Visibility Algorithms

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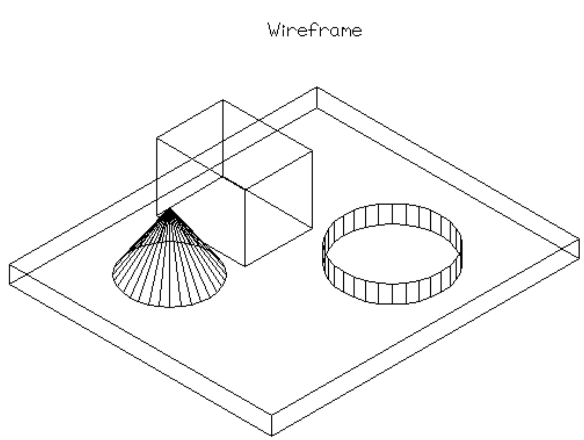
CIS 781

This set of slides reference slides used at Ohio State for instruction by Prof. Machiraju and Prof. Han-Wei Shen.

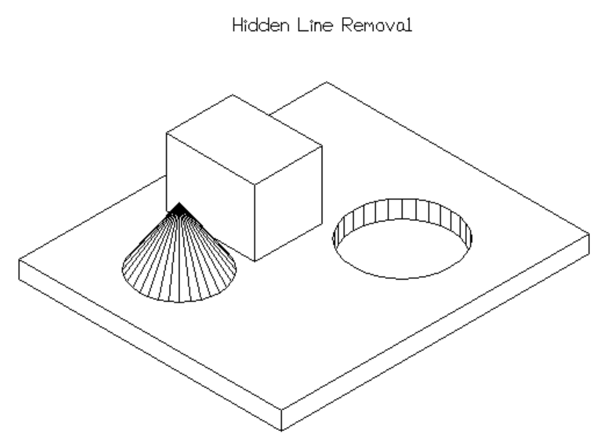
Visibility Determination

- AKA, hidden surface elimination

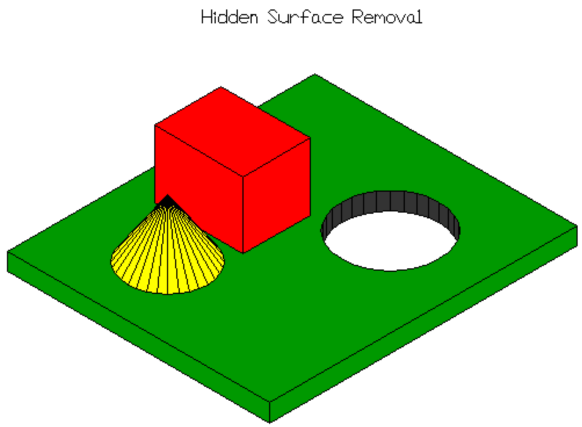
Hidden Lines



Hidden Lines Removed



Hidden Surfaces Removed

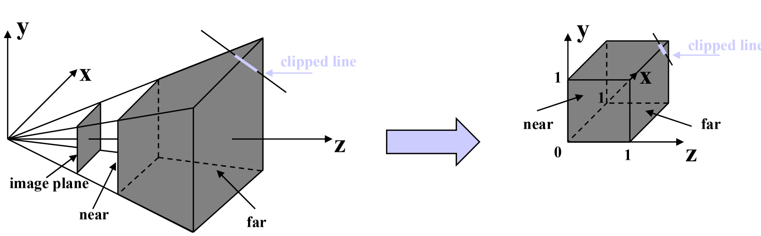


Topics

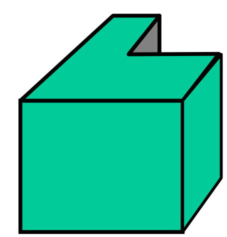
- Backface Culling
- Hidden Object Removal: Painters Algorithm
- Z-buffer
- Spanning Scanline
- Warnock
- Atherton-Weiler
- List Priority, NNA
- BSP Tree
- Taxonomy

Where Are We ?

- Canonical view volume (3D image space)
- Clipping done
- division by w
- z > 0



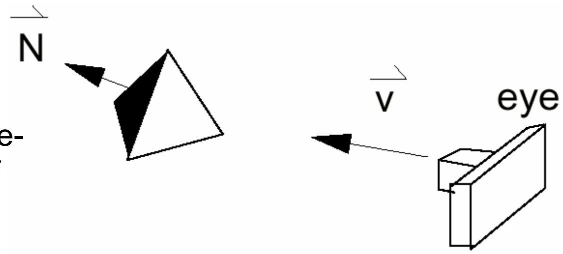
Back-face Culling



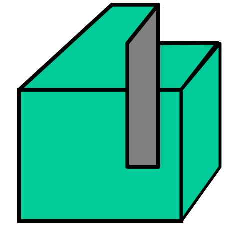
- Problems ?
- Conservative algorithm
- Real job of visibility never solved

Back-face Culling

- If a surface's normal is pointing in the same general direction as our eye, then this is a back face
- The test is quite simple: if N * V > 0 then we reject the surface
- If test is in eye-space, then if $N_z > 0$ reject.



Back-face Culling

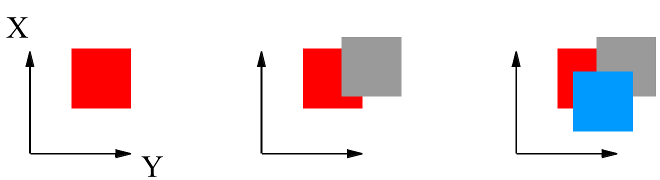


- Only handles faces oriented away from the viewer:
  - Closed objects
  - Near clipping plane does not intersect the objects
- Gives complete solution for a single convex polyhedron.
- Still need to sort, but we have reduced the number of primitives to sort.

Painters Algorithm

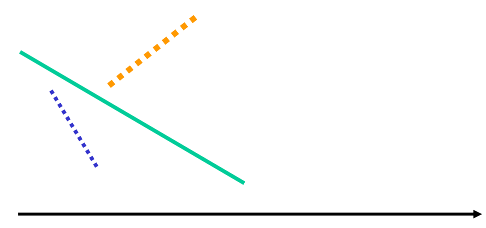
- Sort objects in depth order
- Draw all from Back-to-Front (far-to-near)
  - Simply overwrite the existing pixels.
- Is it so simple?

at z = 22, at z = 18, at z = 10,



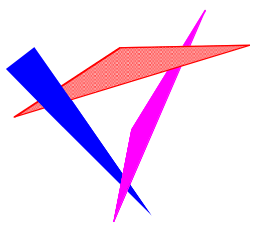
Point sorting vs Polygon Sorting

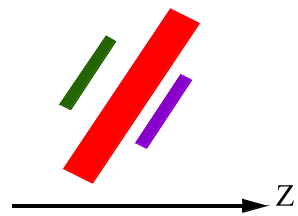
- What does it mean to ***sort*** two line segments?
  - Zmin?
  - Zmax?
  - Slope?
  - Length?



3D Cycles

- How do we deal with cycles?
- How do we deal with intersections?
- How do we sort objects that overlap in Z?





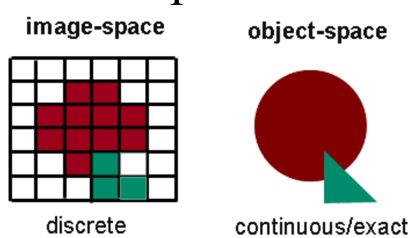
Form of the Input

Object types: what kind of objects does it handle?

- convex vs. non-convex
- polygons vs. everything else - smooth curves, non-continuous surfaces, volumetric data

Form of the output

Precision: image/object space?



- Object Space
  - Geometry in, geometry out
  - Independent of image resolution
  - Followed by scan conversion
- Image Space
  - Geometry in, image out
  - Visibility only at pixel centers

Object Space Algorithms

- Volume testing – Weiler-Atherton, etc.
  - input: convex polygons + infinite eye pt
  - output: visible portions of wireframe edges

Image-space algorithms

- Traditional Scan Conversion and Z-buffering
- Hierarchical Scan Conversion and Z-buffering
  - input: any plane-sweepable/plane-boundable objects
  - preprocessing: none
  - output: a discrete image of the exact visible set

Conservative Visibility Algorithms

- Viewport clipping
- Back-face culling
- Warnock's screen-space subdivision

Z-buffer

- Z-buffer is a 2D array that stores a depth value for each pixel.
- ***InitScreen***:

```
for i := 0 to N do
   for j := 1 to N do
     Screen[i][j] := BACKGROUND_COLOR;  Zbuffer[i][j] := ∞;
```

- ***DrawZpixel*** (x, y, z, color)

```
if (z <= Zbuffer[x][y]) then
     Screen[x][y] := color; Zbuffer[x][y] := z;
```

Z-buffer: Scanline

```
I.  for each polygon do
      for each pixel (x,y) in the polygon's projection do
         z := -(D+A*x+B*y)/C;
         DrawZpixel(x, y, z, polygon's color);

II. for each scan-line y do
      for each "in range" polygon projection do
         for each pair (x1, x2) of X-intersections do
             for x := x1 to x2 do
                 z := -(D+A*x+B*y)/C;
                 DrawZpixel(x, y, z, polygon's color);
```

If we know $z_{x,y}$ at (x,y) then: $z_{x+1,y} = z_{x,y} - A/C$

Incremental Scanline

$$Ax + By + Cz + D = 0$$

$$z = \frac{-(Ax + By + D)}{C}, C \neq 0$$

On a scan line Y = j, a constant

Thus depth of pixel at ($x_1 = x + \Delta x, j$)

$$z_1 - z = \frac{-(Ax_1 + Bj + D)}{C} + \frac{-(Ax + Bj + D)}{C}$$

$$z_1 - z = \frac{A(x - x_1)}{C}$$

$$z_1 = z - (\frac{A}{C})\Delta x \quad \text{, since } \Delta x = 1,$$

$$z_1 = z - \frac{A}{C}$$

Incremental Scanline (contd.)

- All that was about increment for pixels on each scanline.
- How about across scanlines for a given pixel ?
- Assumption: next scanline is within polygon

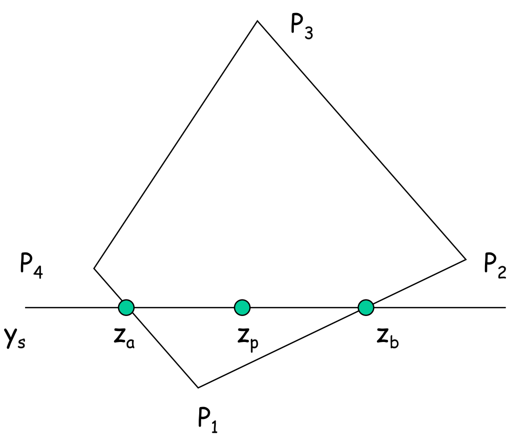
$$z_1 - z = \frac{-(Ax + By_1 + D)}{C} + \frac{(Ax + By + D)}{C}$$

$$z_1 - z = \frac{A(y - y_1)}{C}$$

$$z_1 = z - (\frac{B}{C})\Delta y \quad \text{, since } \Delta y = 1,$$

$$z_1 = z - \frac{B}{C}$$

Non-Planar Polygons



$$z_a = z_1 + (z_4 - z_1)\frac{(y_1 - y_s)}{(y_1 - y_4)}$$

$$z_b = z_1 + (z_2 - z_1)\frac{(y_1 - y_s)}{(y_1 - y_2)}$$

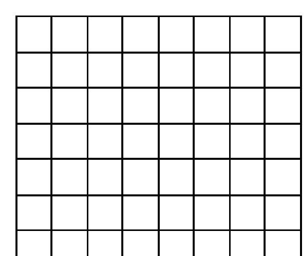
$$z_p = z_a + (z_b - z_a)\frac{(x_a - x_p)}{(x_a - x_b)}$$

Bilinear Interpolation of Depth Values

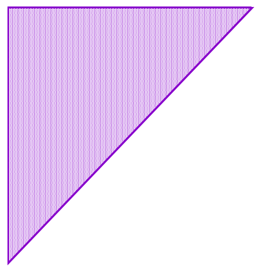
Z-buffer - Example

∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞

Z-buffer



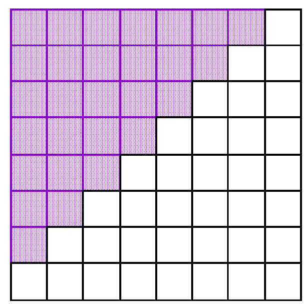
Screen



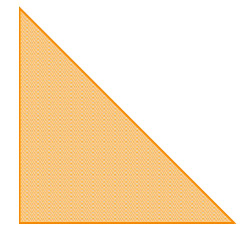
[0,1,5]

5	5	5	5	5	5	5
5	5	5	5	5	5	
5	5	5	5	5		
5	5	5	5			
5	5	5				
5	5					
5						

5	5	5	5	5	5	5	∞
5	5	5	5	5	5	∞	∞
5	5	5	5	5	∞	∞	∞
5	5	5	5	∞	∞	∞	∞
5	5	5	∞	∞	∞	∞	∞
5	5	∞	∞	∞	∞	∞	∞
5	∞	∞	∞	∞	∞	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞



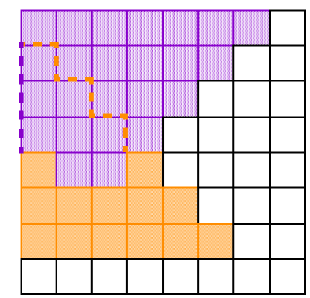
[0,6,7]



[0,1,2] [5,1,7]

7					
6	7				
5	6	7			
4	5	6	7		
3	4	5	6	7	
2	3	4	5	6	7

5	5	5	5	5	5	5	∞
5	5	5	5	5	5	∞	∞
5	5	5	5	5	∞	∞	∞
5	5	5	5	∞	∞	∞	∞
4	5	5	7	∞	∞	∞	∞
3	4	5	6	7	∞	∞	∞
2	3	4	5	6	7	∞	∞
∞	∞	∞	∞	∞	∞	∞	∞



Non Trivial Example ?

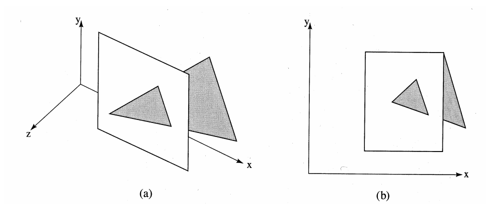


Figure 4–57 Penetrating triangle. (a) Three-dimensional view; (b) two-dimensional projection.

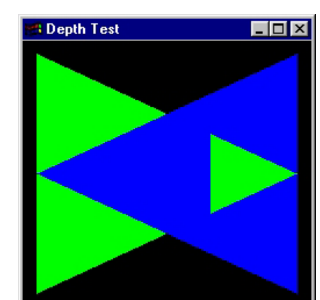
Rectangle: P1(10,5,10), P2(10,25,10), P3(25,25,10), P4(25,5,10)

Triangle: P5(15,15,15), P6(25,25,5), P7(30,10,5)

Frame Buffer: Background 0, Rectangle 1, Triangle 2

Z-buffer: 32x32x4 bit planes

Example



Z-Buffer Advantages

- Simple and easy to implement
- Amenable to scan-line algorithms
- Can easily resolve visibility cycles
- Handles intersecting polygons

Z-Buffer Disadvantages

- Does not do transparency easily
- Aliasing occurs! Since not all depth questions can be resolved
- Anti-aliasing solutions non-trivial
- Shadows are not easy
- Higher order illumination is hard in general

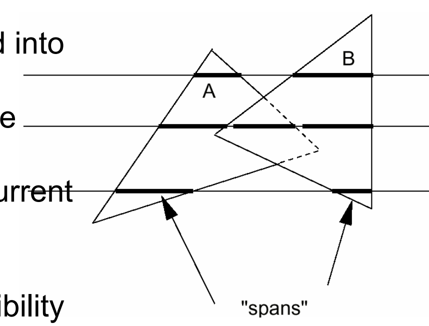
Spanning Scan-Line

Can we do better than scan-line Z-buffer ?

- Scan-line z-buffer does not exploit
  - Scan-line coherency across multiple scan-lines
  - Or span-coherence !
  - Depth coherency
- How do you deal with this – scan-conversion algorithm and a little more data structure

Spanning Scan Line Algorithm

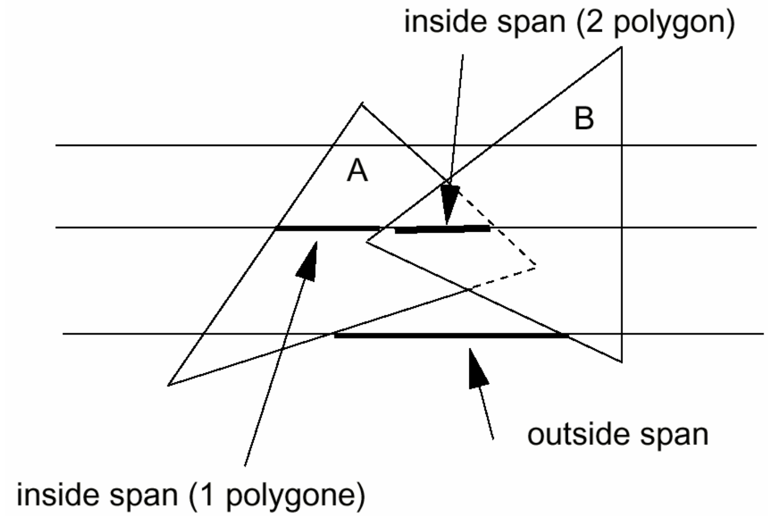
- Use no z-buffer
- Each scan line is subdivided into several "spans"
- Determine which polygon the current span belongs to
- Shade the span using the current polygon's color
- Exploit "span coherence" :
- For each span, only one visibility test needs to be done
  - Assuming no intersecting polygons.



Spanning Scan Line Algorithm

- A scan line is subdivided into a sequence of spans
- Each span can be "inside" or "outside" polygon areas
  - "outside": no pixels need to be drawn (background color)
  - "inside": can be inside one or multiple polygons
- If a span is inside one polygon, the pixels in the span will be drawn with the color of that polygon
- If a span is inside more than one polygon, then we need to compare the z values of those polygons at the scan line edge intersection point to determine the color of the pixel

Spanning Scan Line Algorithm



Determine a span is inside or outside (single polygon)

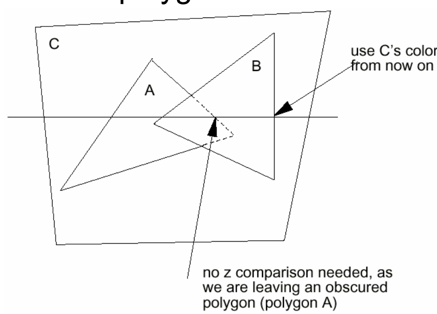
- When a scan line intersects an edge of a polygon
  - for a 1st time, the span becomes "inside" of the polygon from that intersection point on
  - for a 2nd time, the span becomes "outside" of the polygon from that point on
- Use a "in/out" flag for each polygon to keep track of the current state
- Initially, the in/out flag is set to be "outside" (value = 0 for example). Invert the tag for "inside".

When there are multiple polygons

- Each polygon will have its own in/out flag
- There can be more than one polygon having the in/out flags to be "in" at a given instance
- We want to keep track of how many polygons the scan line is currently in
- If there is more than one polygon "in", we need to perform z value comparison to determine the color of the scan line span

Z value comparison

- When the scan line intersects an edge, leaving the top-most polygon, we use the color of the remaining polygon if there is now only 1 polygon "in".
- If there is still more than one polygon with an "in" flag, we need to perform z comparison, but only when the scan line leaves a non-obscured polygon.



Many Polygons !

ET

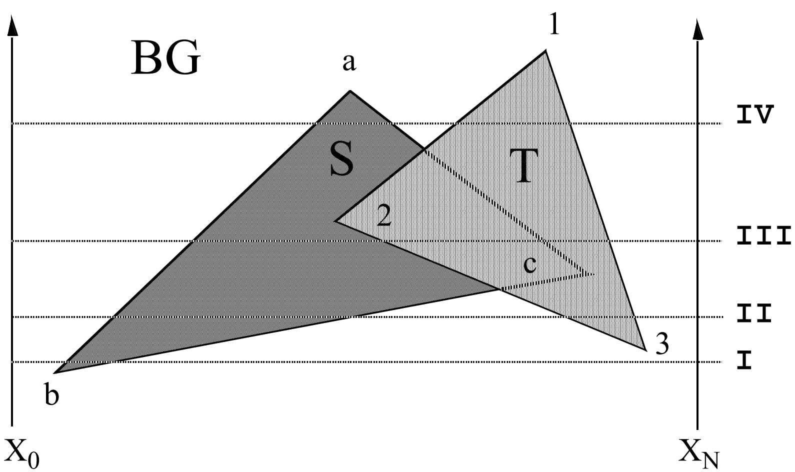
x	y_{max}	Δx	poly-ID	

PT

poly-ID	A,B,C,D	color	in/out flag	

- Use a PT entry for each polygon
- When polygon is considered, Flag is true
- Multiple polygons can have their flags set to true
- Use IPL as active In-Polygon List !

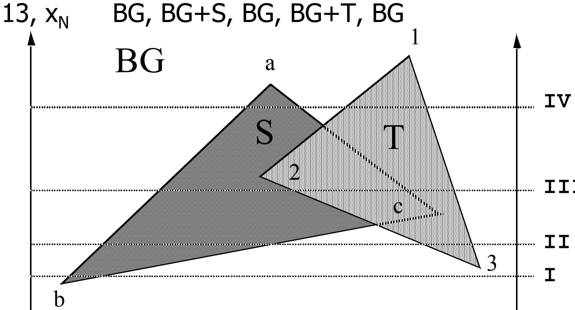
Example



Think of ScanPlanes to understand !

Spanning Scan-Line: Example

Y	AET	IPL
I	x_0, ba , bc, x_N	BG, BG+S, BG
II	x_0, ba , bc, 32, 13, x_N	BG, BG+S, BG, BG+T, BG
III	*x_0, ba , 32, ca, 13, x_N*	*BG, BG+S, BG+S+T, BG+T, BG*
IV	x_0, ba , ac, 12, 13, x_N	BG, BG+S, BG, BG+T, BG



Some Facts !

- Scan Line I: Polygon S is in and flag of S=true
- ScanLine II: Both S and T are in and flags are disjointly true
- Scan Line III: Both S and T are in simultaneously
- Scan Line IV: Same as Scan Line II

Spanning Scan-Line

```
build ET, PT              -- all polys+BG poly
AET := IPL := Nil;
for y := y_min to y_max do
    e1 := first_item ( AET );IPL := BG;
    while (e1.x <> MaxX) do
        e2 := next_item (AET);
        poly := closest poly in IPL at [(e1.x+e2.x)/2, y]
        draw_line(e1.x, e2.x, poly-color);
        update IPL (flags); e1 := e2;
    end-while;
    IPL := NIL; update AET;
end-for;
```

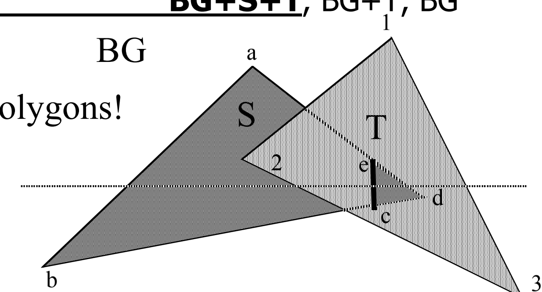
Depth Coherence

- Depth relationships may not change between polygons from one scan-line to the next scan-line.
- These can be kept track using the (active edge table) AET and the (polgon table) PT.
- How about penetrating polygons?

Penetrating Polygons

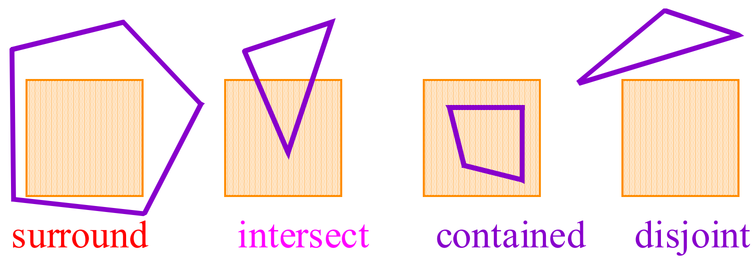
Y	AET	IPL
I	x_0, ba , 23, ad, 13, x_N	BG, BG+S, S+T, BG+T,BG
I'	x_0, ba , 23, **ec**, ad, 13, x_N	BG, BG+S, BG+S+T, **BG+S+T**, BG+T, BG

False edges and new polygons!



Area Subdivision 1 (Warnock's Algorithm)

Divide and conquer: the relationship of a display area and a polygon after projection is one of the four basic cases:



Warnock : One Polygon

```
if it surrounds then
        draw_rectangle(poly-color);

else begin

        if it intersects then
                poly := intersect(poly, rectangle);
        draw_rectangle(BACKGROUND);
        draw_poly(poly);

end else;
```

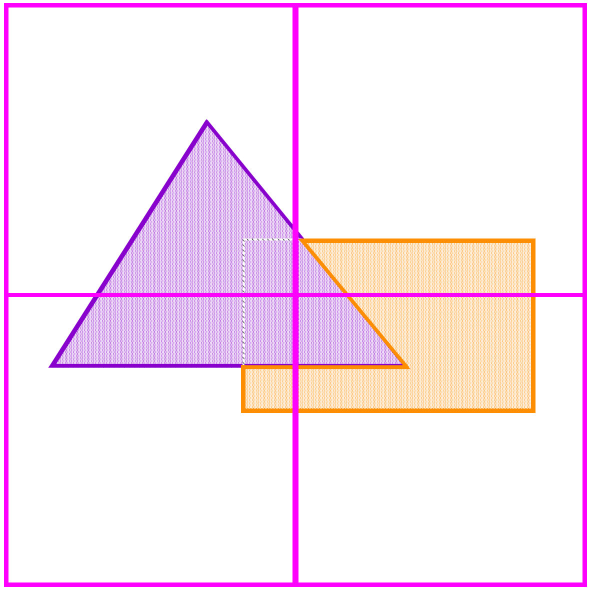
What about contained and disjoint ?

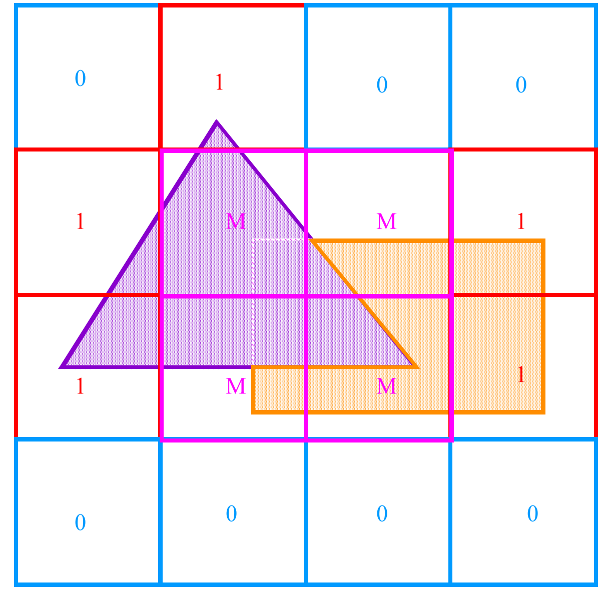
Warnock's Algorithm

- Starting with the entire display, we check the following four cases. If none hold, we subdivide the area and repeat, otherwise, we stop and perform the action associated with the case
  1. All polygons are disjoint wrt the area -> draw the background color
  2. Only 1 intersecting or contained polygon -> draw background, and then draw the contained portion of the polygon
  3. There is a single surrounding polygon -> draw the entire area in the polygon's color
  4. There are more than one intersecting, contained, or surrounding polygons, but there is a front surrounding polygon -> draw the entire area in the polygon's color
- The recursion stops at the pixel level

At A Single Pixel Level

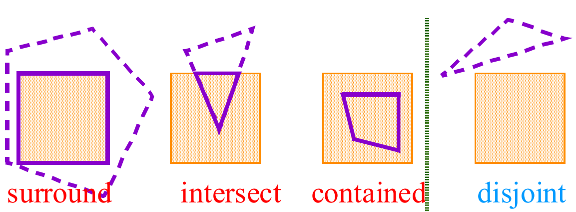
- When the recursion stops and none of the four cases hold, we need to perform a depth sort and draw the polygon with the closest Z value
- The algorithm is done at the object space level, except scan conversion and clipping are done at the image space level





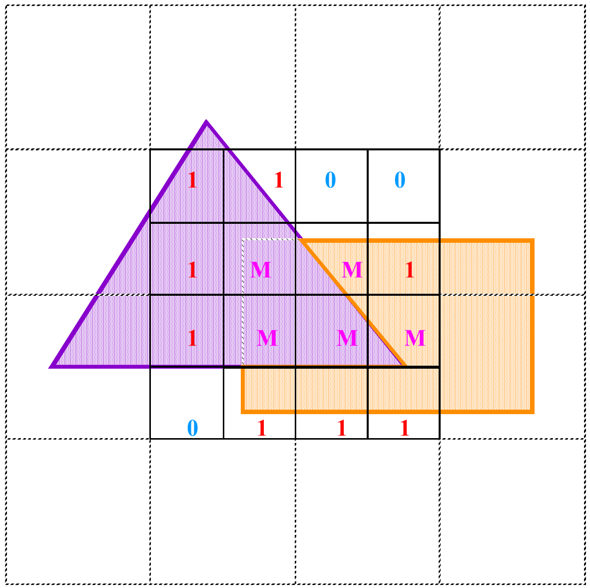
Warnock : Zero/One Polygons

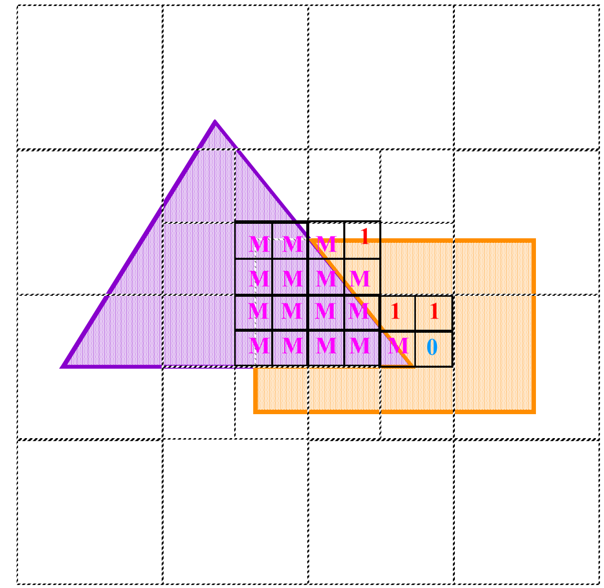
```
warnock01(rectangle, poly)

    new-poly := clip(rectangle, poly);

    if new-poly = NULL then

        draw_rectangle(BACKGROUND);

    else
        draw_rectangle(BACKGROUND);
        draw_poly(new-poly); return;
```

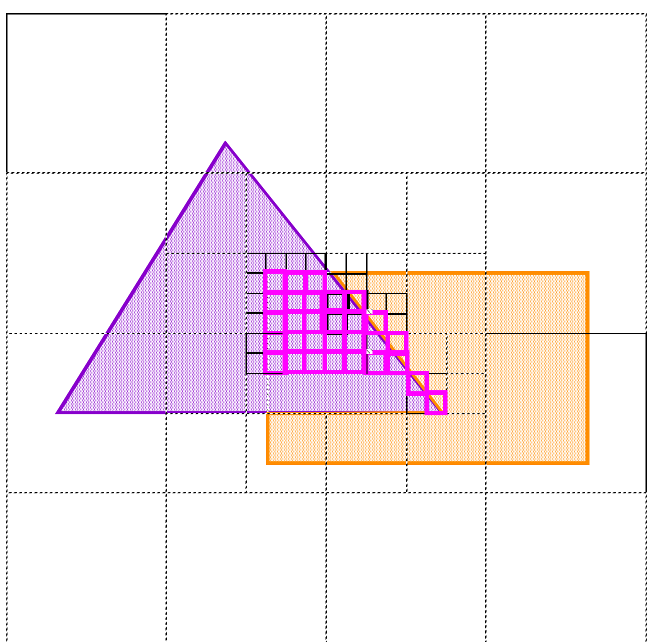


Warnock(rectangle, poly-list)

```
new-list := clip(rectangle, poly-list);
if length(new-list) = 0 then
    draw_rectangle(BACKGROUND); return;

if length(new-list) = 1 then
        draw_rectangle(BACKGROUND);
    draw_poly(poly); return;

if rectangle size = pixel size then
  poly := closest polygon at rectangle center
  draw_rectangle(poly color); return;

warnock(top-left quadrant, new-list);
warnock(top-right quadrant, new-list);
warnock(bottom-left quadrant, new-list);
warnock(bottom-right quadrant, new-list);
```







Area Subdivision 2

Weiler -Atherton Algorithm

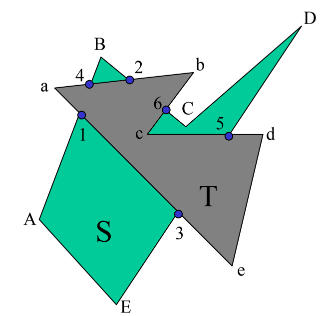
- Object space
- Like Warnock
- Output – polygons of arbitrary accuracy

Weiler-Atherton Clipping

- General polygon clipping algorithm
- Allows one to clip a concave polygon against another concave polygon.

Weiler-Atherton Clipping

- First, find all of the intersection points between edges of the two polygons.

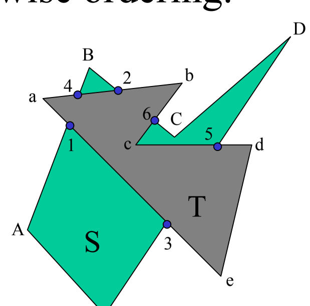


S: A,B,C,D,E

T: a,b,c,d,e

Weiler-Atherton Clipping

- Now, rebuild the polygon's such that they include the intersection points in their clock-wise ordering.

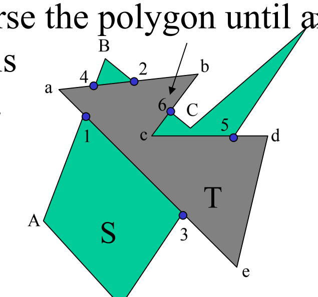


S: A,1,4,B,2,6,C,D,5,3,E

T: a,4,2,b,6,c,5,d,e,3,1

Weiler-Atherton Clipping

- Find an intersecting vertex of the polygon to be clipped that starts outside and goes inside the clipping region.
- Traverse the polygon until another intersection point is found.



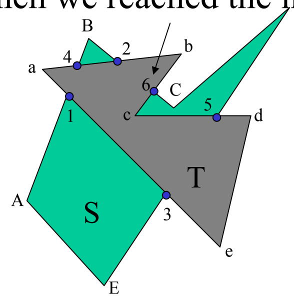
S: A,1,4,B,2,6,C,D,5,3,E

T: a,4,2,b,6,c,5,d,e,3,1

Clip: 6,c,5,…

Weiler-Atherton Clipping

- Switch from walking around the polygon 1, to walking around polygon 2, when an intersection is detected.
- Stop when we reached the initial point.



S: A,1,4,B,2,6,C,D,5,3,E

T: a,4,2,b,6,c,5,d,e,3,1

Clip: 6,c,5,3,1,4,2,6

Weiler -Atherton Algorithm

- Subdivide along polygon boundaries (unlike Warnock's rectangular boundaries in image space);
- Algorithm:
  1. Sort the polygons based on their minimum z distance
  2. Choose the first polygon P in the sorted list
  3. Clip all polygons left against P, create two lists:
     – Inside list: polygon fragments inside P (including P)
     – Outside list: polygon fragments outside P
  4. All polygon fragments on the inside list that are behind P are discarded. If there are polygons on the inside list that are in front of P, go back to step 3), use the 'offending' polygons as P
  5. Display P and go back to step (2)

Weiler -Atherton Algorithm

WA_display(*polys* : **ListOfPolygons**)

sort_by_minZ(*polys*);
while (*polys* <> NULL) **do**
WA_subdiv(*polys->first, polys*)
end;

WA_subdiv(*first*: **Polygon**; *polys*: **ListOfPolygons**)

inP, outP : **ListOfPolygons** := NULL;

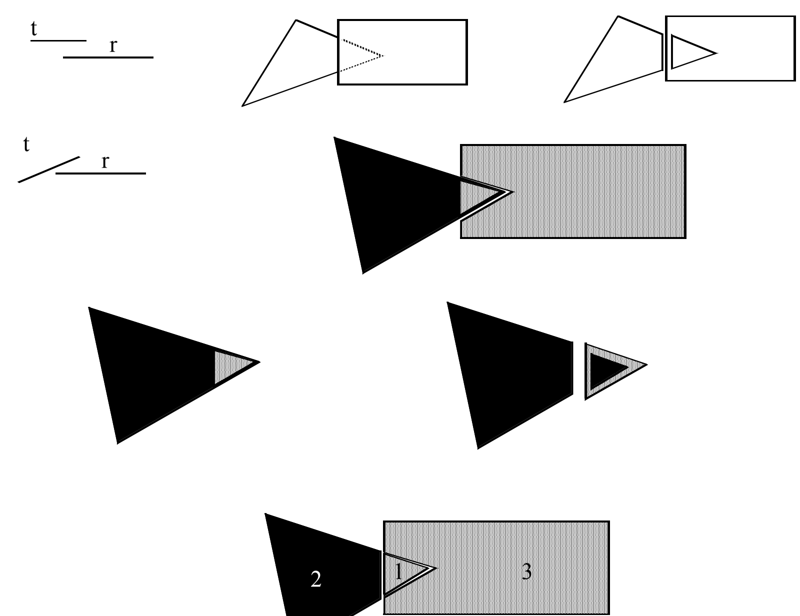
for each *P* **in polys do** Clip(*P, first->ancestor, inP, outP*);

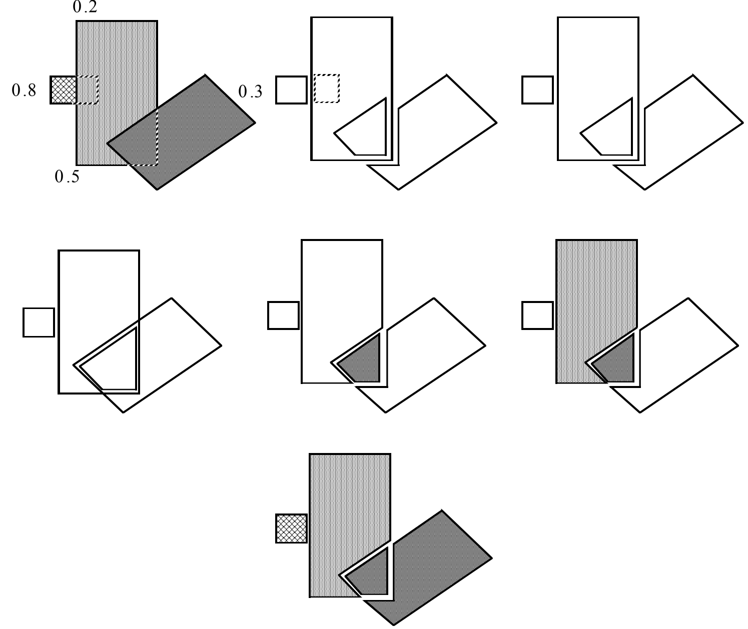
for each *P* **in** *inP* **do if** *P* is behind (min z)*first* **then** discard *P*;

for each *P* **in** *inP* **do**
if P is not part of *first* **then** WA_subdiv(*P, inP*);

for each *P* **in** *inP* **do** display_a_poly(*P*);

polys := *outP*;



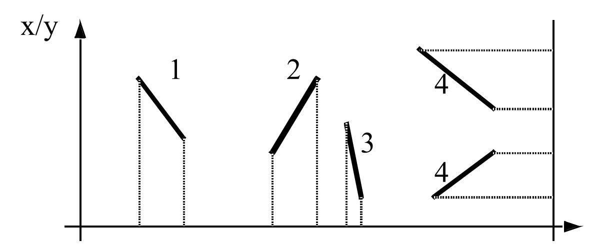


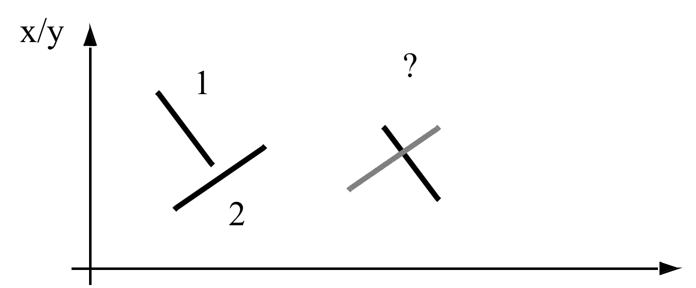
List Priority Algorithms

- Find a valid order for rendering.
- Only consider cases where the sort matters.

List Priority Algorithms

- If objects do not overlap in X or in Y there is no need for hidden object removal process.
- If they do not overlap in the Z dimension they can be sorted by Z and rendered in back (highest priority)-to-front (lowest priority) order (Painter’s Algorithm).
- It is easy then to implement transparency.
- How do we sort ? – different algorithms differ





Newell, Newell, Sancha Algorithm

1. Sort by [min_z..max_z] of each polygon
2. For each group of unsorted polygons G
   resolve_ambiguities(G);
3. Render polygons in a back-to-front order.

resolve_ambiguities is basically a sorting algorithm that relies on the procedure rearrange(P, Q):

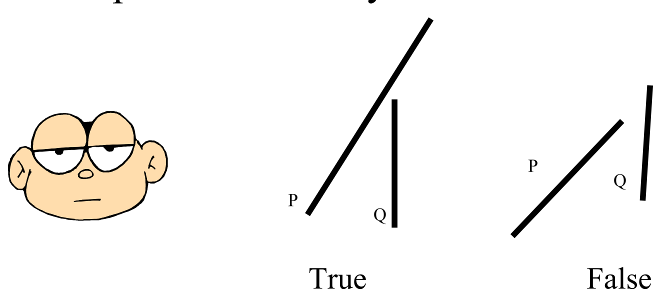
resolve_ambiguities(G)
    not-yet-done := TRUE;
    while (not-yet-done) do
        not-yet-done := FALSE;
        for each pair of polygons P, Q in G do --- *bubble sort*
            L := rearrange(P, Q, not-yet-done);
            insert L into G instead of P,Q

Newell, Newell, Sancha Algorithm

rearrange(P, Q, flag)
    if (P and Q do not have overlapping x-extents, **return** P, Q
    if (P and Q do not have overlapping y-extents, **return** P, Q
    if all Q is on the *opposite* side of P from the eye **return** P, Q
    if all P is on the *same* side of Q from the eye **return** P, Q
    if not overlap-projection(P, Q) **return** P, Q
    flag := **TRUE**; // more work is needed
    if all Q is on the *same* side of P from the eye **return** Q, P
    if all P is on the *opposite* side of Q from the eye **return** Q, P
    split(P, Q, p1, p2);   -- *split P by Q*
    return (p1, p2, Q);

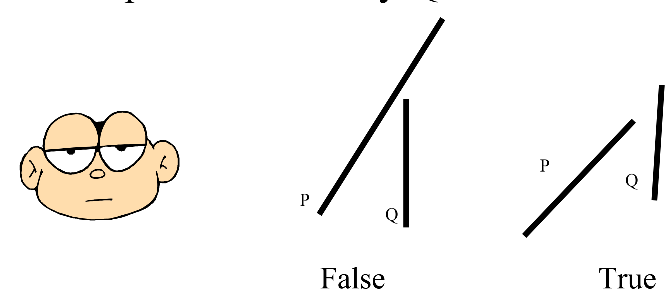
Newell-Newell-Sancha Sorting

- Q is on the ***opposite*** side of P.
- Means, all of Q's vertices are behind the half-plane defined by P.



Newell-Newell-Sancha Sorting

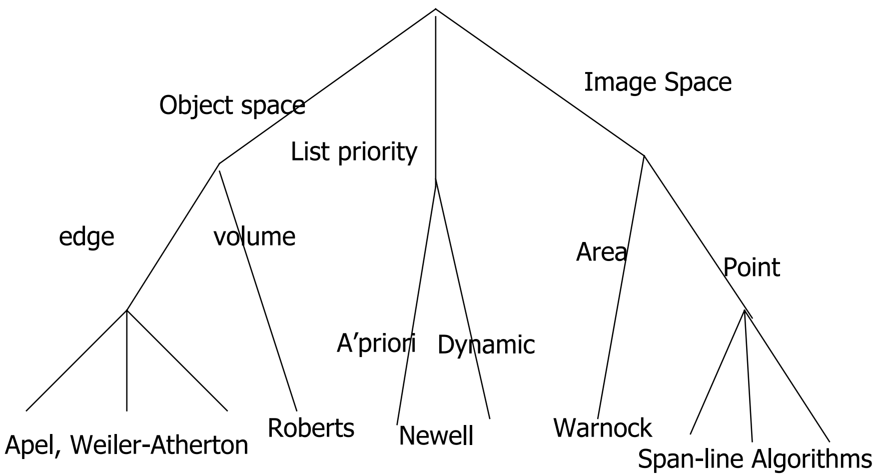
- P is on the ***same*** side of Q.
- Means, all of P's vertices are in front of the half-plane defined by Q.



Taxonomy

A characterization of 10 Hidden Surface Algorithm:

Sutherland, Sproull, Schumaker (1974)



Spatial Subdivision

- Uniform grid
- Octrees
- K-d Trees
- BSP-trees
- Non-overlapping polyhedra
  - Axis-Aligned Bounding Boxes (AABB's)
  - Oriented Bounding Boxes (OBB's)
  - Useful for non-static scenes

Back-to-front Traversals

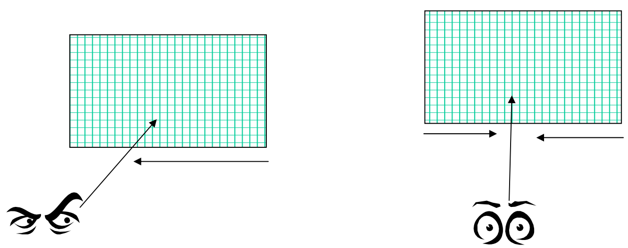
- For the first four, you can develop either a front-to-back or back-to-front traversal order explicitly.
- Thereby, solving the visibility sort efficiently.
- For the polyhedra, use a Newell-Newell-Sancha sort.

Sorting for Uniform Grid

- Parallel Projection
  - Can always proceed along the x-axis, then y-axis then z-axis or any combination.
  - Simply need to decide whether to go forward or backward on each axis.
    - Look at the z-value of the transformed x-axis, …
    - Positive, go forward for back-to-front sort.
  - Better ordering would choose the axis most parallel to the viewing direction to traverse last.

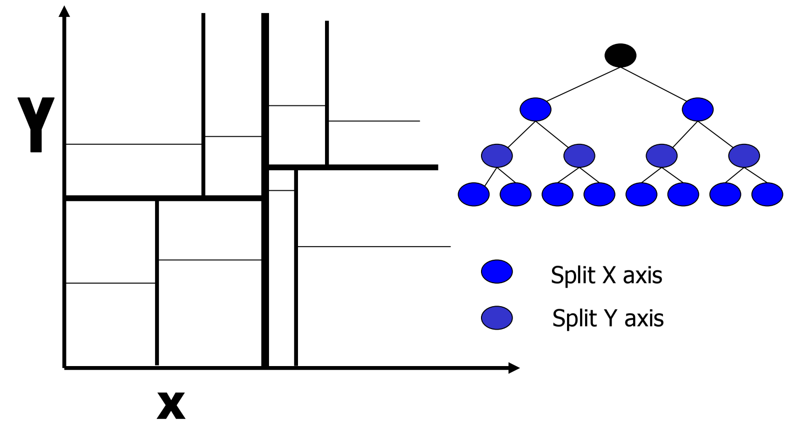
Sorting for Uniform Grid

- Perspective projection
  - May need to proceed forward for part of the grid and backwards for the other.



K-d Trees

- **Alternate splits in each direction**



K-d Trees

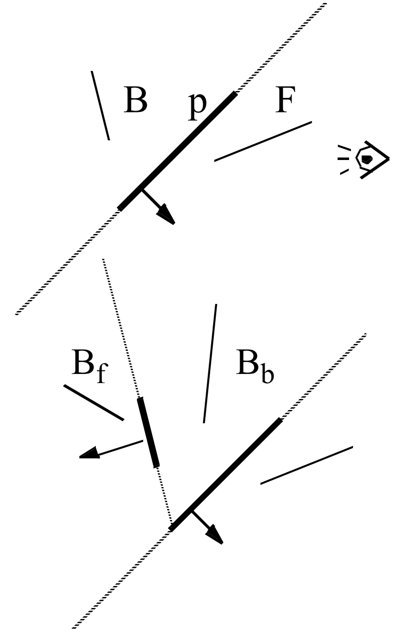
- Extend to any dimension *d*
- In 3D, the splits are done with axis-aligned planes.
  - Test is simple, is x-value (for nodes splitting the x-axis) greater than the node value?

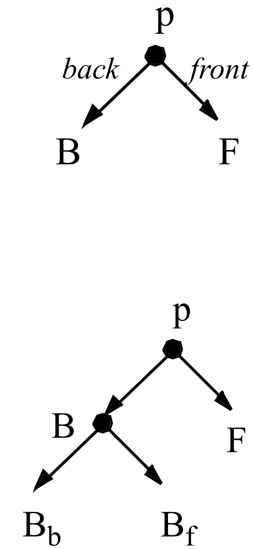
K-d Trees

- A subset of BSP-trees.
- Sorting is the same.
- More efficient storage representation.

Binary Space-Partitioning Tree

- Given a polygon p
- Two lists of polygons:
  - those that are behind(p) :B
  - those that are in-front(p) :F
- If eye is in-front(p), right display order is B, p, F
- Otherwise it is F, p, B



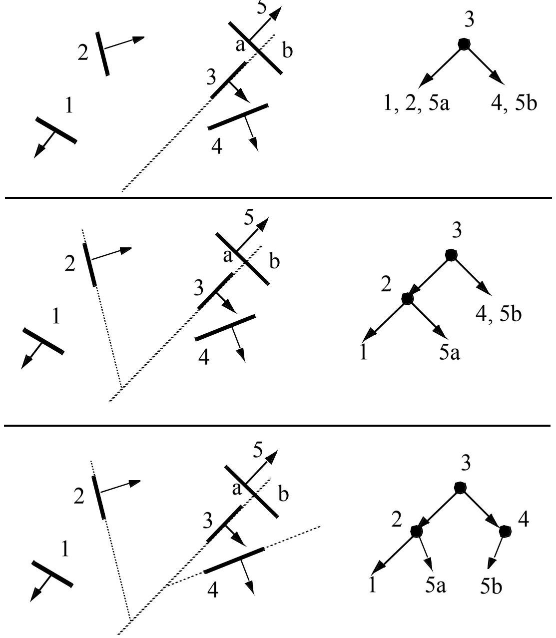


Display a BSP Tree

```
 struct bspnode {
              p: Polygon;  back, front : *bspnode;
} BSPTree;

BSP_display ( bspt )
BSPTree *bspt;
{       if (!bspt) return;
        if (EyeInfrontPoly( bspt->p )) {
         BSP_display(bspt->back);Poly_display(bspt->p);
         BSP_display(bspt->front);
        } else {
              BSP_display(bspt->front); Poly_display(bspt->p);
              BSP_display(bspt->back);
        }
}
```

Generating a BSP Tree

if (*polys* is empty) **then return** NULL;
rootp := first polygon in *polys*;
for each polygon *p* in the rest of *polys* **do**
  if *p* is infront of *rootp* **then**
    add *p* to the *front list*
  else if *p* is in the back of *rootp* **then**
    add p to the *back list*
  else
   split *p* into a back poly *pb* and front poly *pf*
   add *pf* to the *front list*
   add *pb* to the *back list*
end_for;
bspt->back := BSP_gentree(back list);
bspt->front := BSP_gentree(front list);
bspt->p = rootp;return bspt;


1
2
3
4
5
a
b
1, 2, 5a
4, 5b
5a
5b