Implicit Surfaces

CIS 781

Roger Crawfis

Implicit Surfaces

- An *implicit surface* is simply an iso-contour of a scalar function $f(x,y,z)=0$.
- The term is usually used when modeling a surface, whereas iso-contour is used when visualizing a scalar field.
- Same thing!!!!

Implicit Surfaces

- Point-based Modeling primitives
  - Blobbies
  - Meta-balls
  - Soft Objects

Blobbies

- Jim Blinn, 1982

$$f(x,y,z)=\sum_k b_k e^{-a_k\left(x_k^2+y_k^2+z_k^2\right)}-T$$
$$=\sum_k b_k f(r)-T=0$$

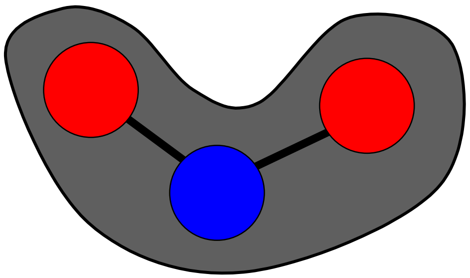
where, if $b_k > 0$ => bump

if $b_k < 0$ => dent

- Smooth function - Gaussian
- Every blobby affects the shape everywhere.
  - No finite support

Molecular Models

- van der Waals surface



Meta-balls

- Nishimura, *et al.*, piecewise quadratic spline

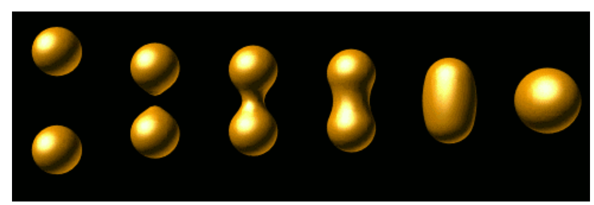
$$f(r)=\begin{cases} 1-\dfrac{3r^2}{d^2} & o<r\le\dfrac{d}{3} \\ \dfrac{3}{2}\left(1-\dfrac{r}{d}\right)^2 & \dfrac{d}{3}<r\le d \\ 0 & r>d \end{cases}$$

Soft Objects

- Wyvill, 1986, 6th order polynomial

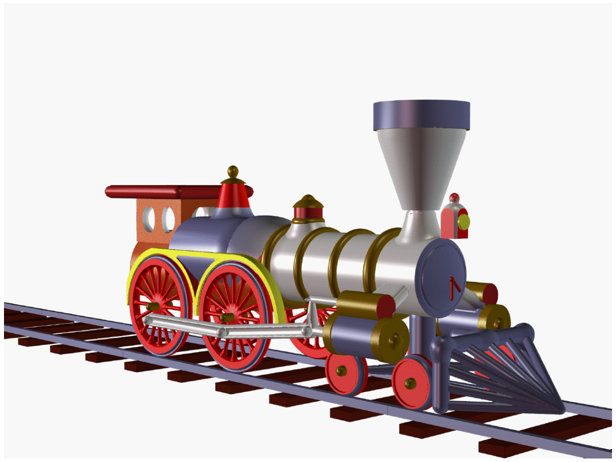
$$f(r)=\begin{cases} 1-\dfrac{22r^2}{9d^2}+\dfrac{17r^4}{9d^4}-\dfrac{4r^6}{9d^6} & 0\le r\le d \\ 0 & r>d \end{cases}$$

Examples



Brian Wyvill, Univ. of Calgary

Examples - CSG



Implicit Surfaces

- How to render?
  - Have an analytical function.
  - Sample it on a regular grid
  - Use marching cubes to extract a polygonal surface.
- Can also use ray-casting/marching to sample the space.
  - Root-finding problem to determine the surface.

Rendering

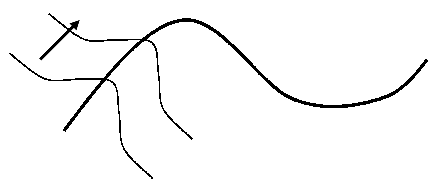
- So, implicit surfaces are a way of defining a 3D volume.
- Can use splatting or any other volume rendering technique to display them by resampling to a regular volume grid.

Convolution Surfaces

- Bloomenthal proposed using curves and 2D surfaces as primitives, rather than simple points.
  - Smooths the transition between points.
  - He proposed a convolution of the basis function with a continuous curve.

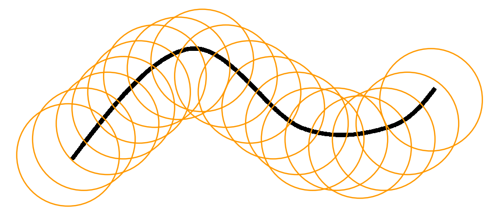
Convolution Surfaces

- Logically

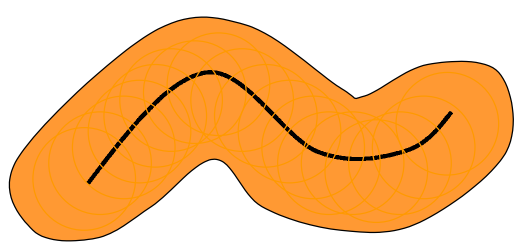


Convolution Surfaces

- Discretely

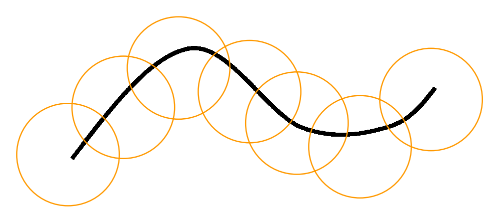


Convolution Surfaces

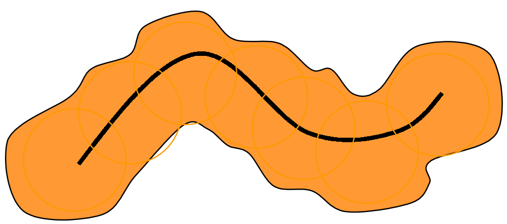


Convolution Surfaces

- Poorly spaced meta-balls



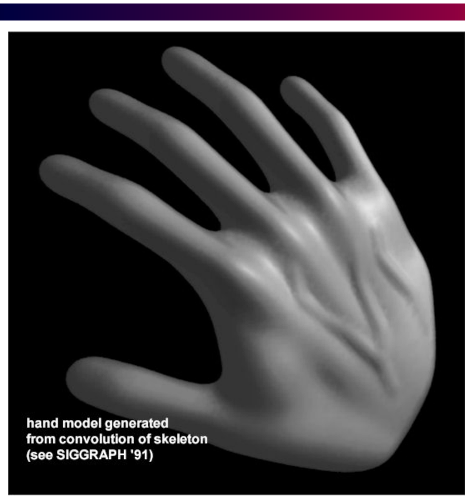
Convolution Surfaces



Convolution Surfaces

- Rule of thumb
  - If spacing is less than radius or basis normalization.
  - As spacing is decreased, the weights need to be adjusted to preserve the thickness.
  - Splatting or reconstruction has a fixed spacing and uses a kernel appropriate for this spacing.

Examples

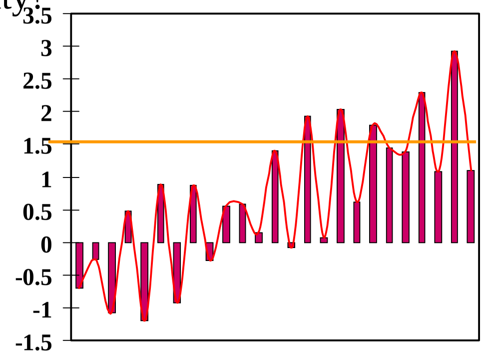


2D Contouring

- Continuous $f(x,y)$
  - Use steepest decent to find zero crossing (root) of the function $f(x,y)$-c
  - Follow contour from this seed point until we reach a boundary or loop back.
  - Direction close to $\nabla f \otimes z$
  - Problems?

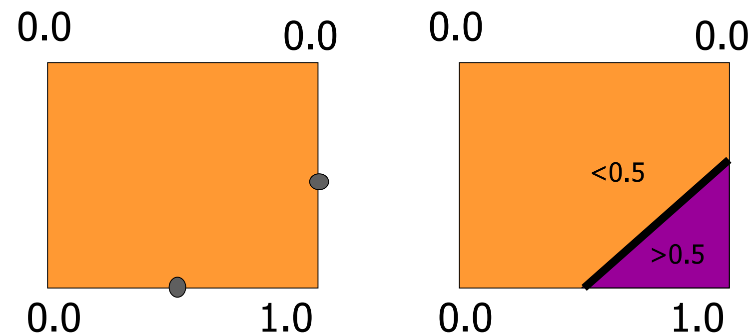
2D Contouring

- Discrete Data
  - Assume the Mean Value Thereom
  - Assume monoticity?
- 1D Analogy
  - 5 Points



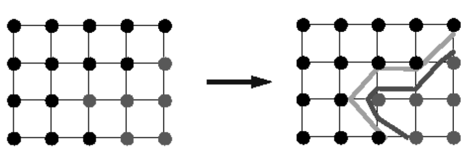
2D Contouring

- Given a quadrilateral
  - $f(x,y) = 0.5$



Marching Cube - ***The Problem***

- Extracting an iso- surface from an implicit function, that is,
- Extracting a surface from volume data (discrete implicit function), $f(x, y, z) = T$



Example: black=0, cyan=10, T= 8, T= 2.

Contouring in 3D

- Treat volume as a set of 2D slices
  - Apply 2D Contouring algorithm on each slice.
  - Or given as a set of hand-drawn contours
- Stitch the slices together.

Contour Stitching

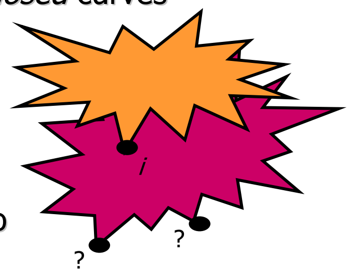
- Problem:

  Given: 2 two-dimensional *closed* curves

  Curve #1 has *m* points

  Curve #2 has *n* points

  Which point(s) does vertex *i* on curve one correspond to on curve two?



Marching Cubes

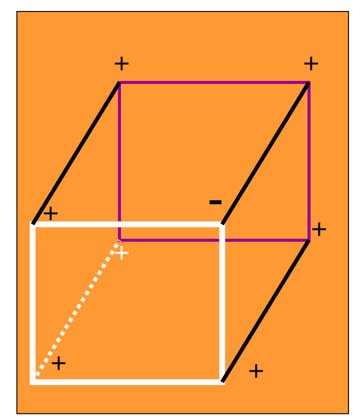
- Lorensen and Cline, SIGGRAPH ‘87
- Predominant method used today.
- Efficient and simple

Marching Cubes

- Treat each cube individual
  - No 2D contour curves
- Allow intersections only on the edges or at vertices.
- Pre-calculate all of the necessary information to construct a surface.

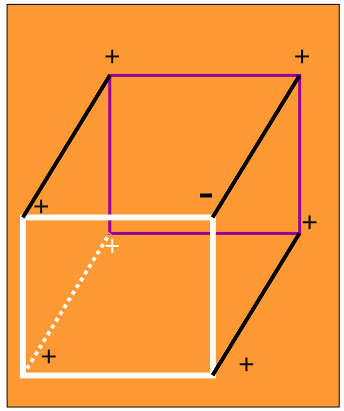
Marching Cubes

- Consider a single cube
  - All vertices above the contour threshold
  - All vertices below
  - Mixed above and below



Marching Cubes

- Binary label each node => (above/below)
- Examine all possible cases of above or below for each vertex.
- 8 vertices implies 256 possible cases.



Marching Cubes

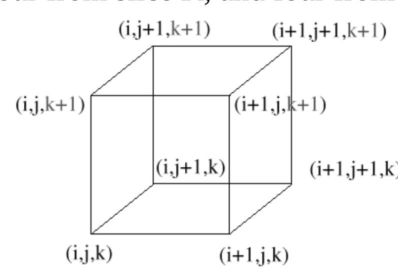
- 14 unique cases
  - +/- symmetry
  - rotational symmetry
  - mirror symmetry

Marching Cube - Summary

- Create a cube
- Classify each voxel
- Build an index
- Lookup edge list
- Interpolate triangle vertices
- Calculate and interpolate normals

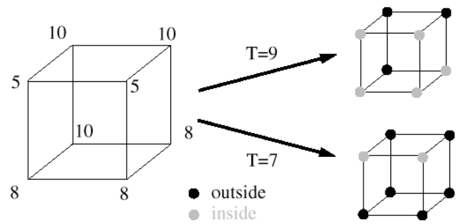
Step 1: Create a Cube

- Consider a cube defined by eight data values: four from slice K, and four from slice K+ 1



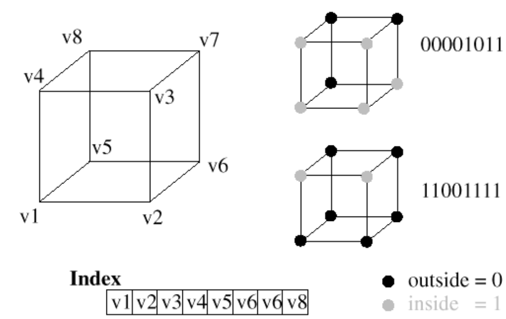
Step 2: Classify Each Voxel

- Binary classification of each vertex of the cube as to whether it lies
  - outside the surface (voxel value < isosurface value)
  - or inside the surface (voxel value <= isosurface value).



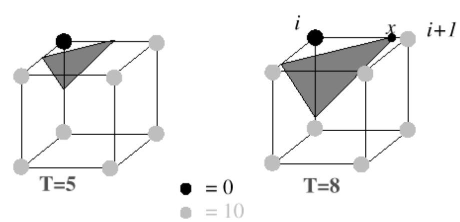
Step 3: Build an Index

- Use the binary labeling of each voxel to create an 8-bit index. (8 vertex - 256 cases)



Step 5: Interpolate Triangle Vertices

- For each edge, find the vertex location along the edge using linear interpolation of the voxel values.

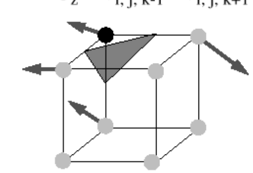


$$x = i + \left(\frac{T - V[i]}{V[i+1] - V[i]}\right)$$

Step 6: Compute Normals

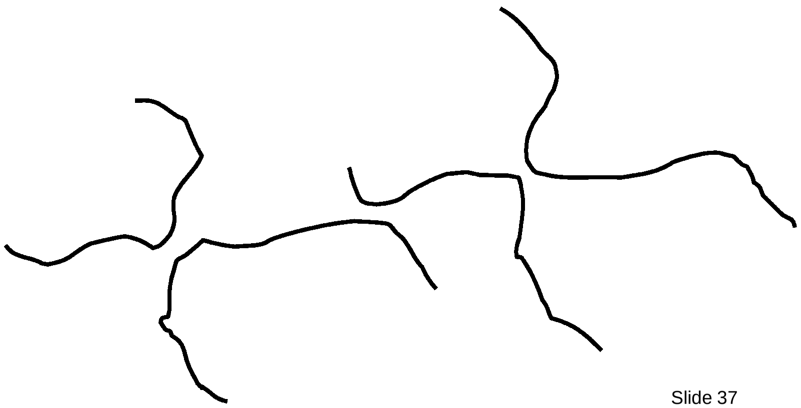
- Calculate the normal at each cube vertex
- Use linear interpolation to interpolate the polygon vertex normal

$$G_x = V_{i-1, j, k} - V_{i+1, j, k}$$
$$G_y = V_{i, j-1, k} - V_{i, j+1, k}$$
$$G_z = V_{i, j, k-1} - V_{i, j, k+1}$$



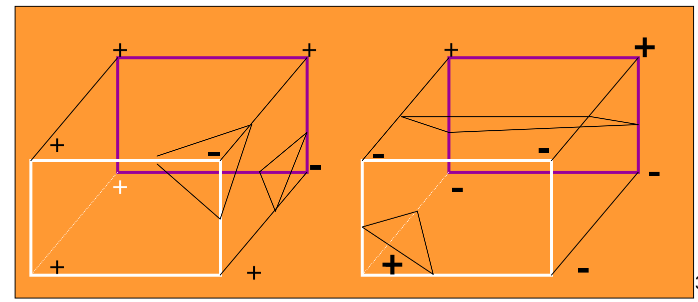
Ambiguities

- Right or wrong?

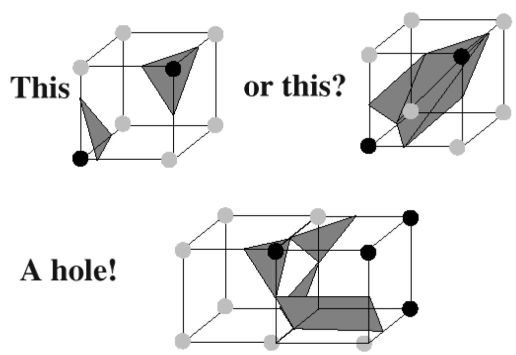


Marching Cubes

- Topological inconsistencies in the 15 cases
  - Turns out positive and negative are not symmetric.

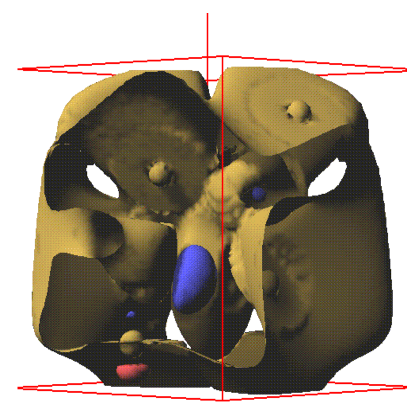


A Bad Example



Marching Cubes

- Animating the contour value
- Special *functions* for contouring
- Varying speeds and numbers of triangles



Marching Cubes

- Data Structures/Tables

```
static int const HexaEdges[12][2] = { {0,1}, {1,2}, {2,3}, {3,0},
                                      {4,5}, {5,6}, {6,7}, {7,4},
                                      {0,4}, {1,5}, {3,7}, {2,6}};

typedef struct {
       EDGE_LIST HexaEdges[16];
} HEXA_TRIANGLE_CASES;

/* Edges to intersect.  Three at a time form a triangle. */

static const HEXA_TRIANGLE_CASES HexaTriCases[] = {
{-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}, /*   0 */
{ 0,  8,  3, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}, /*   1 */
{ 0,  1,  9, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}, /*   2 */
{ 1,  8,  3,  9,  8,  1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1}, /*   3 */
...
```

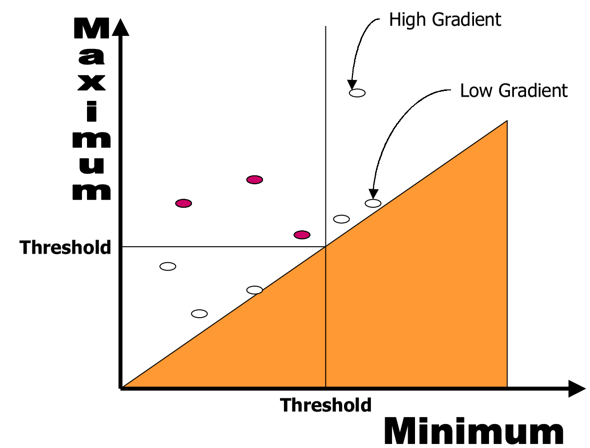

Marching Cubes - How simple

```
/* Determine the marching cubes index */
 for ( i=0, index = 0; i < 8; i++)
     if (val1[nodes[i]] >= thresh)      /* If the nodal value is above the     */
         index |= CASE_MASK[i];         /* threshold, set the appropriate bit. */

 triCase = HexaTriCases + index;        /* triCase indexes into the MC table. */
 edge = triCase->HexaEdges;   /* edge points to the list of intersected edges */

 for ( ; edge[0] > -1; edge += 3 )
 {
   for (i=0; i<3; i++)    /* Calculate and store the three edge intersections */
   {
     vert = HexaEdges[edge[i]];
     n0 = nodes[vert[0]];
     n1 = nodes[vert[1]];
     t = (thresh - val1[n0]) / (val1[n1] - val1[n0]);
     tri_ptr[i] = add_intersection( n0, n1, t ); /* Save an index to the pt. */
   }
   add_triangle( tri_ptr[0], tri_ptr[1], tri_ptr[2], zoneID );       /* Store the
  triangle */
 }
}
```


Efficient Searching

- With < 10% of the voxels contributing to the surface, it is a waste to look at every voxel.
- A voxel can be specified in terms of its interval, its minimum and maximum values.

Span Space



Span Space - Representing

- **K-d Trees**