Curve and Surface Reconstruction: Algorithms with Mathematical Analysis

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Preface

The subject of this book is the approximation of curves in two dimensions and surfaces in three dimensions from a set of sample points. This problem, called *reconstruction*, appears in various engineering applications and scientific studies. What is special about the problem is that it offers an application where mathematical disciplines such as differential geometry and topology interact with computational disciplines such as discrete and computational geometry. One of my goals in writing this book has been to collect and disseminate the results obtained by this confluence. The research on geometry and topology of shapes in the discrete setting has gained a momentum through the study of the reconstruction problem. This book, I hope, will serve as a prelude to this exciting new line of research.

To maintain the focus and brevity I chose a few algorithms that have provable guarantees. It happens to be, though quite naturally, they all use the well known data structures of the Voronoi diagram and the Delaunay triangulation. Actually, these discrete geometric data structures offer discrete counterparts to many of the geometric and topological properties of shapes. Naturally, the Voronoi and Delaunay diagrams have been a common thread for the materials in the book.

This book originated from the class notes of a seminar course "Sample Based Geometric Modeling" that I taught for four years at the graduate level in the computer science department of The Ohio State University. Graduate students entering or doing research in geometric modeling, computational geometry, computer graphics, computer vision and any other field involving computations on geometric shapes should benefit from this book. Also, the teachers in these areas should find this book helpful in introducing materials from differential geometry, topology, and discrete and computational geometry. I have made efforts to explain the concepts intuitively whenever needed, but retained the mathematical rigor in presenting the results. Lemmas and theorems have been used to state the results precisely. Most of them are equipped with proofs that bring out the insights. For most parts, the materials are self-explanatory. A motivated graduate student should be able to grasp the concepts through a careful reading. The exercises are set to stimulate innovative thoughts and the readers are strongly urged to solve them as they read along.

The first chaper describes the necessary basic concepts in topology, Delaunay and Voronoi diagrams, local feature size and ε-sampling of curves and surfaces. The second chapter is devoted to curve reconstruction in two dimensions. Some general results based on ε-sampling are presented first followed by two algorithms and their proofs of correctness. Chapter three presents results connecting surface geometries and topologies with ε-sampling. For example, it is shown that the normals and the topology of the surface can be recovered from the samples as long as the input is sufficiently dense. Based on these results, an algorithm for surface reconstruction is described in chapter four with its proofs of guarantees. Chapter five contains results on undersampling. It presents a modification of the algorithm presented in chapter four. Chapter six is on computing watertight surfaces. Two algorithms are described for the problem. Chapter seven introduces the case where sampling is corrupted by noise. It is shown that, under a reasonable noise model, the normals and the medial axis of a surface can still be approximated from a noisy input. Using these results a reconstruction method for noisy samples is presented in chapter eight. The results in chapter seven are also used in chapter nine where a method to smooth out the noise is described. This smoothing is achieved by projecting the points on an implicit surface defined with a variation of the least squares method. Chapter ten, the last one, is devoted to reconstruction algorithms based on Morse theoretic ideas. Discretization of Morse theory using Voronoi and Delaunay diagrams is the focus of this chapter.

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