Achievable Throughput in Low Power Wireless Networks

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ABSTRACT
Given a duty cycle, we derive bounds on the achievable capacity of random wireless networks for single channel and multiple channel cases. By modeling several state-of-the-art MAC layer protocols, we present bounds on their throughput capacity. In particular, we analyze four abstract models, sender-centric synchrony (e.g. S-MAC, SCP-MAC), receiver-centric synchrony (e.g. O-MAC), sender-centric asynchrony (e.g. X-MAC, BoX-MACs), and receiver-centric asynchrony (e.g. RI-MAC). This enables a numerical comparison on capacity and energy efficiency of these protocols. The results strongly suggest that receiver-centric synchronous scheme has the best performance.

1. INTRODUCTION
Schedulers in low-power wireless networks not only coordinate packet communications to ensure reliable delivery – which is the primary objective for wall-powered wireless networks – they also coordinate the sleep/wakeup of nodes to control the energy consumption. Although a decade of productive research are conducted in low power wireless networks, the throughput capacity of wireless networks in duty-cycled fashion is not easily computed, and methods for its estimation have not received much attention. As a result, we do not know how far the maximum throughput of extant protocols/schedulers at various duty cycle are from what is achievable in theory.

With regard to non-duty-cycled wireless networks, the network capacity has been established by a few researchers [8]. By extending Gupta and Kumar's capacity result to accommodate the duty cycle, we analyze how throughput capacity and energy efficiency scale as duty cycle increases, wherein a node only wakes up its radio to communicate for a fraction of time. Our capacity analysis shows that for random wireless networks, there is a bound on the duty cycle beyond which the maximum throughput of the network does not increase. In other words, there is no capacity gain in operating the network at any duty cycle higher than this duty cycle bound, including the always-on mode. Equally importantly, the capacity that is achievable in an n-node general network at a duty cycle of ψ increases not with $\Theta(\frac{W\psi}{\sqrt{n\log n}})$ but with $\Theta(W\sqrt{\frac{\psi}{n}})$ till the capacity limit is reached, where $\Theta(\frac{W}{\sqrt{n\log n}})$ is the throughput capacity per node in random networks with full duty cycle [8]. Analogous results hold for duty-cycled networks with c channels and one interface, where the achievable capacity is $\Theta(W\sqrt{\frac{\psi c}{n}})$. As for the case of 1 hop MAC traffic, the capacity that is achievable increases linearly in ψ and independently of n till the capacity bound is reached.

We also note that there is a close relationship between capacity and energy efficiency. Energy efficiency —the ratio of the useful communication energy expended to the total energy expended— depends on the ratio of goodput to the overall duty cycle. Now, if a scheduler is able to deliver its traffic load reliably and goodput is equal to the throughput (i.e., duplicates packets are not received), then maximizing the energy efficiency essentially reduces to maximizing the throughput for a given duty cycle or, equivalently, minimizing the duty cycle for a given traffic load.

Outline of the report. Section 2 presents our analysis of duty-cycled capacity for random wireless networks. Section 3 considers the achievable capacity on multi-channel and single-interface model. Section 4 proposes a framework to derive throughput capacity achieved by four extant MAC protocols. Section 5 extends the result to compare energy efficiency of the schedulers. Related work is discussed in Section 6. We make conclusion and point out the future work in Section 7.

2. ANALYSIS OF DUTY-CYCLED RANDOM WIRELESS NETWORKS
In this section, we establish the throughput capacity of duty-cycled random wireless networks in terms of duty cycle and network density for both multi-hop and 1-hop traffic under the Protocol Model [8]. We first illustrate the system model. Second, the theory of duty-cycled capacity for multi-hop traffic is introduced. In the proof, we show the upper bound of throughput capacity and the feasibility of achieving the capacity limit by constructing a scheduler. At last, two corollaries are proposed to restrict our analysis for the case where communication is for senders and receivers that are 1 hop apart.

Symbol	Meaning	Value
W	maximum transmission rate	250 Kbps
q	uniform probability of backoff	1/16
u	beacon length relative to data length	0.4
E_{bit}	energy for sending a bit	0.217 μW
E_{radio}	energy consumed by radio per second	54.3 mW
n	total # of nodes in network	-
λ	throughput capacity	-
ψ	radio duty cycle	-
r_n	communication range	-
Δ	guard zone factor for interference-free communication	-

Table 1: Model parameters

2.1 System Model

Consider a random network where n nodes are uniformly and independently distributed in a unit square. Each node, X_i, $i \in 1, ..., n$ has a random destination node to which it sends data. We consider the case where the transmission range, r_n, and the traffic pattern are homogeneous for each node. Each node has a maximum bandwidth of W bps over a channel and wakes up on average to communicate for ψ fraction of time. We assume only one channel is available in the network and every node has one interface.

We adopt the Protocol Model which postulates a geometric condition for successful transmission [8]. Suppose node X_i transmits over the channel to a node X_j. Then this transmission at rate W bps is successfully received by X_j if the following inequality holds:

$$|X_k - X_j| \geq (1 + \Delta)|X_i - X_j|, \tag{1}$$

for all other nodes X_k, $k \neq i, j$, that are concurrently transmitting over the same channel. A circle of radius $(1 + \Delta)|X_i - X_j|$, $\Delta > 0$, quantifies a guard zone required around the receiver within which there is no destructive interference from neighboring nodes transmitting on the same channel at the same time. Table 1 summarizes the notation along with representative values for subsequent numerical comparisons.

2.2 Capacity of the Optimal Scheduler

We have the following main result for multi-hop traffic in duty-cycled random wireless networks. The proof contains two portions. The upper bound on capacity is derived, followed by a constructive lower bound on capacity that is achievable.

THEOREM 1. *The throughput capacity of duty-cycled random wireless networks in the Protocol Model is*

$$\Theta(\frac{W}{\Delta}\sqrt{\frac{\psi}{n}})bps \tag{2}$$

until it reaches network capacity of $\Theta(\frac{W}{\Delta^2\sqrt{n \log n}})$ *when* $\psi \geq \frac{32}{\Delta^2 \log n}$.

Two comments are in order about the result. For one, below the limiting duty cycle, the capacity scales better than linearly in ψ and better than the inverse of $\sqrt{n \log n}$. For two, the factor $\sqrt{n}$ reflects the average number of hops between source and destination, and $\sqrt{\psi}$ reflects the cumulative duty cycle to forward the source traffic along the route.

2.2.1 Upper Bound on Throughput Capacity

Recall that r_n is the common communication range of nodes in a random network with n nodes. Since each node needs to communicate with some other node, no node can be allowed to be an isolated node. It has been shown by [8] that r_n should be asymptotically larger than $\sqrt{\frac{\log n}{n\pi}}$. Furthermore, disks of radius $\frac{\Delta}{2}r_n$ centered at every transmitter should be disjoint. The area of each disk equals $\frac{\pi\Delta^2 r_n^2}{4}$, and at least 1/4 portion [1] is within the unit square. Hence, the maximum number of concurrent transmissions feasible in the network is no more than

$$\frac{1}{\frac{1}{4} \cdot \frac{\pi\Delta^2 r_n^2}{4}} = \frac{16}{\Delta^2 r_n^2 \pi}. \tag{3}$$

Let $\bar{L}$ be the expected distance between two uniformly and independently chosen points within the unit square. Then the expected length from a node to its destination is $Z = \bar{L} - o(1)$ since there is always a node within $\Theta(\sqrt{\frac{\log n}{n}})$ distance from any point (Lemma 5.7 [8]). As a consequence, each packet needs on average $\frac{Z}{r_n}$ hops to reach its destination. Each node generates packets at rate λ, which indicates that the bit rate the network needs so as to accommodate its traffic is at least $n\lambda\frac{Z}{r_n}$, where $0 < Z < 1$. In an optimal schedule, at most $n/2$ senders can be simultaneously active in any given slot. Suppose each sender on average wakes up for t slots out of T slots, the maximum number of potential transmissions in the network is $nt/2$.On the other hand, the maximum number of simultaneous transmissions the network can support is no more than $\frac{16}{\Delta^2 r_n^2 \pi}$. Therefore, the number of achievable transmissions during a period of T is $\frac{16}{\Delta^2 r_n^2 \pi}T$. As long as the number of potential transmissions does not exceed network capacity, i.e.,

$$\frac{nt}{2} \leq \frac{16T}{\Delta^2 r_n^2 \pi} \Rightarrow \frac{n}{2}\psi \leq \frac{16}{\Delta^2 r_n^2 \pi} \Rightarrow \psi \leq \frac{32}{n\Delta^2 r_n^2 \pi} \leq \frac{32}{\Delta^2 \log n}, \tag{4}$$

an optimal scheduler can accommodate the traffic. Shown in Eq. (4), r_n can be substituted in inequality $\psi \leq \frac{32}{n\Delta^2 r_n^2 \pi}$ since r_n is proven to be asymptotically larger than $\sqrt{\frac{\log n}{n\pi}}$ [8]. When the network capacity is reached, extra duty cycle cannot be effectively utilized. Thus, we have

$$n\lambda\frac{Z}{r_n} \leq W \cdot \min\{\frac{n\psi}{2}, \frac{16}{\Delta^2 r_n^2 \pi}\}. \tag{5}$$

Consider the inequality in Eq. (5) with respect to the second term in the *min* function. Since r_n is asymptotically larger than $\sqrt{\frac{\log n}{n\pi}}$, based on Eq. (5) we can derive

$$\lambda \leq \frac{16W}{n\pi\Delta^2 r_n Z} \leq \frac{c_1 W}{\Delta^2\sqrt{n \log n}}. \tag{6}$$

Now the first term in the *min* function represents throughput before the maximum capacity shown in Eq. (6) is reached. By plugging in the constraint $r_n^2 \leq \frac{32}{\Delta^2 n\pi\psi}$ derived from

[1]Previously in [8], it applies $1/4\pi$ portion instead of 1/4 without an explicit explanation. Typically, when a node is located at the corner of the square, only 1/4 fraction of its disk area is within the unit square, which is the smallest exclusion area around a transmitter.

Eq. (4), we complete the proof for Theorem. 1 with:

$$\lambda \leq \frac{W\psi r_n}{2Z} \leq \frac{c_2 W}{\Delta}\sqrt{\frac{\psi}{n}}\,. \quad (7)$$

In summary, the upper bound for the throughput capacity of duty-cycled random wireless networks is as follows.

$$\lambda \leq \min\{\frac{c_2 W}{\Delta}\sqrt{\frac{\psi}{n}},\ \frac{c_1 W}{\Delta^2\sqrt{n\log n}}\}. \quad (8)$$

2.2.2 Lower Bound on Throughput Capacity

In previous subsection, we establish the upper bound on throughput capacity in terms of both network density and duty cycle. To prove the tightness of the bound in Theorem. 1, we present a scheme that achieves throughput $\lambda = \frac{c_1 W}{(1+\Delta)}\sqrt{\frac{\psi}{n}}$ for every node in this subsection.

Consider the constructive lower bound on throughput capacity illustrated in Section 5.3 of [8], which achieves $\lambda = \frac{c_2 W}{(1+\Delta)^2\sqrt{n\log n}}$ bps for every node in the network to its chosen destination, with probability approaching one as $n \to \infty$. We only sketch its proof of lower bound for reasons of space. First, the unit square is divided into small cells of such a size that each of them holds at least one but no more than $O(\log n)$ nodes. Second, these cells are grouped into a finite number of non-interfering sets which can take turns in transmission without causing interference. Finally, they show that a simple routing strategy - following a packet from cell to cell "along" the line connecting the originating cell to the destination cell - can fulfill the job.

It has been shown that a transmitting schedule exists such that in every $M^2 = (c_3(1+\Delta))^2$ time slots, each cell gets one slot to transmit at rate W bps with transmission range r_n. Considering each cell's role of serving routes, Lemma 5.11 of [8] states that each cell needs to transmit at a rate less than $\lambda c_4\sqrt{n\log n}$, with probability approaching one, which leads to the following inequality of capacity

$$\lambda c_4\sqrt{n\log n} \quad \leq \quad \frac{W}{(c_3(1+\Delta))^2}. \quad (9)$$

The consumption of duty cycle in this schedule can be determined accordingly. Every cell needs to wake up at a duty cycle of $1/M^2$, i.e., $1/(c_3(1+\Delta))^2$ where $c_3 \geq 1$ to ensure non-interfering transmission. Since for any $K > 1$, with probability going to one, there are at least one but no more than $Ke\log n$ nodes in a cell and only one node is allowed to transmit in every cell each time, the duty cycle for each node to transmit is

$$\psi = \frac{1}{M^2 Ke\log n} = \frac{1}{c_3^2(1+\Delta)^2 Ke\log n}. \quad (10)$$

By reorganizing Eq. (9) and filling in ψ, the throughput capacity can be presented in terms of duty cycle and network density as follows.

$$\lambda c_4\sqrt{n} \quad \leq \quad \frac{W}{c_3(1+\Delta)}\cdot\frac{1}{c_3(1+\Delta)\sqrt{\log n}} \quad (11)$$

$$\lambda c_4\sqrt{n} \quad \leq \quad \frac{c_5 W}{(1+\Delta)}\sqrt{\psi}, \quad (12)$$

$$\lambda \quad \leq \quad \frac{c_1 W}{(1+\Delta)}\sqrt{\frac{\psi}{n}}, \qquad \text{where } \psi < \frac{32}{\Delta^2\log n}. \quad (13)$$

Note that achievability of the second term in *min* of Eq. (8) is already shown in [8]. Thus the derivation above completes the achievability part of Theorem 1.

2.3 Capacity of 1-Hop Traffic

Since we are interested in MAC performance, we restrict our analysis for the case of 1-hop traffic scenario and derive capacity of 1-hop traffic for both general and clique network in two corollaries.

For 1-hop traffic, Z/r_n has a constant value of 1, which leads to the following corollary on the throughput of 1-hop traffic in general networks:

COROLLARY 1. *The throughput capacity of a duty-cycled random wireless network with 1-hop traffic is*

$$\lambda = \Theta(W\psi) \quad (14)$$

until it reaches its network capacity of $\Theta(\frac{W}{\Delta^2\log n})$ *when* $\psi \geq \frac{32}{\Delta^2\log n}$.

Proof. We adjust the capacity inequality Eq. (5) by substituting Z/r_n by 1 as follows.

$$n\lambda\cdot 1 \leq W\cdot\min\{\frac{n\psi}{2},\ \frac{16}{\Delta^2 r_n^2\pi}\}. \quad (15)$$

Hence,

$$\lambda \leq W\cdot\min\{\frac{\psi}{2},\ \frac{16}{\Delta^2\log n}\}. \quad (16)$$

Note that the same condition on duty cycle suffices in this case, i.e., $\psi \leq \frac{32}{\Delta^2\log n}$.

For the special case of clique networks, the maximum number of simultaneous transmissions reduces from $\frac{16}{\Delta^2 r_n^2\pi}$ to 1, which leads to our next corollary:

COROLLARY 2. *The throughput capacity of a duty-cycled clique network with 1-hop traffic is*

$$\lambda = \Theta(W\psi) \quad (17)$$

until it reaches its network capacity of $\Theta(\frac{W}{n})$ *when* $\psi \geq \frac{2}{n}$.

Proof. We degenerate Eq. (5) further by replacing $\frac{16}{\Delta^2 r_n^2\pi}$ with 1:

$$n\lambda\cdot 1 \quad \leq \quad W\cdot\min\{\frac{n\psi}{2}, 1\}, \quad (18)$$

$$\lambda \quad \leq \quad W\cdot\min\{\frac{\psi}{2}, \frac{1}{n}\}. \quad (19)$$

Notably, below the limiting duty cycle, nodes in 1-hop traffic networks achieve a throughput that is independent of n and linear with duty cycle.

3. ANALYSIS OF DUTY-CYCLED NETWORKS WITH MULTIPLE CHANNELS

In this section, we explore the duty-cycled capacity of a network with multiple channels. We use the term "channel" to refer to a part of frequency spectrum with some specified bandwidth. There are c channels in total, and we assume that every node is equipped with m interfaces, where $m = 1$. Each node is allowed to switch its interface from one channel to another within short period of time. The c-channel-1-interface assumption is made in accordance with current technology of wireless sensor platforms.

According to Gupta and Kumar [8], the capacity per node with c channel and $m = c$ interfaces per node scales as $\Theta(Wc\sqrt{\frac{1}{n \log n}})$. Here, the aggregate data rate possible by using all c channels is Wc. Previous research on multi-channel capacity [4] shows that the capacity of multi-channel networks exhibits different bounds that are dependent on the ratio between c and m. When the number of interfaces per node is smaller than the number of channels, there is a degradation in the network capacity in many scenarios. However, one exception is a random network with up to $O(\log n)$ channels, wherein the capacity per node remains at the Gupta and Kumar bound, independent of the number of interfaces available at each node. In the model, the number of available channels, c, is a constant value, where $c \geq 1$.

Our results are derived under the assumption that there is no delay in switching an interface from one channel to another. We have the following main result for duty-cycled random wireless networks using multiple channels.

THEOREM 2. *The throughput capacity of duty-cycled random wireless networks with c channels and 1 interface in the Protocol Model is*

$$\Theta(\frac{W}{\Delta}\sqrt{\frac{\psi c}{n}})bps \tag{20}$$

until it reaches network capacity of $\Theta(\frac{Wc}{\Delta^2\sqrt{n \log n}})$ *when* $\psi \geq \frac{32c}{\Delta^2 \log n}$.

The above result implies that the capacity of multi-channel random networks increases square root of c fold before reaching the limit of duty cycle. The limit of duty cycle increases a factor of c as well.

3.0.1 Upper Bound on Throughput Capacity

Given the number of c available channels, the maximum number of concurrent transmissions in the network increases c fold, which equals $\frac{16c}{\Delta^2 r_n^2 \pi}$. Accordingly, Eq. (4) can be modified as follows.

$$\frac{nt}{2} \leq \frac{16Tc}{\Delta^2 r_n^2 \pi} \Rightarrow \frac{n}{2}\psi \leq \frac{16c}{\Delta^2 r_n^2 \pi} \Rightarrow \psi \leq \frac{32c}{n\Delta^2 r_n^2 \pi} \leq \frac{32c}{\Delta^2 \log n}. \tag{21}$$

The network capacity is divided into two regions as follows.

$$n\lambda\frac{Z}{r_n} \leq W \cdot \min\{\frac{n\psi}{2}, \frac{16c}{\Delta^2 r_n^2 \pi}\}. \tag{22}$$

Similarly to previous steps in Eq. (6) and (7), we may obtain the following upper bound of the throughput capacity, where a_1 and a_2 denote constants (insteaf of c_1 and c_2) to avoid overloading variable c.

$$\lambda \leq \min\{\frac{a_1 W}{\Delta}\sqrt{\frac{\psi c}{n}}, \frac{a_2 Wc}{\Delta^2\sqrt{n \log n}}\}. \tag{23}$$

3.0.2 Lower Bound on Throughput Capacity

The lower bound is established by constructing a routing scheme and transmission schedule and representing it through duty cycle. When the lower bound matches the upper bound, it implies that the bounds are tight. We adopt the construction in [4] and show that the upper bound is also achievable via analyzing duty cycle of the schedule.

The transmission schedule is built using a two-step process. In the first step, transmissions are scheduled in "edge-color" slots such that at every node during any edge-color slot, at most one transmission or reception is scheduled. In the second step, the edge-color is divided into $\lceil \frac{k_1 n a(n)}{c} \rceil$ "mini-slot". Each node is assigned a mini-slot for transmission or reception without interfering with others via vertex coloring. As a consequence, the average duty cycle of a node under this schedule is

$$\psi = \frac{1}{\lceil \frac{k_1 n a(n)}{c} \rceil} = \frac{c}{k_1 n a(n)}, \text{ where } c = O(na(n)). \tag{24}$$

The achievable rate for each flow is

$$\lambda = \Omega(\frac{Wc}{k_1 n\sqrt{a(n)}}). \tag{25}$$

By substituting $\sqrt{a(n)}$ with a function of ψ, we may obtain the following capacity

$$\begin{aligned} \lambda &= \Omega(\frac{Wc}{k_1 n\sqrt{\frac{c}{k_1 n\psi}}}) = \Omega(W\sqrt{\frac{\psi c}{k_1 n}}) && (26) \\ &= \Omega(\frac{W}{(2+\Delta)}\sqrt{\frac{\psi c}{n}}). && (27) \end{aligned}$$

Note that the constant k_1 in [4] has the value $a_3(2+\Delta)^2$, where $a_3 \geq 1$. Since in the construction we have $a(n) \geq \frac{100 \log n}{n}$, the scheduled duty cycle ψ satisfies the following restriction

$$\psi \leq \frac{c}{100 k_1 \log n} \leq \frac{c}{100\Delta^2 \log n} \leq \frac{32c}{\Delta^2 \log n}. \tag{28}$$

Thus the derivation above completes the achievability part of Theorem 2.

4. CAPACITY OF EXTANT MACS

Having derived the bounds on achievable capacity, now we proceed to analyze the maximum throughput of extant MACs. We first introduce the framework for computing capacity of CSMA-based MAC protocols. Next, the capacity of four representative MACs are analyzed separately with respect to a given duty cycle. The results can be extended to the many-to-many traffic model. Finally, we take the clique network case and numerically compare these MAC schedulers in terms of achievable throughput capacity.

4.1 Framework for MAC Schedulers

In contrast to an optimal scheduler, which guarantees that only one node transmits in an interference region, the canonical MAC protocols only ensure that the probability of a successful communication for any node during a slot is τ, where $\tau \in [0, 1]$. Hence, the expected number of successful transmissions for $n/2$ senders is $n\tau/2$. Accordingly, by substituting ψ with τ in Eq. (16) and (18), the capacity of these

MACs is

$$\lambda \le W \cdot \min\{\frac{\tau}{2}, \mathcal{C}(n)\}, \tag{29}$$

where

$$\mathcal{C}(n) = \begin{cases} 16/(\Delta^2 \log n), & \text{general network} \\ 1/n, & \text{clique network.} \end{cases} \tag{30}$$

We now present a framework for calculating τ for CSMA-based MAC protocols, which subsumes the representative protocols. In CSMA, typically, when a node attempts to transmit a packet, it first randomly selects one out of C_s contention slots and monitors the channel until that slot to ensure that no other transmission is occurring within its communication range. To avoid overloading the word slot, we henceforth refer to a contention slot as a timeslice. If any transmission is detected before the chosen timeslice, the sender withdraws its transmission attempt; otherwise, it immediately starts the data transmission after the timeslice. Let the probability of selecting any timeslice be q and $\hat{\epsilon}$ be the expected number of contenders in each node's communication range. The probability for a node to successfully access the channel, denoted as p_a, is thus

$$\begin{aligned} p_a &= q + q(1-q)^{\hat{\epsilon}-1} + q(1-2q)^{\hat{\epsilon}-1} + ... + q \cdot q^{\hat{\epsilon}-1}, \\ &= q \sum_{i=0}^{1/q-1} (1-iq)^{\hat{\epsilon}-1}. \end{aligned} \tag{31}$$

Let the expected number of contenders in the interference range be $\hat{\eta}$. Of course, the transmission is guaranteed to succeed when there is no other transmission within the interference range of the receiver. However, in general the probability of successful transmission in any given slot when data is available is equal to $p_a(1-p_a)^{\hat{\eta}-1}$. Thus, the total probability of successful transmission in any slot is

$$\tau = p_d \cdot p_a(1-p_a)^{\hat{\eta}-1}, \tag{32}$$

where p_d indicates the probability of transmitting data.

Now we determine the total number of potential contenders in communication range, named ϵ, and the number in interference range, called η, for a general network. Under the same system model of Section 2, we denote the communication range and interference range of every node by A and B, which are disks of radius r_n and $(1+\Delta)r_n$, respectively. Typically, A is contained within B (for the case $\Delta \ge 1$). Consider the smallest interference area in the unit square network shown in Fig. 1, i.e., at least 1/4 portion of B is within the unit square, where $|B| = \pi((\Delta+1)r_n)^2$. In this scenario, the receiver, X_r, is located at the corner of the square, while its sender, X_s, sits on the edge of the square with a distance r_n to the receiver. Accordingly, the smallest communication area is 1/2 portion of $|A|$, where $|A| = \pi r_n^2$. Since the current model assumes that every sender has its own receiver, at most half of nodes serve as transmitters in communications. We have

$$\epsilon = \frac{1}{2} \cdot \frac{n}{2} \cdot |A| = \frac{\pi r_n^2 n}{4}, \tag{33}$$

$$\eta = \frac{1}{4} \cdot \frac{n}{2} \cdot |B| = \frac{\pi((\Delta+1)^2 r_n^2 n}{8}. \tag{34}$$

For the special case of clique networks, the parameters are simply $\epsilon = \eta = n/2$.

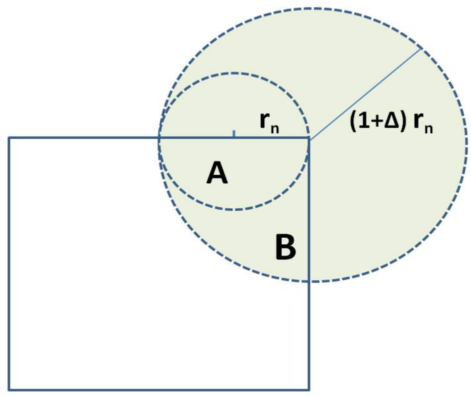

Figure 1: The Smallest Interference and Communication Area

MAC	Duty Cycle Constraint	$\hat{\eta}$ $(= \hat{\epsilon})$
SCP-MAC	$p_t\psi_r + \psi_r = 2\psi$	$p_t\eta$
O-MAC	$p_t\psi_r + \psi_r = 2\psi$	$p_t\psi_r\eta$
BoX-MAC	$(\frac{1}{2\psi_r} + 1) \cdot p_t\psi_r + \psi_r = 2\psi$	$(\frac{p_t}{2} + p_t\psi_r)\eta$
RI-MAC	$(\frac{1}{2\psi_r} + 1) \cdot p_t\psi_r + \psi_r = 2\psi$	$(p_t + u)\psi_r\eta$

Table 2: Capacity framework parameter for protocols

4.2 Analysis of MAC Schedulers

Discussed in previous subsection, Eq. (32) serves as the parameterized framework for analyzing the four representative MAC schedulers. We assume that q follows the same probability distribution for all MAC protocols. Considering in a clique network, the total number of contenders within a communication range, denoted by η, is equal to that within the interference range. In this model, every sender has chosen its own receiver. The total duty cycle of any sender-receiver pair is 2ψ, out of which a node spends ψ_r, $0 \le \psi_r \le 1$, in receiving mode to account for the time over which a node wakes up, polls the channel, possibly receives a packet, and goes to sleep. A corresponding sender may choose to send data with probability p_t once the receiver is known to be awake, which leads to the equation $p_d = p_t \cdot \psi_r$. We use the representative values for the constant parameters of Table. 1 and the key constraints subject to which we optimize the capacity for each MAC in Table 2.

4.2.1 SCP-MAC: Synchronous Sender Centric

SCP-MAC [9] is an extension of the canonical regionally synchronous S-MAC protocol, wherein all nodes in a region wake up simultaneously in each frame. Since receivers poll the channel for activity in an aligned fashion, sender preambles become short *wakeup tones* that are sent just before receiver polls. Such synchronized polling not only reduces the energy cost in sending the long preambles, but also improves channel utilization during the wakeup slots.

When a node spends duty cycle ψ_r, $0 \le \psi_r \le 1$, in receiving mode, the corresponding sender chooses to transmit at duty cycle $p_t\psi_r$, where $0 \le p_t \le 1$. Thus, the total 2ψ is distributed as

$$p_t \cdot \psi_r + \psi_r = 2\psi, \quad (35)$$

$$\psi_r = \frac{2\psi}{p_t + 1}, \quad (36)$$

where the constraint, $0 \leq \psi_r \leq 1$, has to be satisfied. Due to the global synchrony feature, the expected number of active contenders when a node attempts to transmit equals the product of the total number of senders in interference range, η, and the probability that a sender intends to transmit. Thus,

$$\hat{\eta} = p_t \eta. \quad (37)$$

Given each ψ over the range $[0, 1]$, we may determine the optimal τ of SCP-MAC by varying the traffic load p_t in range of $[0, 1]$.

4.2.2 O-MAC: Synchronous Receiver Centric

O-MAC [2] is receiver-centric and locally synchronous: receivers use pseudo-random wakeup schedules to avoid waking up simultaneously and communicate the seed for their schedule to neighboring senders. Senders with pending data thus wakeup just before their intended receiver does. The sharing of the seeds makes the use of probes unnecessary.

Since sender-receiver pair is synchronized, the utilization of duty cycle follows the same equality in Eq. (35). The expected number of interferers with respect to a node, however, reduces by a fraction of ψ_r due to staggered wake-up times. Thus,

$$\hat{\eta} = p_t \psi_r \eta. \quad (38)$$

4.2.3 BoX-MAC: Asynchronous Sender Centric

X-MAC [1] is an extension of the canonical asynchronous B-MAC protocol [6] that adopts the Low Power Listening (LPL) mechanism, where receivers independently and periodically poll the channel for activity using low power. Each sender wakes up its receiver by sending it a preamble that is at least as long as the receiver's frame length. By embedding destination information in the sender preambles, X-MAC reduces overhearing energy loss by allowing non-intended receivers to return to sleep earlier. Also, by inserting short gaps in between preambles, the sender avoids sending a continuous preamble and initiates data transmission upon receiving an acknowledgement from the intended receiver after some preamble. BoX-MAC [5] further refines X-MAC by sending data packets instead of preambles repeatedly, thus eliminating the sender's energy cost in sending the data packet after the preamble. At the receiver, BoX-MAC conserves energy by adopting an LPL-like channel activity detection mechanism instead of the more costly preamble detection, which consumes an order of magnitude more energy than the former.

The parameters for BoX-MAC are derived as follows. Since a node spends ψ_r, $0 \leq \psi_r \leq 1$, in receiving mode, the frame length is calculated as $1/\psi_r$ slots. In order to guarantee a rendezvous with the duty-cycled receiver, the sender has to transmit back-to-back packets for at most a period of the frame length. Hence, the expected number of slots to wait until the receiver waking up is $\frac{1}{2\psi_r}$ units. A corresponding sender may choose to send data with probability p_t when the receiver wakes up. Thus, the total duty cycle of 2ψ is divided into

$$(\frac{1}{2\psi_r} + 1) \cdot p_t \cdot \psi_r + \psi_r = 2\psi, \quad (39)$$

$$\psi_r = \frac{4\psi - p_t}{2(p_t + 1)}, \quad (40)$$

where the constraint, $0 \leq \psi_r \leq 1$, has to be satisfied. When a node is about to transmit data in a slot, the expected number of active contenders equals the product of the total number of senders in interference range and the probability that they are awake. Thus,

$$\hat{\eta} = (\frac{1}{2\psi_r} + 1) \cdot p_t \cdot \psi_r \cdot \eta = (\frac{p_t}{2} + p_t \psi_r)\eta. \quad (41)$$

4.2.4 RI-MAC: Asynchronous Receiver Centric

RI-MAC [7] is receiver-centric, where the rendezvous is initiated by the receiver. Receivers periodically broadcast a preamble to their neighbors. Receivers choose their periods independently and their wakeups are thus likely to not overlap, thereby attempting to avoid contention among senders for different receivers. Once senders have data to send they keep their radio active in receive mode and contend for the channel upon receiving a preamble from their intended receiver.

The rendezvous scheme is analogous to BoX-MAC as in Eq. (39) except that senders wait quietly for the short preamble from the correspondent receiver. We count the constant length of receiver preamble for u slot, where $0 < u \leq 1$. Accordingly, $\hat{\eta}$ now becomes

$$\hat{\eta} = (p_t + u)\psi_r \eta. \quad (42)$$

4.3 Many-to-many Traffic Model

Now we extend the MAC analysis for many-to-many traffic model, where every node acts as both sender and receiver. Assume when any neighboring receiver is on, the probability of transmission is p_t, then the probability of transmitting to one particular receiver is $p_t/(\epsilon - 1)$, where ϵ is the number of nodes in each node's communication range. η denotes the number of nodes in each node's interference range. As for the case of clique network of size n, we have the equality $\epsilon = \eta = n$.

Due to the similarity of analysis on different MACs, we only explain O-MAC as an example. Let a node independently spend duty cycle ψ_r in receiving mode. The duty cycle at which a node transmits to any neighbor is $\frac{p_t}{\epsilon-1}\psi_r$, thus the total transmit duty cycle of a node equals $(\epsilon - 1)\frac{p_t}{\epsilon-1}\psi_r$. Since every node is assigned total duty cycle of ψ, we have the following constraint on each node:

$$\psi_r + (\epsilon - 1) \cdot \frac{p_t}{\epsilon - 1}\psi_r = \psi. \quad (43)$$

At any slot of time, each node may transmit to any of its $\epsilon - 1$ neighbors at probability $\frac{p_t}{\epsilon-1}\psi_r$. Therefore, the expected number of interferers is

$$\hat{\eta} = (\epsilon - 1) \cdot \frac{p_t}{\epsilon - 1} \cdot \psi_r \cdot \eta = p_t \psi_r \eta. \quad (44)$$

Recall that equality $p_d = p_t \cdot \psi_r$ still holds in this scenario.

MAC	Duty Cycle Constraint	$\hat{\eta}\ (= \hat{\epsilon})$
SCP-MAC	$p_t\psi_r + \psi_r = \psi$	$p_t\eta$
O-MAC	$p_t\psi_r + \psi_r = \psi$	$p_t\psi_r\eta$
BoX-MAC	$(\frac{1}{2\psi_r} + 1) \cdot p_t\psi_r + \psi_r = \psi$	$(\frac{p_t}{2} + p_t\psi_r)\eta$
RI-MAC	$(\frac{1}{2\psi_r} + 1) \cdot p_t\psi_r + \psi_r = \psi$	$(p_t + u)\psi_r\eta$

Table 3: Capacity framework parameter for Uniform Traffic

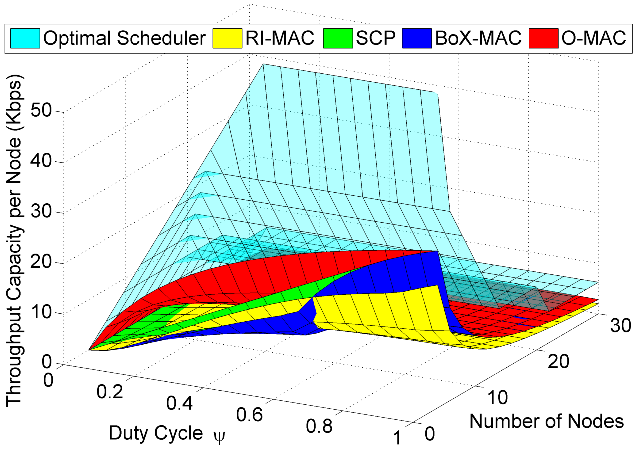

Figure 2: Comparison of throughput capacity in clique networks

The key constraints for each MAC under many-to-many uniform traffic model is listed in Table 3.

4.4 Comparison of MAC Capacity

Fig. 2 shows the MATLAB simulation throughput capacity results at various duty cycles for a network size ranging from 4 to 30 nodes. We observe that of the four protocols, O-MAC approximates the optimal scheduler best, although the performance gap decreases at high duty cycles. At low density, SCP-MAC outperforms RI-MAC as inter-receiver contention —contention caused by traffic destined to different receivers— is low and the synchrony in SCP-MAC substantially reduces the overhead in probe detection. As inter-receiver contention increases with density, RI-MAC takes over in performance. At full duty cycle, all MAC protocols converge to a pure CSMA scheme except for RI-MAC whose use of probes becomes a major constraint.

5. ENERGY EFFICIENCY OF EXTANT MACS

Provided that a MAC can schedule all source traffic within its throughput capacity, its energy efficiency, denoted by e, is the following:

$$e = \frac{\lambda \cdot t \cdot E_{bit}}{\psi \cdot t \cdot E_{radio}}, \tag{45}$$

where E_{bit} is the energy cost of sending one data bit, E_{radio} is the energy consumption rate for active radio, and t is the period of time considered.

Fig. 3 shows the MATLAB simulation results on energy efficiency of different MACs with the same configurations as in Fig. 2. The energy efficiency of all protocols is higher at low network density. As duty cycle increases, the efficiency

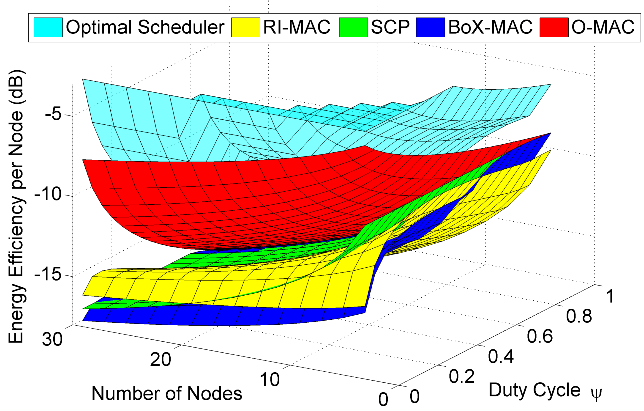

Figure 3: Comparison of energy efficiency in clique networks

of receiver-centric protocols decreases while the efficiency of BoX-MAC increases. Among the representative MACs, O-MAC remains the most energy efficient protocol under all configurations, with a maximum gap of $10dB$ over BoX-MAC and SCP-MAC, and a gap over RI-MAC ranging from $3dB$ to $8dB$.

6. RELATED WORK

Gupta and Kumar [3, 8] derived the capacity of wireless networks. Their results are applicable to single channel, or multi-channel wireless networks where each node is equipped a dedicated interface per channel. In contrast to their studies in wireless networks, we are more interested in capacity in a duty-cycled network, wherein a node only wakes up its radio to communicate for a fraction of time. To our knowledge, this work is the first one that explores the impact of duty cycle on capacity in wireless networks. Several researchers have extended the results of Gupta and Kumar to those multi-channel networks where nodes may have less interfaces than available channels [4]. We also consider the impact of multiple channels on the achievable capacity in duty-cycled network.

As for low-power wireless sensor networks, duty cycled MAC protocols have gained increasingly attention during the past decade. The MAC schedulers not only ensure reliable communication, they also coordinate the sleep/wakeup schedule of nodes to reduce energy cost. These protocols can be categorized by the centricity and asynchrony. Sender-centric asynchronous MACs include B-MAC [6], X-MAC [1], and BoX-MACs [5]. In these protocols, receivers periodically wakes up to check the channel for incoming data and senders transmit "preamble" or back-to-back short packets till the receiver is up. As for sender-centric synchronous MACs, e.g., SCP-MAC [9], the channel polling mechanism of LPL has been further refined, wherein all nodes are synchronized to poll at the same time. Thus, senders only transmit a short preamble slightly before the channel polling, resulting in less overhead in preamble transmission and more efficiency. Receiver-centric asynchronous MACs, such as RI-MAC [7], let a receiver periodically wake up to send out beacons as invitation for data. Whenever a sender has pending data, it monitors the channel until the beacon arrives and transmit the data after winning the channel contention. In contrast, in receiver-centric synchronous protocols such as O-MAC [2], pairwise synchronous communication enables minimal energy waste on rendezvous between senders and receivers. More description of each protocol can be found in section 4.2. To understand the capacity limits of MACs in different categories, we present a framework to derive the capacity of each representative MAC at given duty cycles.

7. CONCLUSIONS

In this paper, we have derived the upper and lower bounds on the capacity of duty cycled wireless networks. We have shown that in random wireless network, the throughput capacity increases with square root of ψc over n till the capacity limit is reached, where ψ denotes the given duty cycle, c is the number of available channels, and n is the number of nodes in network. As for the case of 1 hop MAC traffic model, we have found that the achievable capacity increases linearly in ψ and independently of n till it reaches the capacity limit. For the purpose of understanding the capacity of extant MAC protocols, we next have analyzed four state-of-the-art MACs given different duty cycles and network densities. The results show that receiver-centric and synchronous MAC protocol such as O-MAC approximates the capacity limit as well as energy efficiency most compared to other MAC schemes. As part of our future work, we are interested in applying the insights obtained from this work to design algorithms that approach the capacity and energy efficiency limit.

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