Midterm – Sample

CSE 680: Introduction to Algorithms and Data Structures

Name: ____________________

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1. Solve the following recurrences by giving tight $\Theta$-notation bounds using the Master Method. If I thought you would need the actual value of a logarithm, I have given it to you.
   
   (a) $T(n) = 8T(n/2) + n^3$

   (b) $T(n) = T(n/5) + \lg^2 n$
2. Write a recursion formula for the running time $T(n)$ of the function $\text{Foo}$, whose code is below. Find a solution to the recurrence and prove it using the substitution (or iterative) method.

```c
int Foo(int n) {
    if (n < 3) return 0;
    int sum = 0;
    for(i = 1; i < n; i=2*i)
        for( j = 1; j < n; j++)
            sum = j*j;
    sum += Foo(n/3);
}
```
3. **Show each execution step of the Counting sort for the following** set of numbers: 45, 23, 12, 23.
4. What is the average case Big-O of the following code as a function of N?

```java
for(int i = 0; i < N; i++)
    for(int k = 1; k < 5*N; k *= 4)
        total += 1;
```
5. The diagrams below depict a simple minimum heap.

(a) Insert the integer 2 into the heap. If a node’s contents change, write beside the node what its new contents are. Show all work, but clearly mark the final result.

(b) Remove the minimum from the heap (starting with the original). If a node’s contents change, write beside the node what its new contents are. Show all work, but clearly mark the final result.
6. Describe and compare Quicksort and Mergesort. What are the complexities of each algorithm, what does each do (high level only – points will be taken off if you give me code), how do they differ and when do they work well or not so well.
Consider the following heap. Insert the value 42 assuming quadratic probing and the hash function key = value mod 17. Show all work: