Functional Languages

Chapter 10

Functional Programming Paradigm

- The program is a collection of functions
  - A function computes and returns a value
  - No side-effects (i.e., no changes to state)
  - No program variables whose values change
    - Basically, no assignments
- Languages: LISP, Scheme (dialect of LISP from MIT, mid-70s), ML, Haskell, ...
- Functions as first-class entities
  - A function can be a parameter of another function
  - A function can be the return value of another function
  - A function could be an element of a data structure
  - A function can be created at run time

Outline

- Language elements:
  - Atoms and lists
- Evaluating expressions
  - Function application
  - Quoting an expression
  - Conditionals
  - Defining functions
- Examples
- S-expressions
- Function call semantics & higher-order functions
- More examples and features

Data Objects in Scheme

- Atoms
  - Numeric constants: 5, 20, -100, 2.788
  - Boolean constants: #t (true) and #f (false)
  - String constants: "hi there"
  - Character constants: #\a
  - Symbols: f, x, +, *, null?, set!
    - Roughly speaking, equivalent to identifiers in imperative languages
  - Empty list: ()
- Lists
  - (e₁ e₂ ... eᵣ) where eᵢ is an atom or list

Examples of Lists

- (A B C)
- ((A B) C)
- ((3) (4) 5)
- (A B (C D))
- ([A])
- ()
- (())

Lists

- List elements can be atoms or other lists
  - ( (3 4) 5 (6) ) is a list with 3 elements
  - Thus, lists are heterogeneous: the elements do not have to be of the same type
- Empty list (): has zero elements
  - Operations car and cdr are not defined for an empty list – run-time error
Lists

- **car** for a list produces the first element of the list (the list head)
  - e.g. for ((A B) (C D) E) will produce (A B)
- **cdr** produces the tail of the list: a list containing all elements except the first
  - e.g. for ((A B) (C D) E) will produce ((C D) E)
- **cons** adds to the beginning of the list
  - cons of A and (B C) is (A B C)
  - e.g., cons of car of x and cdr of x is x

Outline

- **Language elements:**
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- **Evaluating expressions**
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Data vs. Code

- Interpreter for an imperative language: the input is code+data, the output is data (values)
- Everything in Scheme is an S-expression
  - The “program” we are executing is an S-expression
  - The intermediate values and the output values of the program are also S-expressions
- Data and code are really the same thing
- Example: an expression that represents function application (i.e., function call) is a list (f p1 p2 ...)
  - f is an S-expression representing the function we are calling; p1 is an S-expression representing the first actual parameter, etc.

Using Scheme

- **Read:** you enter an expression
- **Eval:** the interpreter evaluates the expression
- **Print:** the interpreter prints the resulting value
- **stdlinux:** at the prompt, type scheme48
  - > type your expression here
  - the interpreter prints the value here
  - > help
  - > exit

Evaluation of Atoms

- Numeric constants, string constants, and character constants evaluate to themselves
  - > 4.5 > #t
  - 4.5 #t
  - > "This is a string" > #f
  - "This is a string" #f
- Symbols do not have values to start with
  - They may get “bound” to values, as discussed later
    - > x
    - Error: undefined variable x
- The empty list () does not have a defined value
Function Application

• (+ 5 6)
  – This S-expression is a “program”; here + is a symbol “bound” to the built-in function for addition
  – The evaluation by the interpreter produces the S-expression 11
• Function application: (f p1 p2 ...)
  – The interpreter evaluates S-expressions f, p1, p2, etc.
  – The interpreter invokes the resulting function on the resulting values

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Quoting an Expression

• When the interpreter sees a non-atom, it tries to evaluate it as if it were a function call
  – But for (5 6), what does it mean?
    • “Error: attempt to call a non-procedure”
• We can tell the interpreter to evaluate an expression to itself
  – (quote (5 6)) or simply '(5 6)
  – Evaluates to the S-expression (5 6)
  – The resulting expression is printed by the Scheme interpreter as '(5 6)

Examples

> (+ (+ 3 5) (car (7 8)))
Errors
1> Ctrl-D
> (+ (+ 3 5) (car (7 8)))
15
> (car (7 10))
Errors
1> (car (7 10))
7
1> (+ (car (7 10)) (cdr (7 10)))
Errors
2> (+ (car (7 10)) (car (cdr (7 10))))
17

More Examples

> (cons (car '7 10)) (cdr '7 10))
'(7 10)
> a > 'a > (car (A B))
Error
'a
> (cdr (A B)) > (cons 'a '(b)) > (cons 'a 'b)
'(b)
'(a b)
'(a b)

More Examples

> (equal? #t #f) > (equal? '() #f)
#t
> (equal? #t #f) > (equal? (+ 5 7) (+ 5 7))
#t
> (equal? (cons 'a '(b)) '(a b))
#t
> (pair? (7 10)) > (pair? 7) > (pair? '())
#t
> (null? '(l)) > (null? #f) > (null? '(b))
#t
#f
#f
#f
More Examples

> (even? 7)  > (even? 8)
#t  #t
> (even? (+ 7 7))  > (even? 7)  > (even? 'a)
#t  Error  Error
> (= 5 6)  > (< 5 6)  > (> 5 6)
#t  #t  #f
> (> 4.5 4.5)  > (> 4.5 4.7)
#t  #f
> (= 'a 'b)
Error

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Conditional Expressions

• (if b e₁ e₂)
  — Evaluate b. If the value is not #f, evaluate e₁ and this is the value to the expression
  — If b evaluates to #f, evaluate e₂ and this is the value of the expression
• (cond (b₁ e₁₁) (b₂ e₂₂) ... (bₙ eₙₙ))
  — Evaluate b₁. If not #f, evaluate e₁ and use its value. If b₂, evaluates to #f, evaluate b₃, etc.
  — If all b evaluate to #f, unspecified value for the expression; so, we often have #t as the last b
  — Alternative form: (cond (b₁ e₁₁) (b₂ e₂₂) ... (else eₙₙ))

Member of a List?

In text file mbr.ss create the following:

; this is a comment
; (mbr x list): is x a member of the list?
(define (mbr x list)
  (cond
    ( (null? list) #f )
    ( #t (cond
      ( (equal? x (car list)) #t )
      ( #t (mbr x (cdr list)) )
    )
  )
)

Or we could use just one "cond" ...
Member of a List?
In the interpreter:
> (load "mbr.ss") or ,load mbr.ss
mbr.ss
; no values returned
> (mbr 4 '(5 6 4 7))
#t
> (mbr 8 '(5 6 4 7))
#f

Union of Two Lists
(define (uni s1 s2) 
  (cond 
    (null? s1) s2 
    (null? s2) s1 
    ( #t (cond 
      ( (mbr (car s1) s2) (uni (cdr s1) s2)) 
      ( #t (cons (car s1) (uni (cdr s1) s2))))))))
> (uni '(4) '(2 3))
'(4 2 3)
> (uni '(3 10 12) '(20 10 12 45))
'(3 10 12 45)

How about using "if" in mbr and uni?

Removing Duplicates
; x: a sorted list of numbers; remove duplicates ...
(define (unique x) 
  (cond 
    (null? x) x 
    (null? (cdr x)) x 
    ( equal? (car x) (cdr x) ) (unique (cdr x)) 
    ( #t (cons (car x) (unique (cdr x)))) )
)
> (unique '(2 2 3 4 4 5))
'(2 2 3 4 5)

Largest Number in a List
; max number in a non-empty list of numbers
(define (maxlist L) 
  (cond 
    ( null? (cdr L) ) (car L) 
    ( (> (car L) (maxlist (cdr L))) (car L) ) 
    ( #t (maxlist (cdr L)) ) )
)
What is the running time as a function of list size? How can we improve it?

A Different Approach
; max number in a non-empty list of numbers
(define (maxlist L) (mymax (car L) (cdr L)))
(define (mymax x L) 
  (cond 
    (null? L) x 
    (> x (car L)) (mymax x (cdr L)) 
    ( #t (mymax (car L) (cdr L)) )
  )
)
What is the running time as a function of list size?

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Extracts from the Document:
- Member of a List?
  - In the interpreter:
    > (load "mbr.ss") or ,load mbr.ss
    mbr.ss
    ; no values returned
    > (mbr 4 '(5 6 4 7))
    #t
    > (mbr 8 '(5 6 4 7))
    #f
- Union of Two Lists
  - (define (uni s1 s2)
    (cond
      (null? s1) s2
      (null? s2) s1
      ( #t (cond
        ( (mbr (car s1) s2) (uni (cdr s1) s2))
        ( #t (cons (car s1) (uni (cdr s1) s2))))))
    )
  - > (uni '(4) '(2 3))
    '(4 2 3)
  - > (uni '(3 10 12) '(20 10 12 45))
    '(3 20 10 12 45)
- How about using "if" in mbr and uni?
- Removing Duplicates
  - ; x: a sorted list of numbers; remove duplicates ...
    (define (unique x)
      (cond
        (null? x) x
        (null? (cdr x)) x
        (equal? (car x) (cdr x)) (unique (cdr x))
        ( #t (cons (car x) (unique (cdr x))))
      )
    )
  - > (unique '(2 2 3 4 4 5))
    '(2 2 3 4 5)
- Largest Number in a List
  - ; max number in a non-empty list of numbers
    (define (maxlist L)
      (cond
        ( null? (cdr L) ) (car L)
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      )
    )
  - What is the running time as a function of list size? How can we improve it?
- A Different Approach
  - ; max number in a non-empty list of numbers
    (define (maxlist L) (mymax (car L) (cdr L)))
    (define (mymax x L)
      (cond
        (null? L) x
        (> x (car L)) (mymax x (cdr L))
        ( #t (mymax (car L) (cdr L)) )
      )
    )
  - What is the running time as a function of list size?
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Data Objects in Scheme

• **Atoms**
  – Numeric constants: 5, 20, -100, 2.788
  – Boolean constants: #t (true) and #f (false)
  – String constants: "hi there"
  – Character constants: #\a

  • Symbols: f, x, +, *, null?, set!
    • Roughly speaking, equivalent to identifiers in imperative languages
  – Empty list: ( )
• **S-expressions**
  – A list is a special case of an S-expression

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S-expressions

• Every atom is an S-expression
• If s1 and s2 are S-expressions, so is ( s1 . s2 )
  – Essentially, a binary tree: left child is the tree for s1, and right child is the tree for s2
  – Atoms are leaves of the tree
    • (3 . 5)
    • (3 . 4) . (5 . 6)
    • (3 . (5 . 8))

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Primitive Functions for S-expressions

• **car**: unary; produces the S-expression corresponding to the left child of the argument
  – Not defined for atoms
• **cdr**: unary; produces the S-expression corresponding to the right child of the argument
  – Not defined for atoms
• **cons**: binary; produces a new S-expr with left child = 1st arg and right child = 2nd arg

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Examples of Lists

• ( (3 . 4) 5 ) is ( (3 . 4) . (5 . ( )) )
• ( (3) (4) 5 ) is ( (3 . ( )) . (4 . ( )) . (5 . ( )))
• (A B C) is (A . (B . (C . ()))))
• ((A B) C) is ((A . (B . ())) . (C . ()))
• (A B (C D)) is (A . (B . ((C . (D . ())) . ()))))
• ((A)) is ((A . ()) . ())
• (A (B . C)) is (A . ((B . C) . ()))

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Lists

• Special category of S-expressions
• Recursive definition
  – The empty list ( ) is a list; length is 0
  – If the S-expression Y is a list, the S-expression ( X . Y ) is also a list; length is 1 + length of Y
    • ((3 . 4) . (5 . 6)) is not a list
    • (3 . (5 . ())) is a list, with length 2
• Notation: ( e1 . ( e2 . ( ... ( en . ( ))) ) ) is written as ( e1 e2 ... en )

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• ( (3) (4) 5 ) is ( (3 . ( )) . (4 . ( )) . (5 . ( )))
• (A B C) is (A . (B . (C . ())))
• ((A B) C) is ((A . (B . ())) . (C . ()))
• (A B (C D)) is (A . (B . ((C . (D . ()))) . ())))
• ((A)) is ((A . ()) . ())
• (A (B . C)) is (A . ((B . C) . ()))

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Lists

• Another view of lists: a binary tree in which
  – the rightmost leaf is ( )
  – the S-expressions hanging from the rightmost "spine" of the tree are the list elements
• List elements can be atoms, other lists, and general S-expressions
  – ( (3 4) 5 (6) ) is a list with 3 elements
  – Thus, lists are heterogeneous: the elements do not have to be of the same type
• Empty list ( ) - has zero elements
  – Operations car and cdr are not defined for an empty list – run-time error
Lists
- **car** for a list produces the first element of the list (the list head)
  - e.g. for ((A B) (C D) E) will produce (A B)
- **cdr** produces the tail of the list: a list containing all elements except the first
  - e.g. for ((A B) (C D) E) will produce ((C D) E)
- **cons** adds to the beginning of the list
  - cons of A and (B C) is (A B C)
  - e.g., cons of car of x and cdr of x is x

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Semantics of Function Calls
- Consider (F p1 p2 ...)
- Evaluate p1, p2, ... using the current bindings
- “Bind” the resulting values v1, v2, ... to the formal parameters f1, f2, ... of F
  - add pairs ((f1,v1), (f2,v2), ...) to the current set of bindings
- Evaluate the body of F using the bindings
  - if we see p1 in the body, we evaluate it to value v1
- After coming back from the call, the bindings for p1, p2, ... are destroyed

Higher-Order Functions
- (define (double x) (+ x x))
- (define (twice f x) (f (f x)))
- (twice double 2) Returns 8
- (define (mymap f list)
  (if (null? list)
    list
    (cons (f (car list))
      (mymap f (cdr list)))))
- (mymap double '(1 2 3 4 5)) Returns '(2 4 6 8 10)

Higher-Order Functions
- (define (makelist f n)
  (if (= n 0) '()
    (cons f (makelist f (- n 1)))))
- (makelist double 4)
  Returns '(procedure double, procedure double, procedure double, procedure double)

Higher-Order Functions
- (define (newmap x list)
  (if (null? list)
    list
    (cons ((car list) x) (newmap x (cdr list)))))
- What does this function do?
- (newmap 11 (makelist double 7))
  What is the result of this function application?
- (define (f n) (newmap n (makelist double 5)))
  (twice f 9)
  How about here?
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Recursion for Iterating

; Factorial function
(define (fact n)
  (if (= n 0) 1 (* n (fact (- n 1)))))

Equivalent computation in imperative languages
f := 1;
for (i = 1; i <= n; i++) f := f * i;

Quicksort
Sort list of numbers (for simplicity, no duplicates)

Algorithm:
  – If list is empty, we are done
  – Choose pivot n (e.g., first element)
  – Partition list into lists A and B with elements < n in A and elements > n in B
  – Recursively sort A and B
  – Append sorted lists and n

Constructing the Two Sublists

(define (ltlist n list)
  (if (null? list)
      list
      (if (< (car list) n)
          (cons (car list) (ltlist n (cdr list)))
          (ltlist n (cdr list)))))

Similarly we can define function gtlist

Sorting

(define (qsort list)
  (if (null? list)
      list
      (append
       (qsort (ltlist (car list) (cdr list)))
       (cons (car list) '()))
       (qsort (gtlist (car list) (cdr list))))

(qsort '(4 3 5 1 2 8 7))
Returns '(1 2 3 4 5 6 7 8)

A Few Other Language Features

• (lambda (x y ...) body) : evaluates to a function
  – ((lambda (x) (+ x x)) 4) evaluates to 8
  – (define (f x y ...) body) is equivalent to (define f (lambda (x y ...) body))
  – Comes from the λ-calculus, the theoretical foundation for functional languages (Alonzo Church)
• let bindings – give names to values
  – (let ((x 2) (y 3) (* x y)) produces 6
  – (let ((x 2) (y 3)) (let ((z) (x y)) (* x z))) is 35
• (define x expr) and (define (f x y ...) body) create global bindings for these names