Surfaces without boundaries

As we saw in the last class a rectangular array of control points generate a $B$-spline surface with four boundary curves. However, we can identify the boundary curves to produce surfaces without boundaries.

If we identify two boundary curves only, we get a cylinder. This can be achieved, for example, by taking the $w = constant$ curves to be closed. Then we have:

$$s \in [1 : m + 1], t \in [1 : n + 2 - R]$$

$$i \in [(s - 1)mod(m + 1) : (s + D - 2)mod(m + 1)]$$

$$j \in [t - 1 : t + R - 2]$$

Similar expressions hold when the $u = constant$ curves are closed. See Figure 9.6 for an example of a cylinder. Here $D = 4$ and $R = 3$, with $m = 3$ and $n = 4$ to produce $4 \times 3$ patch array.

If both $u = constant$ and $w = constant$ curves are closed we get a closed surface.

Number of control points

The number of control points needed for a patch is related to the degree of the basis functions. If we have $D = R$, we require $D^2$ control points to produce a single patch. For producing $n_a \times n_a$ array of patches we need $(n_a + D - 1)^2$ control points. In general, to produce $n_a \times n_b$ array of patches, one needs $(n_a + D - 1) \times (n_b + R - 1)$ control points. If the surface is closed in the $w = constant$ direction then we need $n_a(n_b + R - 1)$ control points.

NURBS

The NURBS surface is like the rational Bézier surface. It is expressed as:

$$p(u, w) = \frac{\sum_{i=0}^{m} \sum_{j=0}^{n} h_{ij} p_{ij} N_{i,D}(u) N_{j,R}(w)}{\sum_{i=0}^{m} \sum_{j=0}^{n} h_{ij} N_{i,D}(u) N_{j,R}(w)}$$

where $h_{ij}$ are the weights. As with rational curves, increasing the weights pulls the surface towards the corresponding control point.

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1Note by Tamal K. Dey, Ohio State U.