Sample Midterm 2 for CSE 541

Roger Crawfis

Name:_____________________

You will be graded on how you obtain your answer, not the answer itself. Check your answers to see if they are correct, once you have found a solution.

Problem 1.
Using Romberg integration, compute $R(5,2)$ from $R(3,0)=R(4,0)=16$ and $R(5,0)=-8$.

Problem 2.
Approximate the integral below using the composite trapezoid rule for $x=0,1,2,3,4,5$.

$$\int_0^5 x^2dx$$

Problem 3.
Write the following system of equations in Matrix form using the order as given:

$$3x_1 - 4x_2 + 5x_4 = 4$$
$$x_1 + 2x_2 - x_3 = 2$$
$$3x_2 + 4x_3 + x_4 = 8$$
$$x_2 - 3x_4 = -2$$

Problem 4.
What is the residual vector is an approximate solution $x^* = (1,0,-1,1)^T$ for the above system of equations?

Problem 5.
Give the LU factorization of the matrix $A$ given below and solve of $x$ given the two different $b$ vectors given below.

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \quad b_1 = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \quad b_2 = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

Problem 6.
Consider the linear system
\[
\begin{bmatrix}
10 & -2 \\
-2 & 10
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
18 \\
6
\end{bmatrix}
\]

**Problem 7.**
Perform two iterations of Gauss-Seidel, starting with the zero vector. Consider the linear system
\[
\begin{bmatrix}
100 & 1 \\
1 & 100
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
11 \\
100.1
\end{bmatrix}
\]

**Problem 8.**
Perform two iterations of Gauss-Seidel, starting with the zero vector.

**Problem 9.**
Discuss how you would find the area of the figure listed below. Be very specific in your answer. Also, give an estimate (and justification) as to what is needed to achieve a relative error of 1%.