $\tau_S > N_B \tau_A$. It is not hard to see that this will be the case if the parameter γ is sufficiently large. This can also be achieved by choosing I^+ so that the left knee of an uncoupled oscillator is close to the y -nullcline. This condition guarantees that there must be some time during each cycle when every oscillator lies in the silent phase on the left branch of C . Hence, the next oscillator which jumps up must do so by reaching a left knee. The block containing this oscillator must then separate itself from the remaining blocks.

We must still show that once a block, say B , separates itself from the remaining blocks, it must always remain bounded away from the other blocks. This is proved in Terman and Wang (1995, Theorem 4.3.1). The basic idea of the proof is the following. First consider a time, say t_0 , when B jumps down to the silent phase after being in the active phase with no other block. At this time, B is bounded away from the other oscillators by some distance determined by how long B spent in the active phase. This time is bounded below by some constant T_A which does not depend on the particular block B . It is actually more convenient to consider the time metric. That is, let $\tau(y_1, y_2)$ be as defined in Section 4.1. If $e_i = (x_i, y_i)$ is any oscillator in B and $e_j = (x_j, y_j)$ is any oscillator not in B , then when $t = t_0$, $y_j < y_i$ and $\tau(y_j, y_i) > T_A$. The time metric is convenient because it remains invariant as long as two oscillators remain on the same left branch in the silent phase. One technical difficulty is that oscillators may jump back and forth between different left branches while they are in the silent phase. These jumps occur whenever other oscillators jump to and from the active phase. We require that the times of excursion on different left branches are not too much different from each other. This will be the case if the parameter β in (1b) is sufficiently small. With this assumption, we can show that if e_j and e_i are as above, then $\tau(y_j, y_i) > T_A/2$ as long as the oscillators in B remain in the silent phase. Hence, B remains bounded away from the other blocks as long as it remains in the silent phase. Eventually the oscillators in B jump up to the active phase, and this entire analysis repeats itself over the next

cycle. The condition $\tau_S > N_B \tau_A$ is needed to insure that once B is separated from the remaining blocks it will not jump up at the same time with any other block because of rebound.

4.6 Minor Blocks

We assume throughout this subsection that e_i is a loner which belongs to a minor block B . It is possible that e_i is the only oscillator in B . We first demonstrate that e_i can jump up to the active phase for at most a finite number of cycles. After that, e_i must remain in the silent phase.

We prove that the oscillators in B eventually remain silent by showing that there is some time after which every oscillator in B is excitable. After this time the oscillators in B will not be able to jump to the active phase for the following reason. If a stimulated oscillator is excitable, then during the silent phase it lies on either $L(0, 0, 0)$ if no other oscillator is in the active phase, or on $L(0, 0, W_Z)$ if some other oscillator is in the active phase. The y -nullcline intersects both of these branches. Hence, the oscillators in B will always be attracted towards one of these intersection points. They will never be able to jump up by reaching the left knee of one of these cubics. It may be possible that the oscillators in B jump up because of rebound, but the left knee of $C(0, 0, 0)$ is below the fixed point on $L(0, 0, W_Z)$. This implies that if the oscillators in B are released from inhibition because some other oscillator jumps down, the oscillators in B must jump from $L(0, 0, W_Z)$ to $L(0, 0, 0)$. They will not be able to jump up to the active phase.

It remains to prove that there is some time after which all of the oscillators in B are excitable. The difficulty here is that while B is in the active phase, other blocks may also be in the active phase. Hence, oscillators in B may achieve a high potential by receiving input from other oscillators besides those in B (cf. the discussion regarding $N_1(i)$ vs. $N_2(i)$ in Sect. 3). In the preceding subsection, however, we saw that after N_B cycles, all of the blocks must be separated from each other; there can be at most one block in the active phase at any time. In particular, after N_B cycles, B can only be in the active phase by itself. Since B , by itself, cannot develop leaders, after this time, the potential of each oscillator in B must decay by a rate determined by μ . Eventually, these potentials must fall below threshold and the oscillators change from enabled to excitable.

4.7 Stronger Result

We now prove that if the parameters are chosen more carefully, then the Theorem holds with T corresponding to $N_M + 1$ cycles. From our analysis, this will be the case if the minor blocks are not able to jump up to the active phase after the first cycle. This, in turn, will follow if all of the loners become excitable after the first cycle. Referring to (1a), we see that there are two ways in which a loner can be oscillatory. The first is if $\exp(-\alpha t) > \theta$. The second is if the loner has a high potential.

We choose α so that if t corresponds to some time after the first cycle, then $\exp(-\alpha t) < \theta$. More precisely, consider the singular periodic solution of (4.1). Again, let τ_S and τ_A equal to the times that this singular solution spends in the silent and active phases, respectively. In addition, let τ_F equal to the time between the system start and when the first oscillator jumps to the active phase. Choose α so that

$$\theta < \exp(-\alpha(\tau_S + \tau_A)) \quad \text{and} \quad \exp(-\alpha(\tau_S + \tau_A + \tau_F)) < \theta \quad (4.6)$$

This guarantees that all the stimulated oscillators remain enabled during the first cycle, but only those oscillators with high potential remain enabled when they are ready to jump up during the second cycle.

It follows that the only way that a loner can jump up during the second cycle is if it has a high potential. Recall that the only way that this can happen is if the loner receives enough input from oscillators not in its own block. We now show how to choose the parameters so that this is impossible.

We assume that the network is weakly isotropic in the following sense. Suppose that the quantity $\sum_{k \in N_1(i)} T_{ik} \equiv \Omega$ is independent of the oscillator i . We then let $\theta_p = \Omega - \delta$, where δ is sufficiently small. In particular, we assume that $\delta < \min_{k \in N_1(i)} \{T_{ik}\}$. It then follows that the oscillator i which belongs to the block B is a leader if and only if the entire neighborhood $N_1(i)$ belongs to B. Moreover, if i is in the active phase and there is at least one oscillator in $N_1(i)$ which is not in the active phase, then $\sum_{k \in N_1(i)} T_{ik} H(x_k - \theta_x) < \theta_p$. Clearly, this is the case if i is a loner. It follows that a loner can never have a high potential. From the above discussion, this completes the proof of the Theorem.

4.8 Choosing the Parameters

We have now demonstrated that the Theorem holds if the parameters in the model are chosen appropriately. The analysis is constructive in the sense that it leads to precise estimates that the parameters must satisfy. It also shows that the Theorem holds for a robust range of parameter values. Moreover, the analysis does not depend on the precise forms of the nonlinear functions $f(x, y)$ and $g(x, y)$. Here, we summarize what estimates the parameters must satisfy.

First consider the parameters in (1a). We require that $0 < \theta < 1$, and α satisfies (4.6). Recall that $I_i = I^+ > 0$ if i corresponds to a stimulated oscillator, and $I_i = I^- < 0$ if i corresponds to an unstimulated oscillator. The conditions needed on I^+ and I^- will be discussed shortly.

Next consider (1b). We require that β is sufficiently small so that the times of excursion on different left branches are not too much different from each other. We also need that γ is sufficiently large so that the time of excursion in the active phase is much shorter than that in the silent phase. More precise conditions on these parameters are given earlier in this section.

We next turn to (2). We simply require that λ is $O(1)$, with respect to ε , and μ is $O(\varepsilon)$. The threshold θ_x must be chosen to lie between the values of x at the left and right branches of the cubics. Finally, we assume that $0 < \theta_x < 1$.

Now consider (3) and (4). The weights W_{ik} , W_Z , and W_T must satisfy (4.3), (4.4), and (4.5). These also give conditions on I^+ and I^- . We require that $0 < \theta_{xz} < 1$. The parameter η in (4) and the parameter ϕ in (5) simply need to be $O(1)$, and ν to be $O(\varepsilon)$.

5. Computer Simulation

To illustrate how the LEGION network is used for image segmentation while eliminating fragmentation, we have simulated a 25x25 grid of oscillators with a global inhibitor as defined by (1)-(5). In the simulation, $N_1(i)$ is simply the four nearest-neighbors without boundary wrap-around, and $N_2(i)$ is set equal to $N_1(i)$. We arbitrarily selected an image with four binary objects (patterns): two **O**'s, one **H**, and one **I**; and they form the word **OHIO** as shown in Fig. 5A (see

also Wang and Terman 1995). We then injected 10% noise to the image: each uncovered box has a 10% chance of becoming covered (stimulated). The resulting image is shown in Fig. 5B. For all the stimulated oscillators $I = 0.2$, while for the others $I = 0$. Notice that if oscillator i is unstimulated, $W_{ik} = W_{ki} = 0$ for all k , and I_i does not need to be negative to prevent i from oscillating. Thus the oscillators under stimulation become enabled, and those without stimulation become excitable and unable to oscillate. The amplitude ρ of the Gaussian noise was set to 0.02. This represents a 10% noise level compared to the external stimulation. We observed during the simulations that noise facilitated the process of desynchronization.

The differential equations (1)-(5) were solved using both a fourth-order Runge-Kutta method and the adaptive grid o.d.e. solver LSODE. Permanent connections between any two neighboring oscillators were set to 2.0, and for total dynamic connections (see (4b)), $W_T = 6.0$. Dynamic weights W_{ik} were set up at the beginning according to (4). The following values for the other parameters in (1)-(5) were used for the simulation results shown in Fig. 5: $\varepsilon = 0.02$, $\alpha = 0.005$, $\beta = 0.1$, $\gamma = 6.0$, $\theta = 0.9$, $\lambda = 0.1$, $\theta_x = -0.5$, $\theta_p = 5.0$, $W_z = 1.5$, $\eta = 1.0$, $\mu = \nu = 0.01$, $\phi = 3.0$, and $\theta_{zx} = \theta_{xz} = 0.1$. The value of θ_p is chosen so that, in order to achieve a high potential, an oscillator must have at least three active neighbors out of four possible neighbors on a 2-D grid. The simulation results were robust to considerable changes in the parameter values.

The phases of all the oscillators on the grid were randomly initialized. Fig. 5C-5G show the instantaneous activity (snapshot) of the network at various stages of dynamic evolution. The diameter of each black circle represents the x activity of the corresponding oscillator. Specifically, if the range of x values of all the oscillators is given by x_{min} and x_{max} , then the diameter of the black circle corresponding to one oscillator is set to be proportional to $(x - x_{min}) / (x_{max} - x_{min})$.

Fig. 5C shows the snapshot at the beginning of the dynamic evolution. This is included to illustrate the random initial conditions. Fig. 5D shows a snapshot shortly after Fig. 5C. One can clearly see the effect of synchrony and desynchrony: all the stimulated oscillators which belong to or are the coupling neighbors of the right **O** are entrained and have large activities (in the active phase). At the same time, the oscillators stimulated by the rest of the noisy image have very small activities (in the silent phase). Thus the noisy right **O** is segmented from the rest of the image. A short time later, as shown in Fig. 5E, the oscillators in the group representing the noisy **H** reach their active phase and are separated from the rest of the image. Fig. 5F shows another snapshot after Fig. 5E. At this time, the noisy left **O** has its turn to be activated and separates from the rest of the input. Finally, in Fig. 5G, the oscillators representing the noisy **I** are in the active phase and the rest of the scene remains inactive. This successive "pop-out" of the segments continues in a stable periodic fashion until the input image is withdrawn.

To illustrate the entire segmentation process, Figure 6 shows the temporal evolution of every stimulated oscillator. The activities of the oscillators stimulated by each object are combined together as one trace in the figure, and so are for the background. If any group of oscillators synchronizes, then together they appear like a single oscillator. Since the oscillators receiving no external stimulation remained excitable and unable to oscillate throughout the simulation process, they are excluded from the display of Fig. 6. The four upper traces represent the activities of the four oscillator blocks corresponding to the four objects, and the fifth one represents the background consisting of all of the scattered dots. Because of low potentials, these oscillators quickly become excitable even though they are enabled at the beginning. The bottom trace represents the activity of the global inhibitor. Shortly after the start, the pattern **H** and the right **O** are in their active phase, together with some of the background oscillators. Then the left **O** and **I** jump up together. After about one cycle, the loners fade away, and **H** and the right **O** start to separate. After about three cycles, the left **O** and **I** start to separate. The noisy image of Fig. 5B is completely segmented into four objects, corresponding to each of the four letters in **OHIO**, in about four cycles. The speed of segmentation is consistent with the analysis of Sect. 4. After complete segmentation, the quality of synchrony within the right **O** improves markedly. The

dynamic evolution process is well indicated by the activity of the global inhibitor, which is activated when an oscillator block reaches its active phase.

As a comparison with the alternative definition of (3.2) for a single oscillator, we have simulated the same network for segmenting the same noisy image of Fig. 5B. The simulation was done in exactly the same way. Here, for all stimulated oscillators $I = 0.2$, while for unstimulated oscillators $I = -7.0$. In addition, $W_T = 6.0$, $\theta_S = 1.0$, and $W_p = 0.2$. The other parameters have the same values as in the previous simulation. Figure 7 shows the simulation result, in the same format as in Fig. 6. For this task, the network takes about four cycles to fully segment all four noisy objects. This rate of segmentation is again consistent with our analysis in Sect. 4. Also, the simulation reveals that after full separation the quality of synchrony within each oscillator block is not as good as in Fig. 6. Note that the lateral input each oscillator receives is a binary value, whereas in (1) the lateral input is a graded value depending on how many oscillators in $N_2(i)$ are in the active phase. Graded lateral input tends to promote synchrony in jumping down and thus overall synchrony, because an oscillator can immediately feel reduced lateral input (see Sect. 4.4). During the simulation, we also observed that the definition (3.2) seems to limit the range of parameter values for lateral interactions, again because of binary lateral interaction.

With a fixed set of parameters, the dynamical system of LEGION can segment only a limited number of patterns because of the reasons given in Sect. 4.5. The number of patterns depends, to a large extent, on the ratio τ_S/τ_A of the times that a single oscillator spends in the silent and active phases. Let us refer to this limit as the *segmentation capacity* of LEGION. In the above simulation, the number of the major blocks to be segmented is within the segmentation capacity. What happens if this number exceeds the segmentation capacity? From the previous section, we know that the rebound mechanism may synchronize oscillator blocks that have no intrinsic reason to be grouped together. So, intuitively, the system should separate the entire image into as many segments as the capacity allows, where each segment may correspond to one major block (called *simple segment*) or a number of major blocks (called *congregate segment*). This is confirmed by our numerical simulations. To illustrate this point, we show the following simulation results using the system (1)-(5). We present to a 30x30 LEGION network with an arbitrary image containing nine binary patterns, which together form the phrase **OHIO STATE**. These patterns are arranged as shown in Fig. 8A. We use exactly the same parameter values as in the simulation presented in Figs. 5 and 6. For this set of parameters, our earlier experiments showed that the system's segmentation capacity is less than 9. The simulation results are presented in Fig. 8B-G, in the same format as in Fig. 5. Shortly after the start of system evolution, the LEGION network segmented the input of Fig. 8A into five segments, shown in Fig. 8C-8G respectively. Among these five segments, two are simple segments (Figs. 8C and 8D) and three are congregate segments (Figs. 8E, 8F, and 8G). To illustrate the entire segmentation process, Figure 9 shows the temporal activity of every stimulated oscillator, shown in the same format as in Figs. 6 and 7. For this task, the network takes less than three cycles to segment the entire scene into the five segments shown in Fig. 8.

Besides the simulation illustrated in Figs. 8 and 9, many other simulations have been performed for the input of Fig. 8A with different random initial conditions, and the results are comparable with Figs. 8 and 9. There are different ways, however, that the system separates the nine patterns into five segments. For this particular set of parameters, we can say that the segmentation capacity of the LEGION network is 5. In fact, we have not seen a single simulation trial where more than 5 segments are produced. This important property of the system, i.e. it naturally exhibits a segmentation capacity, is in good accord with the well-known psychological principle that there are fundamental limits on the number of simultaneously perceived objects. Imagine that you walk into a new house. Though there are many interesting things in the house, you can attend to only a few things at a time. Some implications of this property, as well as its psychological link, shall be further discussed in Sect. 7.2.

The order of activation among segmented patterns bears no particular semantics. It is determined by the random initial conditions and system noise. The LEGION networks simulated

above can be readily applied to segmenting binary images of much larger sizes. In the next section, we consider gray-level images.

6. Real Images

LEGION can segment gray-level images in a way similar to segmenting binary images. For a given image, a LEGION network of the same size as the image with a global inhibitor is used to perform segmentation. Each pixel of the image corresponds to an oscillator of the network, and we assume that every oscillator is stimulated when the image is applied to the network. The main difference between gray-level and binary images lies in how to set up connections. For gray-level images, the coupling strength between two neighboring oscillators is determined by the disparity of two corresponding pixels.

6.1 Algorithm

To segment real images with large numbers of pixels involves integrating a large number of the differential equations of (1)-(5). To reduce numerical computations on a serial computer, an algorithm is extracted from these equations. The algorithm follows major steps in the numerical simulation of the equations, and it exhibits the essential properties of relaxation oscillators, such as two time scales (fast and slow) and the properties of synchrony and desynchrony in a population of oscillators. Such extraction is quite straightforward because, in a relaxation oscillator network, much of the dynamics takes place when oscillators are jumping up or jumping down. Besides reducing numerical computations, the algorithm also overcomes the segmentation capacity, which may be desired in applications where many patterns need to be segmented from an image. More specifically, the following approximations have been made.

(a) When no oscillator is in the active phase (see Fig. 2), the leader closest to the jumping point (left knee) among all enabled oscillators is selected to jump up to the active phase.

(b) An oscillator takes one time step to jump up to the active phase if the net input it receives from neighboring oscillators and the global inhibitor is positive.

(c) The alternation between the active phase and the silent phase of a single oscillator takes one time step only.

(d) All of the oscillators in the active phase jump down if no more oscillators can jump up. This situation occurs when the oscillators stimulated by the same pattern have all jumped up.

In the above actions, (a) and (d) are particularly effective in cutting down integration time, because they dramatically shorten the time a block stays in the active phase or in the silent phase, the two relatively stable and time-consuming stages in dynamical evolution (see Fig. 2 and Fig. 6). Despite these simplifications, it is easy to see that the behavior of the following algorithm well approximates that of the dynamical system defined in (1)-(5).

In the following, we first present the precise algorithm used in later segmentation tasks. Further simplifications made in the algorithm will be explained in Sect. 6.2.

LEGION algorithm

Only the x value of oscillator i , x_i , is used in the algorithm. $N_1(i)$ is assumed to be the eight nearest neighbors of i on the 2-D network, except on the boundaries where no wrap-around is used. It is further assumed that $N_2(i) = N_1(i)$. LK_x , RK_x , LC_x represent the x values of three of four corner points of a typical limit cycle (see Fig. 2A), where LK and RK denote the left knee and the right knee (see Sect. 4), and LC denotes the (upper) left corner of the limit cycle. By

straightforward calculations, we obtain $LK_x = -1$, $LC_x = -2$, $RK_x = 1$. In the algorithm, I_i indicates the value of pixel i , and I_M indicates the maximum possible pixel value.

1. Initialize

1.1 Set $z(0) = 0$;

1.2 Form effective connections

$$W_{ij} = I_M / (1 + |I_i - I_k|), \quad k \in N_1(i)$$

1.3 Find leaders

$$p_i = H\left[\sum_{k \in N_1(i)} W_{ik} - \theta_p\right]$$

1.4 Place all the oscillators randomly on the left branch. Namely $x_i(0)$ takes a random value between LC_x and LK_x .

2. Find one oscillator j so that (1) $x_j(t) \geq x_k(t)$, where k is currently on the left branch; (2) $p_j = 1$. Then

$$\begin{aligned} x_j(t+1) &= RK_x; \quad z(t+1) = 1 && \{\text{jump up}\} \\ x_k(t+1) &= x_k(t) + (LK_x - x_j(t)), \quad \text{for } k \neq j. \end{aligned}$$

In this step, the leader on the left branch which is closest to the left knee is selected. This leader jumps up to the right branch, and all the other oscillators move towards LK .

3. Iterate until stop

If ($x_i(t) = RK_x$ and $z(t) > z(t-1)$)

$$x_i(t+1) = x_i(t) \quad \{\text{stay on the right branch}\}$$

else if ($x_i(t) = RK_x$ and $z(t) \leq z(t-1)$)

$$x_i(t) = LC_x; \quad z(t+1) = z(t) - 1 \quad \{\text{jump down}\}$$

If ($z(t+1) = 0$) go to step 2

else

$$S_i(t+1) = \sum_{k \in N_2(i)} W_{ik} H(x_k(t) - LK_x) - W_z H(z(t) - 0.5)$$

If ($S_i(t+1) > 0$)

$$x_i(t+1) = RK_x; \quad z(t+1) = z(t) + 1 \quad \{\text{jump up}\}$$

else

$$x_i(t+1) = x_i(t) \quad \{\text{stay on the left branch}\}$$

6.2. Further Remarks

Comparing the above algorithm with the dynamical system of (1)-(5), one can find the following simplifications additionally.

(a) The dynamic weight W_{ij} is directly set to $W_{ij} = I_M / (1 + |I_i - I_k|)$. The intuitive reason for this choice of weights is that the more pixel i and pixel j are similar to each other, the stronger the

connection between the two corresponding oscillators, and thus the easier the two oscillators are to synchronize with each other. This selection of weights promotes gray-level homogeneity as a determining factor in grouping. Compared to binary pixels, it is less straightforward to derive this weight from neural dynamics of the kind in (4). But it is possible given the fact that the frequency of a relaxation oscillator, to a certain degree, increases monotonically when the external stimulation increases (see Fig. 3 of Terman and Wang 1995). The weight W_{ij} can then be computed as a frequency correlation between the presynaptic oscillator and the postsynaptic oscillator. It is also worth noting that the algorithm does not compute normalized weights. Whether an oscillator can be recruited to jump up depends solely on the net input the oscillator receives from its neighboring oscillators and the global inhibitor. As mentioned in Sect. 3, the weight setting in this case does not prevent the selective gating mechanism from taking place.

(b) The leaders are chosen during initialization. According to the dynamics described in Sect. 3, potentials, and thus leaders, are determined during a few initial cycles of oscillatory dynamics. Since every oscillator is stimulated, and W_{ij} is set at the beginning, it can be precisely predicted at the beginning which oscillators will become leaders. Thus, to save computational time, the leaders are determined in the initialization step. Notice that, according to (2), $p_i = H[\sum_{k \in N_1(i)} T_{ik} H(x_k - \theta_x) - \theta_p]$ at equilibrium (μ is ignored here). Given that $N_1(i)$ is 8 nearest neighbors, it is reasonable to assume that T_{ik} is a constant (say 1). Thus, p_i depends on the extent that the oscillators within $N_1(i)$ synchronize, or whether i lies at the center of a homogeneous region of a certain size. The latter is the idea behind the concept of potential, and this is effectively captured by $p_i = H[\sum_{k \in N_1(i)} W_{ik} - \theta_p]$, the formula used in the algorithm. One alternative for computing potentials is $p_i = H[\sum_{k \in N_1(i)} H(W_{ik} - W_z) - \theta_p]$, which computes the number of oscillators in $N_1(i)$ that i can potentially recruit to jump up (note $W_{ik} = W_{ki}$). This formula appears closer to (2), and has been tested during simulations. Our tests indicate that this formula is less flexible than the one used in the algorithm, and tends to need a larger $N_1(i)$ to achieve comparable results.

The condition for the oscillators in the active phase to jump down is an important one, and is based on the observation that the number of oscillators in the active phase increases every time step when an oscillator block is in the process of jumping up to the active phase. Thus, when the oscillators in the block has all jumped up, the number of the oscillators in the active phase does not increase any longer. When all of the oscillators in a single block have jumped to the silent phase, i.e. $z(t) = 0$, another leader is selected to form another block in step 2. In step 2, the system maintains the specific values of x_i to ensure that every block corresponding to a segment has the chance to jump up once before any block jumps up twice. That is, the system keeps an order in jumping up.

For image segmentation applications, the algorithm can be stopped when every leader has jumped up once (A user can of course preset the number of time steps for the algorithm to execute). It should be noticed, however, that some oscillators may jump up more than once in a cycle of oscillations, because of the summation in the coupling term S_i (see the algorithm and (3)). This is illustrated in Fig. 10, with an image consisting of three regions of equal pixel value. It is easy to choose the parameter W_z so that if the left segment with $I = 30$ jumps up, then this segment will recruit the stripe with $I = 50$ to jump up. This is because of the summated excitation from the five oscillators of the left 30-segment. The $I = 50$ stripe, however, will not recruit the right 30-region. When the right 30-segment jumps up, it recruits the middle stripe to jump up similarly. Thus, the stripe with $I = 50$ jumps up twice in one oscillation cycle.

This observation raises an issue, that a region of an image may participate in more than one segment. In Fig. 10, it is possible that the 50-region is part of two segments: one with the left 30-region and one with the right 30-region. It should be noted that this phenomenon is less likely to occur in the original dynamical system of LEGION, because an oscillator has to evolve in the silent for a while before it can jump up again (see Fig. 2A), resembling the refractory period of a real

neuron. Although this phenomenon violates the basic definition of image segmentation in computer vision (Schalkoff 1989), there exist interesting parallels with human perception. The so called *duplex perception* refers to the phenomenon that some sensory features play a double role by participating in two organizations. In speech perception, for example, part of a spectrum can be heard as a separate sound; at the same time, it also joins other parts to form a speech sound (Liberman *et al.* 1981; Repp *et al.* 1983).

There are two critical parameters in the algorithm: W_z and θ_p , where the former is the strength of global inhibition and the latter is the threshold for forming high potentials (leaders). For W_z , higher values make the algorithm more difficult to group pixels into regions. Thus, in order for a region to be grouped together, the algorithm demands a higher degree of homogeneity within the region. Generally speaking, given a gray-level image, higher W_z leads to more and smaller regions. For θ_p , higher values make the algorithm more difficult to develop leaders. Thus fewer leaders will be developed, and fewer regions result from the algorithm. On the other hand, regions produced with a higher θ_p tend to be more homogeneous. The above discussion will become clearer when the test results are presented next.

6.3 Segmenting Real Images

6.3.1 Sum vs. Max: an aerial image

The first image the algorithm is tested on is an aerial image, called Lake, which is shown in Fig. 11A. As in the following images to be used, this is a typical gray-level image, where each pixel is an 8-bit number ranging from 0 to 255 (also called intensity), and pixels with higher values appear brighter. The image has 160x160 pixels, and is presented to a LEGION network of 160x160 oscillators plus a global inhibitor. For this simulation, $W_z = 40$ and $\theta_p = 1200$. Quickly after the image is presented, the algorithm produces different segments at different time steps, as though the segments "pop out" sequentially from the oscillator network. Fig. 11B-11G display the first six segments that have been produced sequentially, where a black pixel corresponds to an oscillator in the active phase and a blank pixel corresponds to an oscillator in the silent phase. As shown in the figure, each segment corresponds to a meaningful region in the original image: a segment is either a lake, a field, or a parkway. The region in Fig. 11B corresponds to a lake. The region in Fig. 11C corresponds to the main lake, except for the lower-left part where the lake region extends to a non-lake part, and for the right side where the lake extends to its bank. The parkway segment in Fig. 11G picks up a partial parkway network in the original image. The other segments match well with the fields of the image.

The entire image is separated into 16 regions and a background. To simplify the display, we put all the segments and the background together into one figure, using gray levels to indicate phases of oscillator blocks. Such a display is called a *gray map*. The gray map of the results of this simulation is shown in Fig. 12, where the background is shown by the black scattered areas. The segments besides those shown in Fig. 11 all have good correspondence with the regions in the image. Generally speaking, the background corresponds to parts of the image that have high intensity variations. Due to the mechanism of fragment removal, these noisy regions stay in the background, as opposed to many segments that would have been the result without fragment removal.

In the above algorithm, an oscillator summates all the input it receives from its coupling neighborhood, and if the overall input is greater than the global inhibition the oscillator jumps to the active phase (see also (3)). Another reasonable way of grouping is to replace summation by maximization when computing S_i :

$$S_i = \text{Max}_{k \in N_2(i)} \{W_{ik} H(x_k - \theta_x)\} - W_z H(z - \theta_{xz}) \quad (6.1)$$

Intuitively, the maximum operation concentrates on the relation of i with the oscillator in $N_2(i)$ that has the strongest coupling with i , but omits the relation between i and $N_2(i)$ as a whole. Thus, grouping by maximization emphasizes pairwise pixel relations, whereas grouping by summation emphasizes pixel relations in a local field.

By using (6.1) in the LEGION algorithm, the Lake image is segmented again. In this simulation, $W_z = 20$ and $\theta_p = 1200$. Fig. 13 shows the result of segmentation by a gray map. The entire image is segmented into 17 regions and a background, which is indicated by black areas. Each segment corresponds well with a relatively homogeneous region in the image. Interestingly, except for the parkway region in the lower part of the image, every region in Fig. 13 has a corresponding one in Fig. 12. A careful comparison between the two figures reveals the difference between summation and maximization in segmentation. The fact that the two figures correspond almost exactly demonstrates that the two schemes produce similar results. A closer comparison, however, indicates that the maximum scheme yields a little more faithful regions. One can see this by comparing the lake regions, the parkway regions, and by the fact that another extended parkway region is segmented in Fig. 13. The problem of the main lake extending to non-lake parts in Fig. 11 is almost eliminated in Fig. 13. On the other hand, regions in Fig. 12 appear smoother and have fewer "black holes" - parts of the background. The smoothing effect of summation is generally positive, but it may lose important details. For example, the sizable hole inside the main lake region of Fig. 13 corresponds to an island in the original image, which is neglected in the main lake region of Fig. 12. Because the maximum scheme appears to produce better results, it will be used in all of the following simulations.

The reason that summation in oscillator coupling has a role of smoothing while maximization does not can be seen from the following example. In Fig. 14A, the center pixel, under the summation scheme, can be grouped by its surrounding pixels with an appropriate W_z . This grouping effectively smoothens the difference between the center pixel and its surround. But, the center pixel cannot be grouped with the surround under the maximum scheme. On the other hand, in Fig. 14B, the center pixel tends to be recruited by the right one with $I = 40$, but it does not under the summation scheme. Another distinction is that the phenomenon of one region belonging to multiple segments (see Fig. 10) does not occur under the maximum scheme, whereby grouping is symmetrical in the sense that if pixel a can recruit pixel b , then b can recruit a as well. This is because effective weights are symmetrical, namely $W_{ij} = W_{ji}$ (see the algorithm).

To show the effects of parameters, we reduce the value of θ_p from 1200 in Fig. 13 to 1000. As a result, more regions are segmented, as shown in Fig. 15 where 23 segments plus a background are produced by the algorithm. Compared with Fig. 13, the notable new segments include an open-theater-like region to the left of the main lake, and its nearby field region. Other newly produced segments are also meaningful regions in the original image. The Lake image has been used in the study of Sarkar and Boyer (Sarkar and Boyer 1993a). As mentioned in Sect. 2.1, their approach is edge-based. One of the results produced by their system is a set of closed boundaries, which implies segmented regions of the image. The reader is encouraged to compare our results with theirs.

6.3.2 MR images

The next image to test our algorithm is a magnetic resonance image (MRI) of a human head, as shown in Fig. 16A. MR images constitute a large class of medical images, and their automatic processing is of a great deal of practical value. This particular image, which we denote as "Brain-1", is a midsagittal section, consisting of 256x256 pixels. Salient regions of this picture include the cerebral cortex, the cerebellum, the brainstem, the corpus callosum, the fornix (the bright stripe below the corpus callosum), the septum pellucidum (the region surrounded by the corpus callosum and the fornix), the extracranial soft tissue (the bright stripe on top of the head), the bone marrow

(scattered stripes under the extracranial tissue), and several other structures (for the nomenclature see p. 318 of Kandel *et al.* 1991). For this image, a LEGION network of 256×256 oscillators plus a global inhibitor is used. The algorithm is performed as specified before. Figure 16B shows the result of one simulation by a gray map. In this simulation, $W_z = 25$ and $\theta_p = 800$. The Brain-1 image is segmented into 21 regions, plus a background which is indicated by the black areas. Of particular interest are two parts of the brain: the upper part, and the brainstem with part of the spinal cord (to be called "brainstem" for short), parts of the extracranial tissue, and parts of the bone marrow. Other interesting segments include the neck part, the chin part, the nose part, and the vertebral segment.

Though it is useful to treat the brain as a whole in some circumstances, it may also be desirable to segment the brain into more detailed structures. To achieve this in LEGION, one can increase W_z . But in order not to produce too many regions, θ_p should usually be increased as W_z increases. To show the combined effects, Fig. 16C displays the result of another run with $W_z = 40$ and $\theta_p = 1000$. With these parameter values, Brain-1 is segmented to 25 regions plus a background. The background is again indicated by black areas. Now, the upper part of the brain is further segmented into the cerebral cortex, the cerebellum, the callosum/fornix region and its surrounding septum. Because of the higher W_z , regions in Fig. 16C tend to contain more background (compare, for example, the two brainstem regions). However, in the cortex segment in Fig. 16C, the noisy stripes actually have physical meanings: they tend to match with various fissures on the cerebral cortex. With the higher θ_p , some segments in Fig. 16B cannot generate any leaders and thus become the background. See, for example, the segments corresponding to the extracranial tissue.

The final segmentation to be presented uses another MRI of a human head, shown in Fig. 17A. This image is denoted as Brain 2, consisting of 256×256 gray-level pixels. Similar to Brain-1, Brain-2 is a sagittal section, but through one eye instead of along the midline. Salient regions of this picture include the cortex, the cerebellum, the lateral ventricle (the black hole within the cortex), the eye, the sinus (the black hole below the eye), the extracranial soft tissue, and the bone marrow. A LEGION network with 256×256 oscillators and a global inhibitor is used for the segmentation task. In the first simulation, $W_z = 20$ and $\theta_p = 800$, and Fig. 17B shows the result by a gray map. Brain-2 is segmented into 17 regions, plus a background which is indicated by black scattered areas. One can see from the figure that the entire brain forms a single segment. Other significant segments include the eye, the sinus, parts of the bone marrow, and parts of the extracranial tissue. The chin and the neck parts are also two large segments. The lateral ventricle is put into the background.

In order to generate finer structures, W_z is raised to 35 in the second simulation. As in Fig. 16, θ_p is increased as well to avoid too many segments, and it is set to 1000. Fig. 17C shows the result of this simulation, where Brain-2 is segmented into 13 regions plus a background, the latter indicated by the black areas. As expected, the segments in Fig. 17B become further segmented or shrunk, and the background becomes more extensive. Worth mentioning is that the brain segment in Fig. 17B is segmented into three segments: one corresponding to the cortex and the other two corresponding to the cerebellum. Due to the increase of θ_p , the segments corresponding to the extracranial tissue and the marrow in Fig. 17B disappear in Fig. 17C.

In the segmentation experiments of this section, our goal was to illustrate the mechanism of oscillatory correlation and the potential effectiveness of derived algorithms. We did not attempt to produce best possible results by fine tuning of parameters. One can easily tell this by the simple rule of setting W_{ij} , the simple choice of $N_1(i)$ and $N_2(i)$, and the values of W_z and θ_p that have been used in the simulations. Therefore, better results can be expected by using more sophisticated schemes of choosing these parameters. For example, a more extended neighborhood of $N_1(i)$ than

8 nearest neighbors can remove small segments that occur occasionally. See Sect. 7 for more discussions on choosing parameters. Indeed, a more elaborate version of the LEGION algorithm has been applied to segment 3-dimensional MR and CT (computerized tomography) images from the medical domain, and good segmentation results have been obtained (Shareef and Wang, in preparation). On the other hand, the parameter values used in the above experiments demonstrate that the results thus achieved are robust to considerable parameter variations.

Two important distinctions between the algorithmic version of LEGION and the dynamical version have not been discussed earlier. One is that the algorithmic version does not have a segmentation capacity; it can produce an arbitrary number of segments. The number of the segments produced by the algorithm only depends on the pixel distributions of an image. On the other hand, as discussed in Sect. 5, with a fixed set of parameters the dynamical system of LEGION can segment only a limited number of patterns. The second distinction is that the algorithm, though having many parallel components, is not a fully parallel algorithm. It is iterative in nature, requiring global operations such as selecting a leader, the test of when a block of oscillators jumps down, and a central clock. These sequential components are introduced to speed up simulations on a serial computer. The dynamical system of LEGION, on the other hand, is fully parallel, and does not require synchronous operations (thus a central clock).

7. Discussion

7.1 Further Remarks on LEGION Computation

The analysis in Section 4 demonstrates that the oscillatory dynamics described in this paper is a robust property of the model. It is insensitive to considerable changes in the parameters and exists for a large class of initial conditions. Moreover, very few assumptions are required on the nonlinear functions in (1). This helps to explain why relaxation-type oscillators, together with synaptic coupling, are more appropriate for scene segmentation than other models which include phase oscillators, for example. These differences are also discussed elsewhere (Wang 1995; Terman and Wang 1995; Somers and Kopell in press).

The analysis is constructive and clearly indicates how the parameters need to be chosen. It also indicates how instabilities can arise as parameters are varied. The discussion in Sect. 4.5, for example, demonstrates that the oscillators within different blocks may not separate if the parameter γ is too small. The analysis does not depend on the fact that we considered a two-dimensional LEGION network. It extends in a straightforward way to arbitrary dimensions, because, in the analysis, oscillator blocks are formed on the basis of local excitation and global inhibition, without reference to the dimension of the network.

In the simulations of Sect. 6, $N_1(i)$ is set to the eight nearest-neighbors of i . Larger $N_1(i)$'s entail more computations for determining leaders. But larger $N_1(i)$'s have more flexibility in specifying the conditions for creating leaders, which tends to produce better results. Also, the order of pop-out of different segments is currently random. In some situations, however, it might be useful to influence the order of pop-out by some criteria, such as the size of each region. This is room in LEGION to incorporate different criteria by ordering leaders accordingly, either by using the global inhibitor in different ways or by introducing other processing units.

The main difference between our approach to image segmentation and other segmentation algorithms reviewed in Sect. 2 is that ours is neurocomputational, building on the strengths and the constraints of neural computation. Our approach relies on emergent behavior of LEGION to embody the computation involved in image segmentation. Methodologically, the computation is performed by a population of active agents - oscillators in this case - that are driven by pixels, whereas, in a typical segmentation algorithm, pixels are data to be processed by a central agent - the algorithm or the neural network trained as a pixel classifier. That LEGION is a massively parallel network of dynamical systems with mainly local coupling makes it particularly feasible for

analog VLSI implementation, the success of which would be a major step towards real time processing of scene segmentation.

7.2 Multi-stage Segmentation

A thorny issue with scene segmentation is that often no unique answer exists. A house, for example, may be grouped into a single segment if viewed afar. The same house, if viewed nearby, may be broken into multiple segments including a door, a roof, windows, etc. This situation demands a flexible treatment of scene segmentation, i.e., a system should be able to easily generate multiple sets of segmentation, each of which should be reasonable. In LEGION, this flexibility is reflected to a certain degree by the effects of the parameters of W_z and θ_p , as discussed in Sect. 6. As noted there, both Fig. 16B and 16C are arguably reasonable results, so are Fig. 17B and Fig. 17C. These parameters are currently determined by simulation trials. It should be an interesting issue of future study to search for an adaptive scheme that can automatically find appropriate values of the parameters.

Psychological and biological evidence suggests that human and animal visual processing involves multiple stages (Marr 1982; Watt 1987; Zeki 1993), where later stages refine the results of earlier ones. The LEGION network used so far has only one layer of oscillators, representing a single stage of processing. We expect that the ability of LEGION improves significantly when multiple layers are used. More specifically, the following method may be used for multistage segmentation from coarse to fine grains. At the first stage, we segment the entire image into a limited number of large regions by choosing a level of global inhibition which we denote by W_{z_1} . Each of the large segments from stage 1 is processed at the next stage by another LEGION layer with the inhibition level W_{z_2} , with $W_{z_2} > W_{z_1}$. Subsequent stages continue in a similar fashion. Note that the size of each segment can be easily detected by a global unit. The background can be treated in the same way, although it is generally discontinuous. With multistage processing, the hierarchical system can provide results of both coarse- and fine-grain segmentation.

In Sect. 5, we mentioned that the oscillatory dynamics of one-layer LEGION has a limited segmentation capacity (see Figs. 8 and 9). It is interesting to note that the human perceptual system is also limited in simultaneously attending to the objects in a scene, the capacity of which seems to be around seven (Miller 1956). The multistage processing outlined above provides a natural way out of this fundamental limitation. In multistage processing, each layer does not need to segregate more than several segments, and yet the system as a whole can segregate many more segments than the segmentation capacity - an idea reminiscent of chunking proposed by Miller (Miller 1956). When the number of segments in an image is greater than the segmentation capacity, one-layer LEGION will produce a number of segments (simple or conglomerate) up to the segmentation capacity (see Figs. 8 and 9). Conglomerate segments, however, can be further segmented with another layer of LEGION, whereas simple segments will not segment further. Thus, we expect that the segments separated from each image in Sect. 6, despite they number higher than the segmentation capacity, will still be valid segments when a multilayer LEGION network is used.

7.3 Biological Relevance

The relaxation-type oscillator used in LEGION is dynamically very similar to numerous other oscillators used in modeling neuronal behavior. Examples include the FitzHugh-Nagumo equations (FitzHugh 1961; Nagumo *et al.* 1962), and the Morris-Lecar model for electrical activity in the barnacle muscle fiber (Morris and Lecar 1981). These can all be viewed as simplifications of the Hodgkin-Huxley equations (Hodgkin and Huxley 1952). In (1), the variable x corresponds to the membrane potential of the neuron, and y corresponds to the channel activation or inactivation state variable which evolves on the slowest time scale. The reduction from a full Hodgkin-Huxley model to the two variable model is achieved by assuming that the other, faster, channel state

variables are instantaneous. Of course, the behavior of a single neuron can be quite complicated. It may, for example, exhibit bursting oscillations in which the dynamics alternates between a silent phase of near steady state behavior and an active phase of rapid spiking. In this case, the two variable model in (1) can be viewed as a model for the envelope of a bursting neuron.

The mechanism of selective gating is consistent with the growing body of evidence that supports the existence of neural oscillations in the visual cortex and other brain regions. In the visual system, synchronous oscillations have been observed in cell recordings of the cat visual cortex (Eckhorn *et al.* 1988; Gray *et al.* 1989). The main discovery of these experiments can be summarized in the following. First, neural oscillations are triggered by sensory stimulation, thus known as stimulus-dependent. These oscillations range from 30 to 70 Hz, often referred to as *40 Hz oscillations* (or γ rhythms). Second, synchronous oscillations (locking with zero phase lag) occur across an extended brain region if the stimulus constitutes a coherent object. Third, no phase locking exists across regions stimulated by different stimuli if they are not related with each other. These basic findings have been confirmed repeatedly in different brain regions and in different animal species (Engel *et al.* 1991b; Engel *et al.* 1991a; Murthy and Fetz 1992; Sanes and Donoghue 1993; Bressler *et al.* 1993; Sillito *et al.* 1994; Buzsáki *et al.* 1994; Whittington *et al.* 1995; Singer and Gray 1995). Similar phenomena have also been observed in the olfactory and auditory systems.

The local excitatory connections assumed in LEGION conform with various lateral connections in the brain. Relating to the visual cortex, these excitatory connections, which link the excitatory elements of oscillators, could be interpreted as the horizontal connections in the visual cortex (Gilbert and Wiesel 1989; Gilbert 1992). It is known that horizontal connections originate from pyramidal cells, which are of excitatory type, and pyramidal cells are also the principal target of the horizontal connections (Gilbert *et al.* 1990; Gilbert 1992). Furthermore, at the functional level, physiological recordings from monkeys suggest that motion-based visual segmentation may be processed in the primary visual cortex (Stoner and Albright 1992; Lamme *et al.* 1993). The global inhibitor (see Fig. 3) receives input from the entire oscillator network, and feeds back inhibition onto the network. It serves to segment multiple patterns simultaneously present in a visual scene, thus exerting a global coordination. Crick has suggested that part of the thalamus, the thalamic reticular complex in particular, may be involved in the global control of selective attention (Crick 1984; Crick 1994). The thalamus is uniquely located in the brain: it receives input from and sends projections to almost the entire cortex. This suggestion and key anatomical and physiological properties of the thalamus prompt us to speculate that the global inhibitor might correspond to a neuronal group in the thalamus. The activity of the global inhibitor should be interpreted as the collective behavior of the neuronal group.

An often-cited theoretical problem with the grandmother-cell representation is exponential explosion in cell use: The number of cells required grows exponentially with the number of features which can be used to construct objects, and yet only a tiny fraction out of all possible feature combinations are actually used to represent patterns (von der Malsburg 1981; Singer 1993; Anderson 1995, Chapter 10). However, as far as memory or learned patterns are concerned, this problem can be solved by some computational trick. For example, in Wang and Yuwono (1995), a self-organization technique is proposed that automatically selects high-level units when needed. So, theoretically speaking, we only need as many high-level units as the number of memory items - no exponential explosion problem. But the problem does arise following the observation that one can segment an image into objects which are not necessarily familiar (learned). Recall from Sect. 1 that for the grandmother-cell representation, every object, whether it is familiar or unfamiliar, would need to use a neuron. Otherwise, there would be no representation for unfamiliar objects. Thus if we allocate neurons to all possible unfamiliar objects that just might occur in the future, the problem of exponential cell use is bound to arise.

7.4 Perceptual Relevance

In a recent theoretical account of perceptual organization, Palmer and Rock attached the special importance to the so-called uniform connectedness, and regarded the formation of uniform and connected regions as the first step in perceptual organization, the result of which is subject to later, possibly interactive, stages of refinement (Palmer and Rock 1994b; Palmer and Rock 1994a). Their emphasis on uniform connectedness agrees well with our emphasis on emergent synchrony based on local connectivity, which reflects both connectedness and uniformity. Selective gating provides a neurocomputational foundation for generating uniform and connected regions, accomplishing a fundamental part of their theoretical framework. On the other hand, their theory provides psychological justifications to our model, and gives useful suggestions about how to improve the initial results by later stages of refinement. The multistage processing discussed above is a step in this direction.

We regard the rates of synchrony and desynchrony as particularly important, not only because speed is critical for real time scene segmentation but also because of the considerations of neural plausibility. It is known that human subjects can separate and identify an object in a visual scene in less than 100 ms (Biederman 1987; Irving Biederman, personal communication, 1994). This suggests that both synchrony and desynchrony must be achieved in just a few cycles if 40 Hz oscillations are taken to be the underlying mechanism. Considering that the visual environment may change quickly and the subject may move around, it is not surprising that perceptual organization must be achieved rapidly in order for the subject to make sense of its visual environment. Physiological recordings of synchronous oscillations also suggest that synchrony seems to be a transient event, lasting for only a few cycles (Murthy and Fetz 1992; Sanes and Donoghue 1993; Bressler *et al.* 1993).

7.5 Figure-Ground Segregation

This paper addresses the problem of fragmentation by introducing the concept of potential for each oscillator, which increases when the oscillator receives significant input from its potential neighborhood. As a result, a scene is separated to a number of major segments and a background, which corresponds to the rest of the scene. The major segments combine to form the foreground, whose corresponding oscillators are oscillatory until the input scene fades away. The oscillators corresponding to the background, after a brief beginning period, become excitable and stop oscillating. This dynamics effectively gets rid of noisy fragments without either preprocessing (such as smoothing) or postprocessing (such as removing small regions), the methods often used in segmentation algorithms. With this dynamics, typical figure-ground segregation can be characterized as a special case, where only one major segment is allowed to be separated from the scene. In this sense, we claim that our dynamics also provides a potential solution to the problem of figure-ground segregation. We allow a foreground to include multiple segments, because this way both scene segmentation and figure-ground segregation are incorporated in a unified framework.

7.6 Future Topics

In the present study, we have not addressed the role of prior knowledge in image segmentation. Knowledge has long been recognized as playing an important role in perceptual organization by Gestalt psychologists (Koffka 1935; Rock and Palmer 1990; see Sect. 1). For example, when people segment the images of Fig. 16A and Fig. 17A, they inevitably use their knowledge of human anatomy, which describes among other things the relative size and position of major brain regions. A more complete system of image segmentation must address this issue. We note that studies have been conducted to segment a mixture of random patterns based on oscillatory associative memories (Wang *et al.* 1990; Horn and Usher 1991; Sompolinsky and Tsodyks 1994).

Besides prior knowledge, many grouping principles outlined in Sect. 1 have not been incorporated into the system. One of the main future topics is to incorporate more grouping cues

into the system. The way to do so in our approach is to embed them in connection weights between oscillators. A general issue in incorporating multiple cues is how to set relative importance of each cue for segmentation. Two simple strategies are winner-take-all and linearly weighted combination. The integration process gets tricky when different cues produce contrasting results of segmentation. The global inhibitory mechanism will play a key role in overall system coordination: it makes various factors compete with each other, and a final segment is formed because of strong binding within the segment. The general criterion for the integration should be that it must enhance the overall system performance.

Our study in this paper focuses exclusively on visual segmentation. It should be noted that neural oscillations occur in other modalities as well, including audition (Galambos *et al.* 1981; Ribary *et al.* 1991) and olfaction (Freeman 1978). Strikingly, these oscillations in different modalities show comparable frequencies, all characterized by γ rhythms. In a recent study, Wang extended LEGION to deal with auditory scene segregation, and modeled a number of psychophysical findings regarding auditory scene analysis (Wang in press). With its computational properties and its biological relevance, the oscillatory correlation approach promises to provide a general neurocomputational theory for scene segmentation and perceptual organization.

8. Conclusion

We have proposed a mechanism for locally excitatory globally inhibitory oscillator networks so that only those oscillators that receive substantial lateral excitation can lead oscillator blocks to jump to the active phase. The stimulated oscillators in the neighborhood of a leader can be recruited to jump up as well, but those stimulated by noisy regions cannot oscillate after a short period of time. This state of oscillatory dynamics is proven to be globally stable, and is demonstrated by computer simulation. The analysis is constructive in nature and shows how the parameters of the dynamical system should be chosen to achieve image segmentation. An algorithm has been abstracted from the model, and it has been applied to real image segmentation. The LEGION algorithm produces a number of major homogeneous regions and puts the rest of the image into a background. We have discussed the model's biological plausibility, its theoretical role in perceptual organization, and its possible extensions. The mechanism of selective gating lays a neurocomputational foundation for the theory of oscillatory correlation, and may provide an effective device for automatic scene segmentation and figure-ground segregation.

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Figure Caption

Figure 1. A caricature of an image with three objects that appear on a noisy background. The noiseless caricature is adapted from (Terman and Wang 1995).

Figure 2. Nullclines and orbits of a single oscillator (We thank S. Campbell for making this figure). **A.** If $I > 0$ and $H = 1$, the oscillator is enabled. The periodic orbit is shown with a bold curve, and its direction of motion is indicated by the arrowheads. The left and the right branches of the x -nullcline are labeled as L and R, respectively. LK and RK indicate the left and the right knees of the cubic, respectively. **B.** If $I \leq 0$ and $H = 1$, the oscillator is excitable. The fixed point P_I on the left branch of the cubic is asymptotically stable.

Figure 3. Architecture of a two dimensional LEGION network with four nearest-neighbor coupling. An oscillator is indicated by an open circle, and the global inhibitor is indicated by the filled circle.

Figure 4. The effects of two neighborhoods. $N_1(i)$ is the potential neighborhood, and $N_2(i)$ is the coupling neighborhood.

Figure 5. **A** An image composed of four patterns which are presented to a 25x25 LEGION network (adapted from Wang and Terman 1995). Each square corresponds to an oscillator. Filled squares indicate stimulated oscillators, while open squares indicate unstimulated oscillators. **B** The image in **A** is corrupted by 10% noise. **C** A snapshot of the activities of the network at the beginning of dynamic evolution. **D** Another snapshot taken shortly after **C**. **E** Another snapshot taken shortly after **D**. **F** Another snapshot taken shortly after **E**. **G** Another snapshot taken shortly after **F**.

Figure 6. Temporal evolution of every stimulated oscillator. The upper four traces show the combined temporal activities of the four oscillator blocks representing the four corresponding patterns indicated by their respective labels. The fifth trace shows the temporal activities of the loners corresponding to the background, and the bottom trace shows the activity of the global inhibitor. The ordinates except for the bottom one in the figure indicate the normalized x -activity of an oscillator, and the inhibitor trace is drawn according to the normalized activity. The simulation took 8,000 integration steps.

Figure 7. Another simulation with an alternative definition for a single oscillator. See the legend of Fig. 6 for explanations. As in Fig. 6, this simulation took 8,000 integration steps.

Figure 8. **A** An image composed of nine patterns which are presented to a 30x30 LEGION network. See the legend of Fig. 5 for explanations. **B** A snapshot of network activity at the beginning of dynamic evolution. **C** Another snapshot taken shortly after **B**. **D** Another snapshot taken shortly after **C**. **E** Another snapshot taken shortly after **D**. **F** Another snapshot taken shortly after **E**. **G** Another snapshot taken shortly after **F**.

Figure 9. Temporal evolution of every stimulated oscillator. The upper nine traces show the combined temporal activities of the oscillator blocks representing the nine corresponding patterns indicated by their respective labels. The bottom trace shows the activity of the global inhibitor. See the legend of Fig. 6 for further explanations. The simulation took 8,000 integration steps.

Figure 10. A scenario for producing overlapping segments. A square indicates a pixel with its value shown in the middle.

Figure 11. **A:** A gray-level image consisting of 160x160 pixels (by courtesy of K. Boyer). The image is presented to a 160x160 LEGION network. **B** One segment pops out from the network shortly after the LEGION algorithm is executed. **C** Another segment pops out shortly after **B**. **D** Another segment pops out shortly after **C**. **E** Another segment popped out shortly after **D**. **F** Another segment pops out shortly after **E**. **G** Another segment pops out shortly after **F**. (We thank E. Cesmeli for his assistance in making this figure)

Figure 12: A gray map showing the result of segmenting Fig. 11A. The algorithm produces 16 segments plus a background. The algorithm was run for 1,000 time steps.

Figure 13. A gray map showing the result of segmenting Fig. 11A. Maximization is used to compute S_i . The algorithm produces 17 segments plus a background. The algorithm was run for 1,000 steps.

Figure 14. Comparison between summation and maximization in computing S_i . **A** and **B** show two different scenarios.

Figure 15. A gray map showing the result of segmenting Fig. 11A with a different value for θ_p . The system produces 23 segments plus a background. The algorithm was run for 1,000 steps.

Figure 16. A A gray-level image consisting of 256x256 pixels (by courtesy of N. Shareef). **B** A gray map showing the result of segmenting the image in **A** by a 256x256 LEGION network. The system produces 21 segments plus a background. **C** A gray map showing the result of segmenting the image in **A** with different values of W_z and θ_p . The system produces 25 segments plus a background. The algorithm was run for 1,200 steps in both **B** and **C**.

Figure 17. A A gray-level image consisting of 256x256 pixels (by courtesy of N. Shareef). **B** A gray map showing the result of segmenting the image in **A** by a 256x256 LEGION network. The system produces 17 segments plus a background. **C** A gray map showing the result of segmenting the image in **A** with different values of W_z and θ_p . The system produces 13 segments plus a background. The algorithm was run for 1,200 steps in both **B** and **C**.

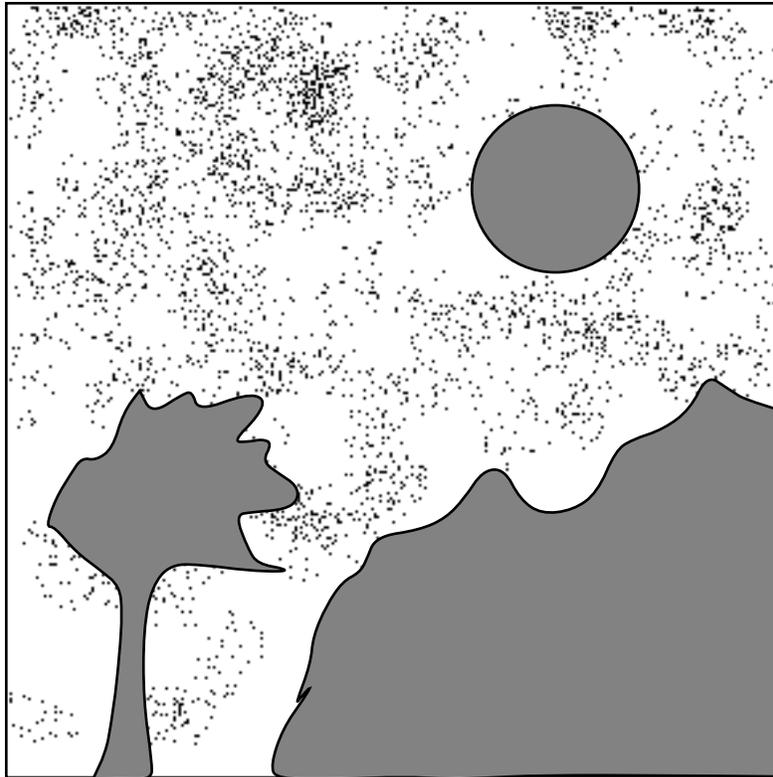


Figure 1

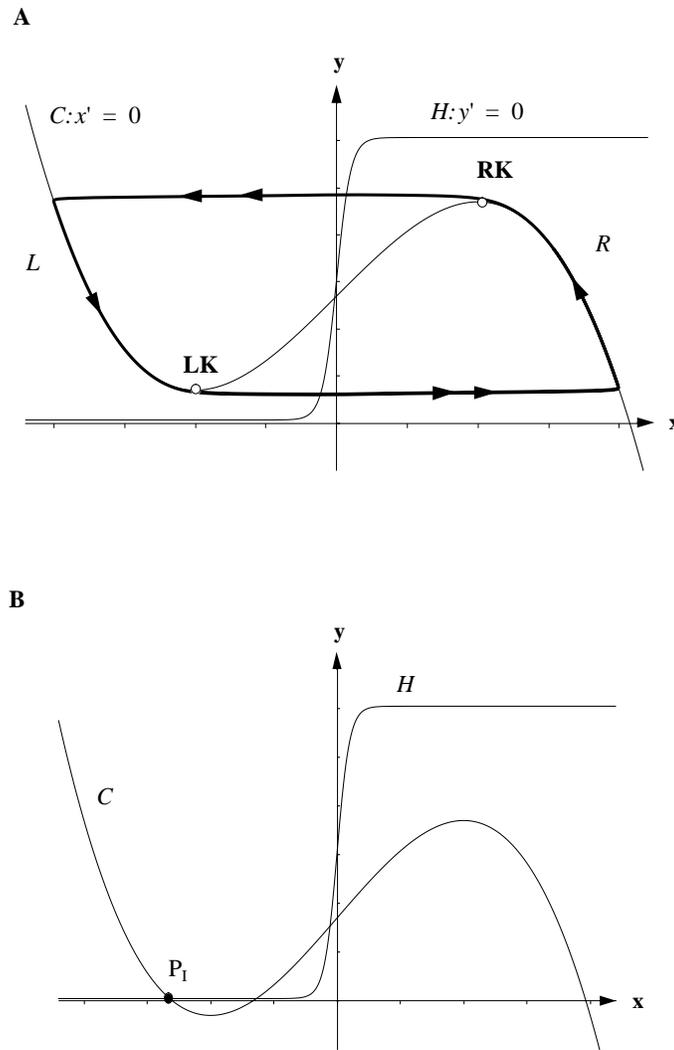


Figure 2

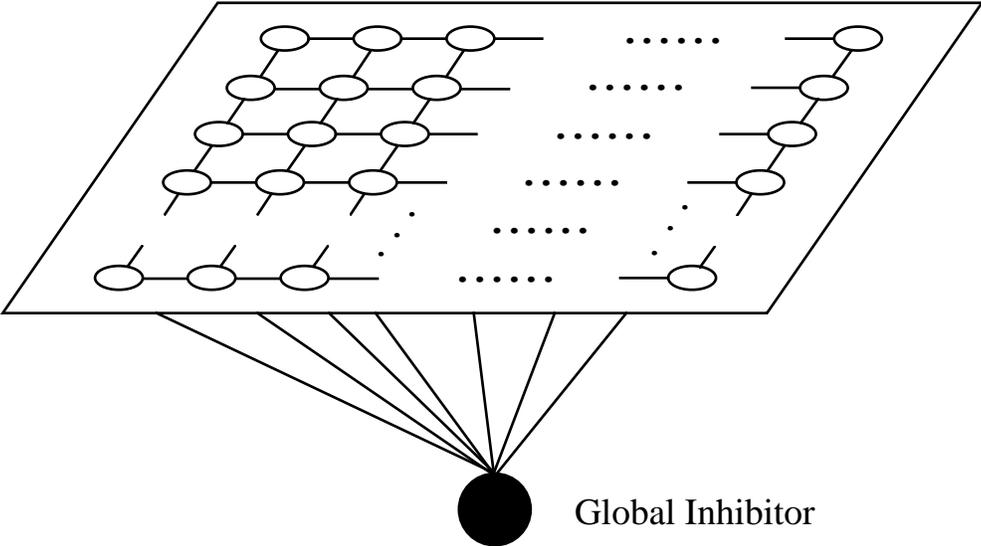


Figure 3

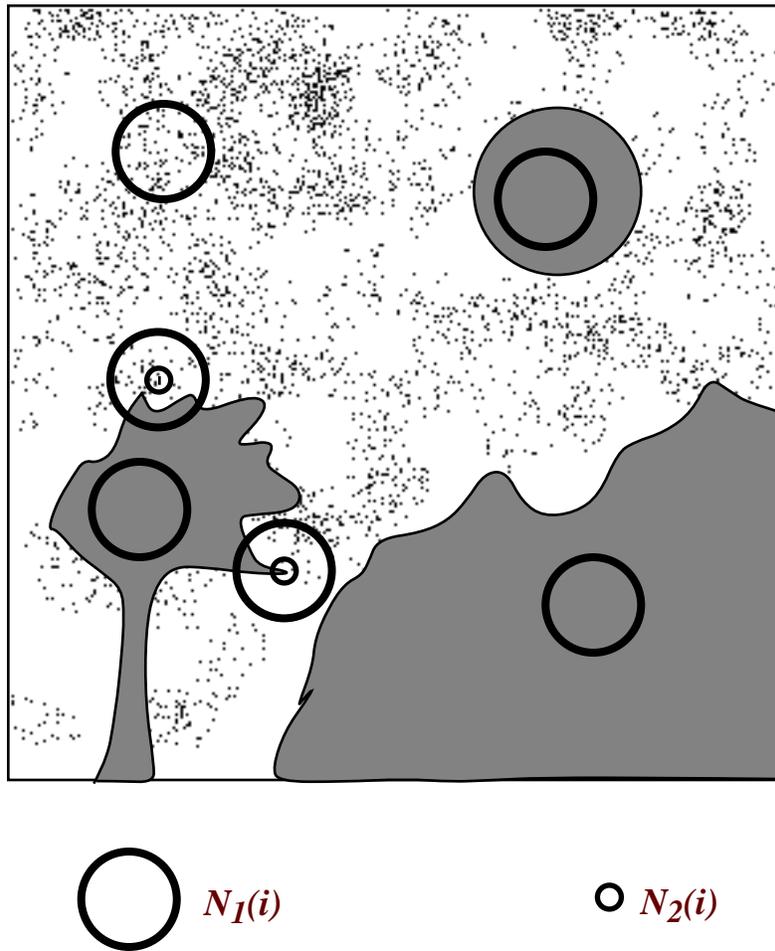


Figure 4

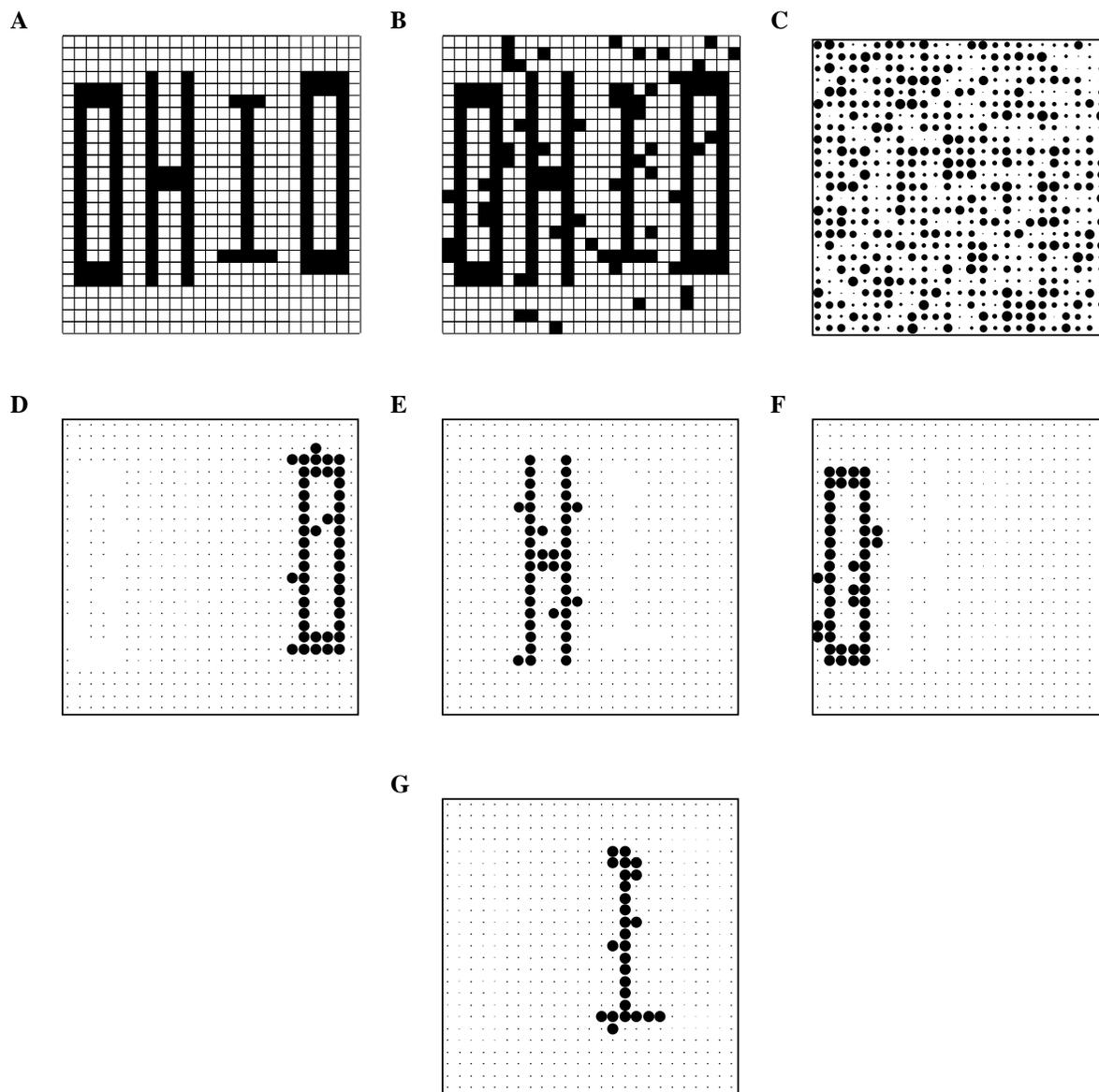


Figure 5

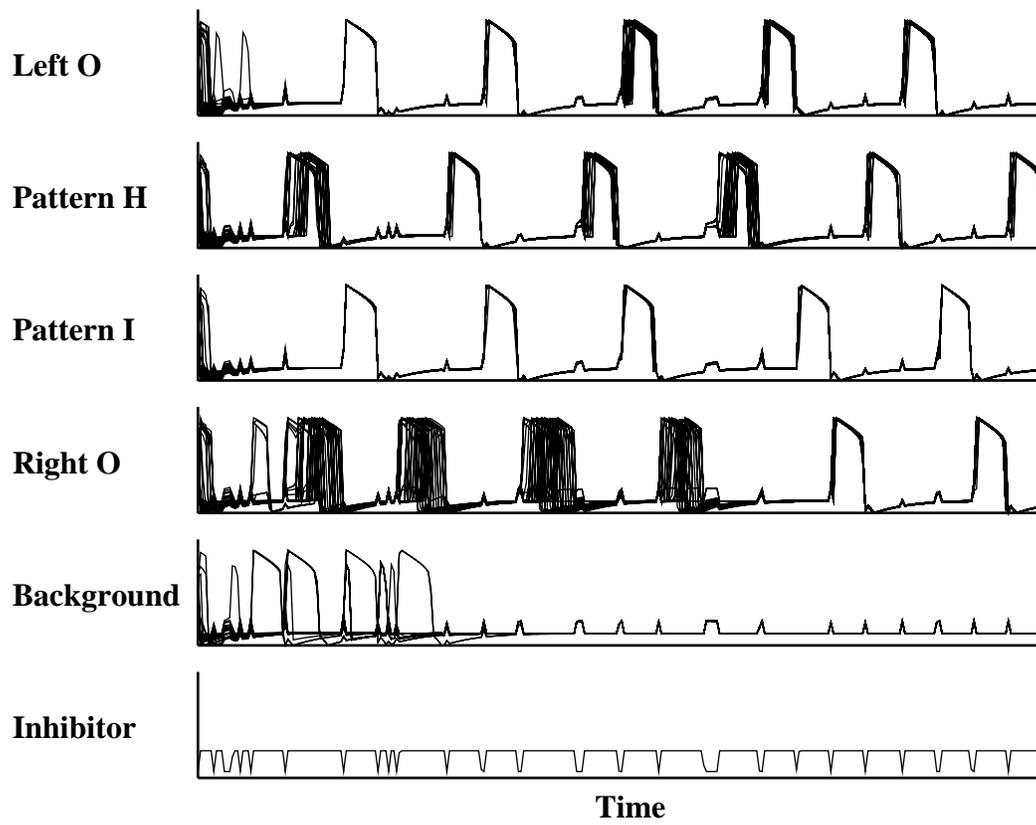


Figure 6

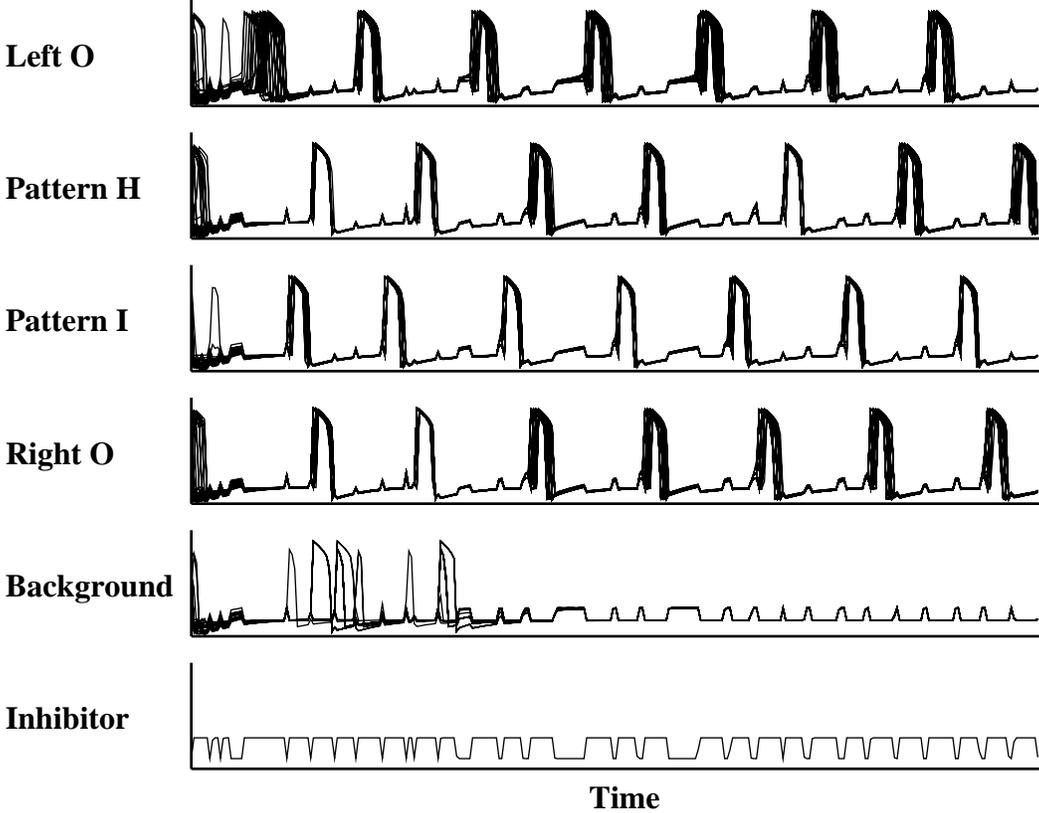


Figure 7

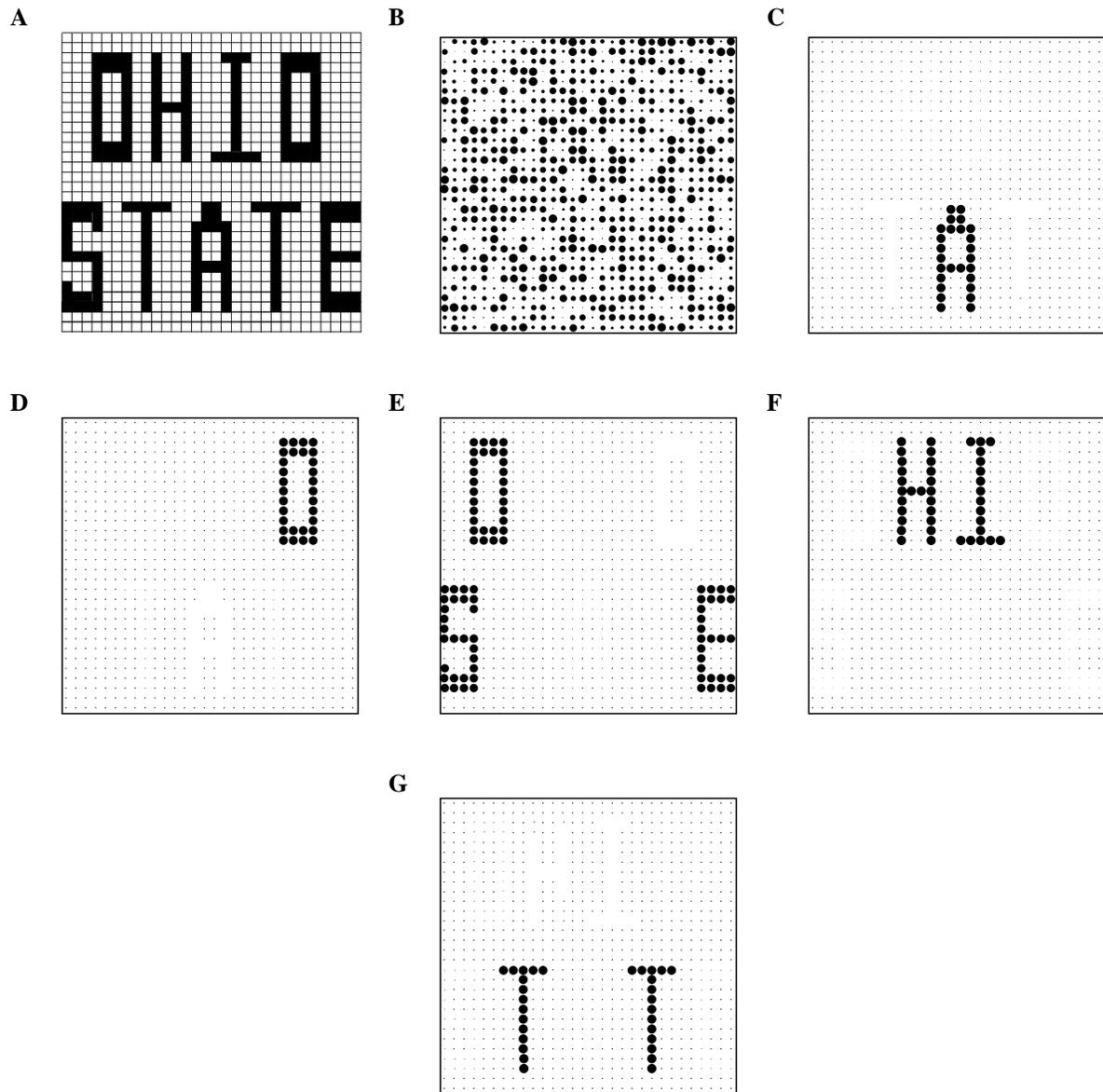


Figure 8

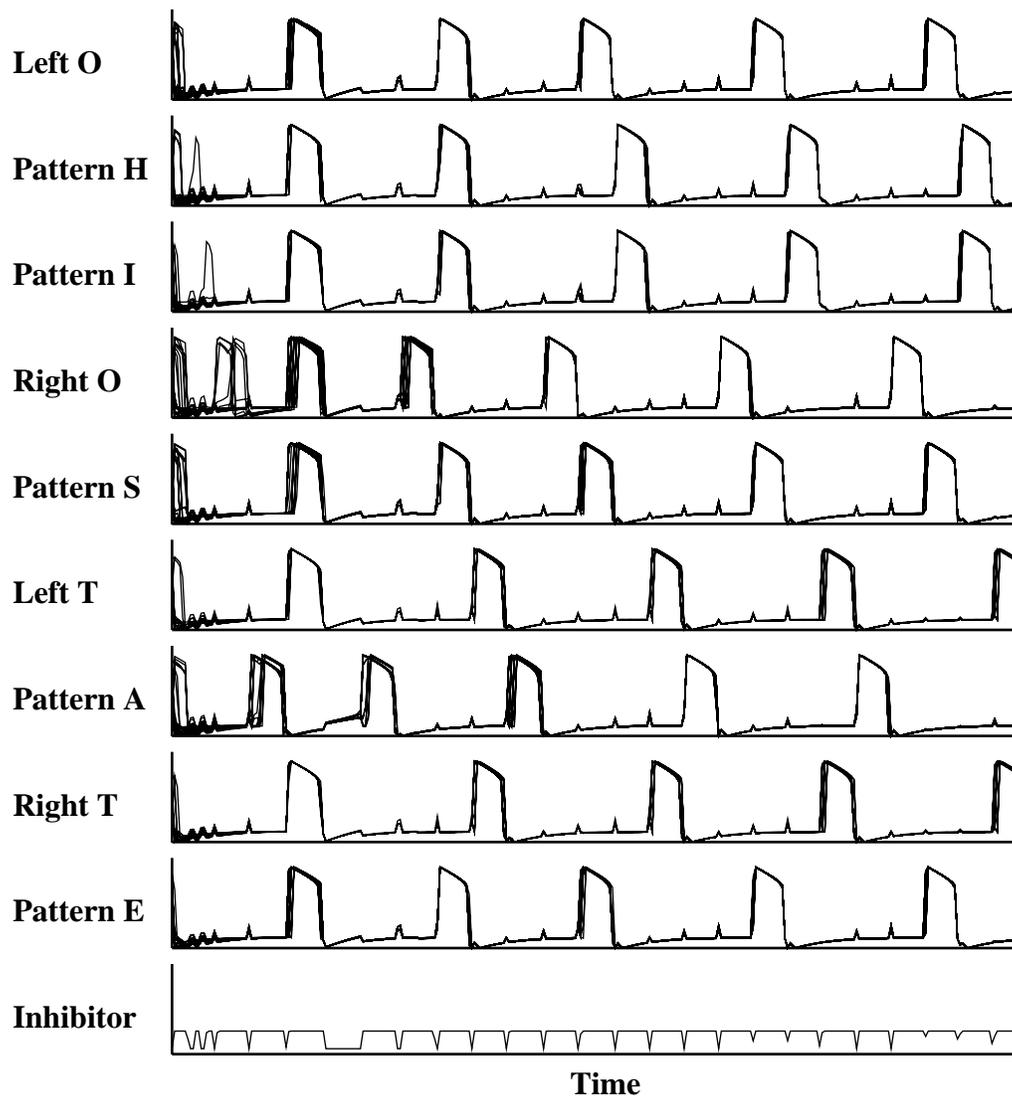


Figure 9

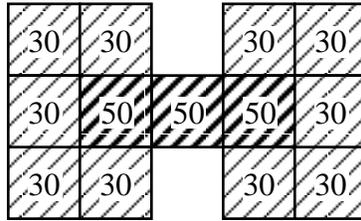


Figure 10

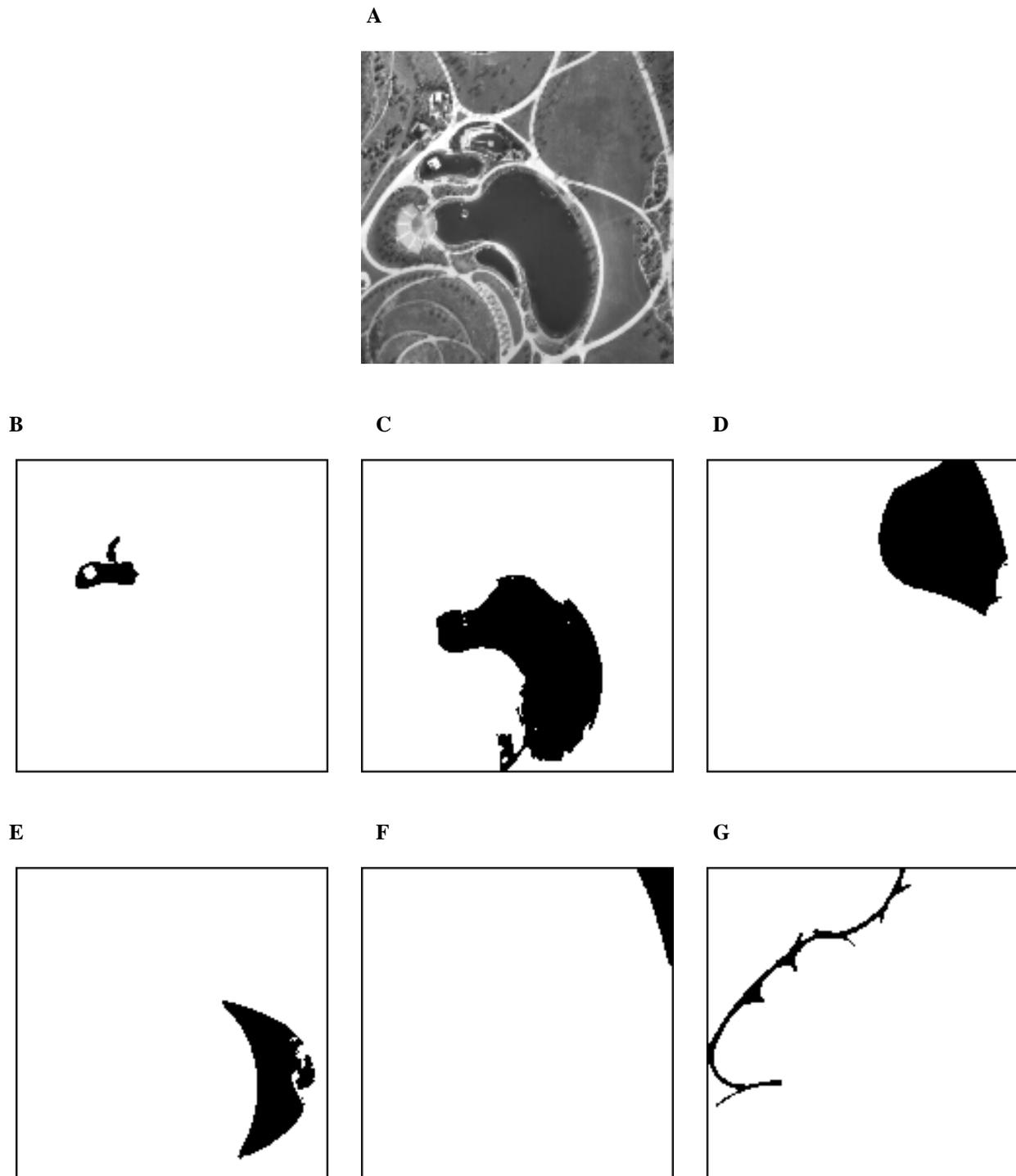


Figure 11



Figure 12



Figure 13



	A	B
Summation	grouping	no grouping
Maximum	no grouping	grouping

Figure 14



Figure 15

A



B

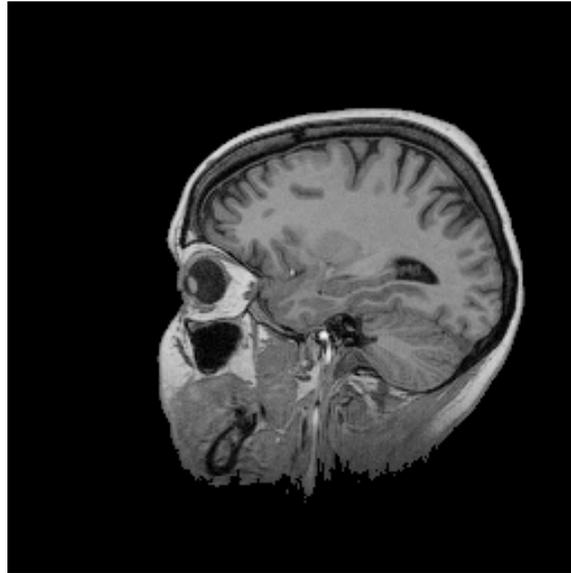


C

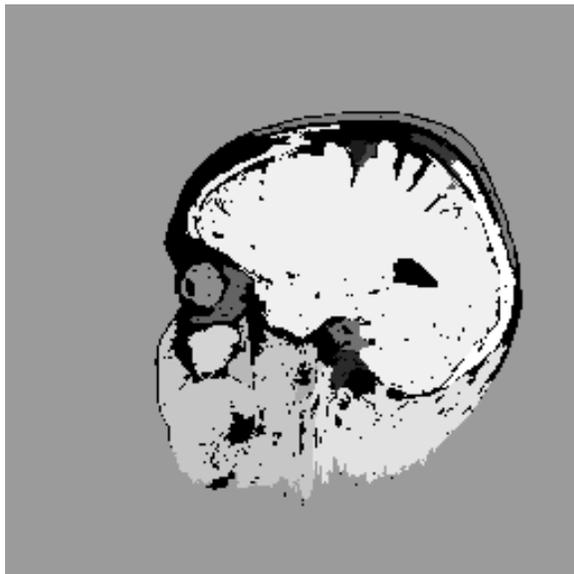


Figure 16

A



B



C

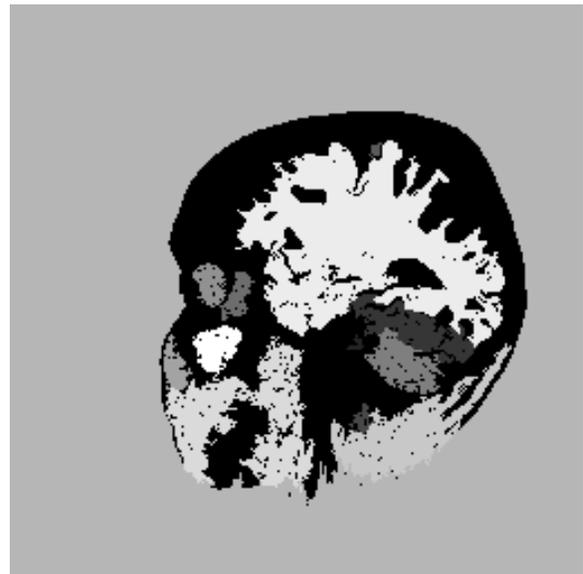


Figure 17