1. Give an example of INFINITE languages $L_1$ and $L_2$ over $\{a, b\}$ where $L_1 \not\subseteq L_1L_2$ and $L_2 \not\subseteq L_1L_2$. Justify your answer by giving strings in $L_1$ and $L_2$ which are not in $L_1L_2$.

2. Give an example of languages $L_1$ and $L_2$ over $\{a, b\}$ where $(L_1 \cup L_2)^* \neq L_1^* \cup L_2^*$. Justify your answer by giving a string in $(L_1 \cup L_2)^*$ which is not in $L_1^* \cup L_2^*$.

3. Give an example of languages $L_1$ and $L_2$ over $\{a, b\}$ where $(L_1 \cup L_2) \neq \overline{\overline{L_1} \cup \overline{L_2}}$. Justify your answer by giving a string in $\overline{\overline{L_1} \cup \overline{L_2}}$ which is not in $(L_1 \cup L_2)$.

4. Give an example of a language $L$ over $\{a, b\}$ where $L = L^*$ but $\overline{L} \neq (\overline{L})^*$. Justify your answer by giving a string which is in $(\overline{L})^*$ but not in $\overline{L}$.

5. Give an example of a language $L$ where $L \neq LL$ but $LL = L^*$. Justify $L \neq LL$ by giving a string which is in $LL$ but not in $L$.

(The grader will only grade a subset of these problems. For more practice, try Martin 1.44, 1.45, 1.46, 1.47, 1.50, 1.51.)