

# Distributed Roadmap Aided Routing in Sensor Networks

Zizhan Zheng, Kai-Wei Fan, Prasun Sinha, Yusu Wang

Department of Computer Science and Engineering, The Ohio State University

Email: {zhengz, fank, prasun, yusu}@cse.ohio-state.edu

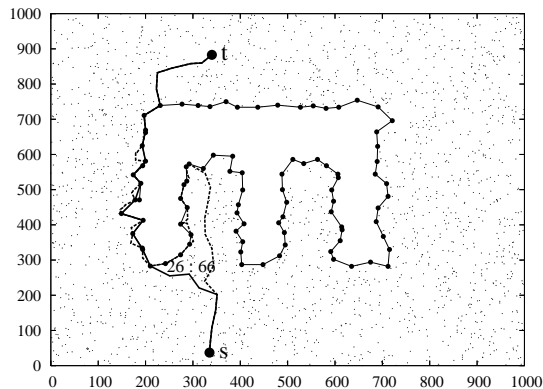
## Abstract

Communication between arbitrary pairs of nodes has become critical to support in emerging sensor networking applications. Traditional routing techniques for multi-hop wireless networks either require high control overhead in computing and maintaining routes, or may lead to unbounded route-stretch. In addition, protocols designed specifically for sensor networks usually consider routing to and from a single sink only. In order to bound the route-stretch, we propose a distributed shortest-path roadmap based routing paradigm that embodies two ideas: routing hole approximation that summarizes the critical information about hole boundaries and controlled advertisement that advertises the boundary information of each hole within limited neighborhoods. Through such techniques, we show how geographic routing can be leveraged to guarantee bounded route stretch. We further show that our approach makes a desired tradeoff between the worst case route-stretch and the message overhead through both analysis and simulations.

## 1 Introduction

With emerging sensor applications where packets may originate from anywhere in the network and may be destined to any node, such as pursuer-evader tracking [4] and battlefield monitoring [14], pairwise communication between arbitrary sensors becomes an essential requirement in sensor networks. New functionalities such as in-network storage [13] also require communication between arbitrary nodes. Stateful routing protocols designed for multi-hop wireless networks can incur high communication and storage overhead (up to  $O(n)$  per node in the worst case where  $n$  is the number of nodes in the network), and therefore are not suitable for resource constrained sensor networks. Although stateless routing protocols based on geographic information have been proposed [2, 7, 8, 10], and perform well in networks with dense deployments, their performance can be severely impacted in presence of holes in the network.

Geographic routing protocols typically operate in two phases. A packet is greedily forwarded towards the destination until a “local minimum” is reached, where the forwarding node has no neighbor closer to the destination due to the incidence of a routing hole. A recovery phase is then followed to bypass the hole until the greedy phase can be continued. Due to the reactive routing decisions upon encountering a hole, the discovered path may be substantially longer than the shortest path in terms of the number of hops, especially when the hole is big. Figure 1 shows an scenario from our simulations. The path found by GPSR [7], a classic geographic routing protocol, has 66 hops. However if the hole is known by the nodes in its neighborhood and bypassed in advance, the path length can be lowered to 26 hops. Large holes may exist in a network due to the presence of large obstacles such as a building or a lake, or an irregular deployment of sensing nodes.



**Figure 1.** Geographic routing can result in sub-optimal routes and high route-stretch. The dashed line is the routing path of GPSR, and the straight line is the routing path if the hole can be bypassed before its boundary is touched. The boundary nodes of the hole are highlighted.

One of the critical metrics for routing in sensor networks is the route-stretch, which is the ratio of the number of hops on the computed route to the number of hops on the shortest route. The route-stretch is closely related to the end-to-end

latency as well as the energy consumption. In Figure 1, as the “concave” regions of the hole can be arbitrarily deep, the stretch is unbounded for GPSR. There do have several recent works that deal with holes explicitly so that packets can bypass holes in advance without getting trapped [1, 6, 15]. However, none of these works ensure a bound on route-stretch and some of them [6, 15] only support “many to one” communications, where all the data are forwarded towards a single sink. The key observation is that *the presence of a hole has to be made known to at least the nodes in the vicinity of the hole to bound the stretch.*

We propose a distributed shortest-path roadmap based routing paradigm where each routing hole is treated as a polygonal obstacle. and explore two ideas, *routing hole approximation* and *controlled advertisement*, to support low-overhead critical information propagation for holes to assist in computing routes with bounded stretch. Instead of handling holes passively, we proactively advertise information on holes, but within a controlled region where route-stretch is most affected by the holes. Our approach is composed of the following two components:

- **Hole approximation and advertisement:** Each routing hole is approximated by its *core*, a simple polygon enclosed by the hole, controlled by a single parameter  $\alpha$ . Each routing core is then flooded to the  $k$ -hop neighborhood of the hole, where  $k$  is proportional to the size of the hole.
- **Hole bypassing routing:** Each node sets up a shortest-path roadmap [9] locally by treating the cores it knows as obstacles, and makes routing decisions based on that map. The real path mimics the planned one and is realized by greedy forwarding and hole traversing, and is further optimized by a local strategy.

The contributions of this paper are:

- We propose a protocol for routing between any pair of nodes in a sensor network, which has low storage and message overhead, and guarantees a bounded route-stretch.
- We prove that the route-stretch of our protocol is bounded by  $1/\cos(\alpha/2)$  if all the cores are advertised to the entire network. We also derive a bound for the route stretch when the cores are advertised locally.

By guaranteeing route-stretch with low message and storage overhead, our approach is suited for sensor networks to support efficient pairwise communication.

The remainder of this paper is organized as follows. In the next section, we present the main idea of the hole bypassing routing protocol. The details on the approximation

and advertisement of routing holes are discussed in Section 3, and the bounds on route-stretch are analyzed in Section 4. In Section 5, the basic routing scheme is further optimized by a local strategy. Our approach is compared with GPSR [7] and GLDR [12] in Section 6 through simulations. The related work are summarized in Section 7. We conclude our work in Section 8.

## 2 Hole Bypassing Routing (HBR)

We assume in this paper that each node knows the positions of itself and its 1-hop neighbors, and each routing hole  $H$  is a closed region bounded by a simple polygon  $\partial H$ , where there is a node at each vertex, and two adjacent boundary nodes are within the transmission ranges of each other. Notice that, routing holes are not necessarily disjoint from each other and may share boundary nodes or edges. In the continuous domain where the network density is so high that we can assume that there is a node at every point in the target field, it is well known from motion planning [9] and computational geometry [3] that if every node  $s$  knows the complete boundaries of all routing holes,  $s$  can build a complete shortest-path roadmap locally by viewing routing holes as polygonal obstacles, and then for a given destination  $t$ ,  $s$  can find an optimal path to  $t$ . Furthermore, an optimal path is composed of line segments connecting convex boundary vertices defined as follows.

**Definition 2.1. Convex vertex, concave vertex:** *A convex (resp. concave) vertex of a polygon  $P$  is a vertex of  $P$  for which the interior angle is less (resp. greater) than  $\pi$ .*

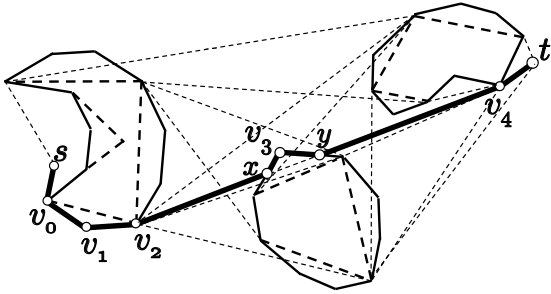
The main problem of applying this approach to a sensor network directly is its high message overhead due to the flooding of the complete hole boundaries to the entire network and the high storage overhead at each node to maintain a complete shortest-path map. The main idea of our approach is to approximate each routing hole  $H$  with a *core*, a simple polygon  $H_c$  enclosed by  $H$  that satisfies special requirements defined precisely in Section 3.2. One of these requirements relevant now is that every convex vertex of  $H_c$  must be a boundary node of  $H$ .

Cores are then advertised to its  $k$ -hop neighborhood where  $k$  is determined by the size of the hole. Every node can build a shortest-path roadmap locally by viewing the cores it knows as obstacles. Since each core is contained in a routing hole, cores do not intersect with each other except possibly at boundaries. Furthermore, a path computed is composed of segments ending at the convex vertices of cores, which are boundary nodes by definition. Therefore, by substituting every such segment with a subpath implemented by either greedy forwarding or hole traversing, a realistic path can be constructed. We will show that such

a hole approximation and advertising approach can significantly reduce the control overhead while still ensuring a desired route-stretch.

**Definition 2.2. Route-stretch:** For a given routing protocol  $\mathcal{R}$ , its route-stretch with respect to a source destination pair  $(s, t)$  is  $\rho(s, t) = |f_{\mathcal{R}}|/|f_{opt}|$ , where  $f_{\mathcal{R}}$  and  $f_{opt}$  are the paths from  $s$  to  $t$  found by  $\mathcal{R}$  and the shortest path, respectively, and  $|f|$  is the length of  $f$  in continuous domain or the number of hops of the path in discrete domain.

In this section, we present the main idea of the routing protocol, and assume that all the routing holes have been discovered, approximated, and advertised. These mechanisms will be presented in Section 3.



**Figure 2.** A shortest-path roadmap and a path from  $s$  to  $t$  computed by HBR in the continuous domain. The three polygons are routing holes and the dashed polygons are their cores. The dotted segments are the bitangent edges of the map. The path computed using cores is  $(s, v_0, v_2, v_4, t)$ . The real path is highlighted.  $x$  and  $y$  are the intersections of line  $v_2v_4$  with the boundary of the hole in the middle.

## 2.1 Building Roadmaps

Once a node  $s$  learns about a set of cores, it builds a shortest-path roadmap locally, which is defined as follows [9]. See Figure 2 for reference.

**Definition 2.3. Shortest-path roadmap:** A shortest-path roadmap at node  $s$  consists of the set of the convex vertices  $V_s$  of the cores that  $s$  knows and the set of edges  $E_s$ . For any two vertices  $a, b \in V_s$ ,  $(a, b) \in E_s$  if  $a$  and  $b$  are visible to each other, and either  $(a, b)$  is an edge of a core or the line going through  $a$  and  $b$  is a bitangent line, that is, a tangent line at  $a$  and at  $b$  with respect to the cores they belong to.

The shortest-path roadmap at node  $s$  can be built in  $O(|V|^2 \log |V|)$  time, where  $V$  is the set of core vertices that  $s$  knows [9].

## 2.2 Routing Protocol

Consider an arbitrary source-destination pair  $(s, t)$ . Algorithm 1 shows how a packet  $p$  is forwarded from  $s$  to  $t$ . See Figure 2 for reference.

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### Algorithm 1 Hole bypassing routing

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1. Initialize:  $v \leftarrow s$ .
  2.  $v$  makes a routing plan locally to select the next convex vertex  $v'$  on the shortest path from  $v$  to  $t$ .
  3.  $p$  is forwarded towards  $v'$  as follows.
    - (a) If  $v$  and  $v'$  are consecutive vertices of a core  $H_c$  for some hole  $H$ ,  $\partial H$  is traversed to reach  $v'$ .
    - (b) Otherwise,  $p$  is forwarded greedily towards  $v'$ . If a hole  $H$  is touched before reaching  $v'$ ,  $\partial H$  is traversed to reach the node that is closest to the intersection of  $\partial H$  and  $\overline{vv'}$  where  $v'$  is visible, then greedy forwarding continues.
  4. Let  $v \leftarrow v'$  and repeat step 2 and 3 until  $t$  is reached.
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At step 2, the routing plan at node  $v$  is made as follows. First, the shortest-path roadmap is extended by connecting  $v$  and  $t$  to all the visible roadmap vertices. Second, Dijkstra's algorithm is applied to find the first convex vertex on a shortest path from  $v$  to  $t$ .

At step 3(b), there are two possible directions when traversing  $\partial H$  is required. Suppose  $p$  touches  $\partial H$  at node  $x$ .  $x$  will make a routing plan towards  $v'$  using only the core of  $H$  to determine which direction to go. If  $xv'$  is disjoint from the core, the packet will follow the direction so that the real path is also disjoint from the core. For instance, in Figure 2, a packet forwarded by  $x$  towards  $y$  traverses the hole boundary in clockwise order, which is planned by  $x$ .

To assist the routing protocol, each data packet header carries the locations and node IDs of the source, the destination, and the next route planning node (a convex vertex).

In the above protocol, a new routing plan is made at each convex vertex on a path from  $s$  to  $t$ . This can be optimized as follows. First, when a planned path includes a sequence of contiguous convex vertices on the same hole, routing plans could be made only at the first and last convex vertices, and let the packet header carry the last one so that the intermediate convex vertices can simply forward the packet towards the last vertex. Second, a packet header could carry part of the routing plan (a sequence of convex vertices) made at the source or an intermediate node. Third, a node could cache the routing plans made for certain destinations. In this case, a protocol that handles outdated plans due to the changes of network topology is needed, which is left for future work.

We will analyze the performance of the routing protocol in Section 4, and a local optimization strategy is presented

in Section 5.

### 3 Hole Approximation and Advertisement

In this section, we discuss how a routing hole is discovered, approximated by its core, and advertised in a controlled way.

#### 3.1 Hole Discovery

We apply the approach proposed in [5] to discover routing holes in a network, which involves a local rule called TENT that finds nodes where packets may get stuck in greedy forwarding and the BoundHole algorithm that discovers routing holes with stuck nodes on their boundaries. The nodes on the same hole boundary then cooperate to elect the node with the smallest node ID as the hole coordinator. That ID is also used as the hole ID. Each boundary node then sends a message containing its position to the coordinator. Furthermore, every boundary node keeps the positions of the two neighboring boundary nodes in each direction. The coordinator then approximates the hole boundary as discussed below.

#### 3.2 Hole Approximation

In this section, we discuss how to approximate a routing hole by its core. Our approach can be viewed as a polygon simplification approach since each routing hole is bounded by a polygon. Although many work on polygon simplification have been done in computational geometry, they can not be directly applied to our scenario because the approximation has to satisfy the following two properties to simplify the routing protocol and bound route-stretch:

1. A core is contained in the original routing hole, and its convex vertices must be the boundary nodes of the hole.
2. A core should be a good approximation of the original routing hole so that the route-stretch of our hole bypassing protocol can be bounded.

We will first consider how to approximate a routing hole bounded by a convex polygon, and then extend the approach to a general polygon. In both cases, we assume a routing hole has at least 4 boundary nodes since a polygon with less than 4 vertices can not be simplified any further.

##### 3.2.1 Holes with Convex Boundary

Consider a routing hole  $H$  bounded by a convex polygon  $\partial H$  with vertices  $v_0, v_1, v_2, \dots, v_{n-1}$  ( $n \geq 4$ ) sorted in coun-

terclockwise order. The idea is to divide  $\partial H$  into chains and replace each chain with a line segment. The approximation has only one parameter  $\alpha$ , which determines how vertices are grouped. Let  $\beta_{kk'} \in [0, \pi)$ ,  $k' \geq k + 1$ <sup>1</sup> denote the angle from line  $v_k v_{k+1}$  to line  $v_{k'-1} v_{k'}$  (see Figure 3). The following algorithm is named as  $\alpha$ -approximation, which begins at a vertex  $v_0$  and traverses the hole boundary in counterclockwise order. See Figure 3 for reference.

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#### Algorithm 2 $\alpha$ -approximation of a convex hole $H$

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- 1:  $H_c \leftarrow \{v_0\}$ ,  $k \leftarrow 0$
  - 2: **repeat**
  - 3:   find the largest  $k'$  s.t.  $k < k' \leq n$  and  $\beta_{kk'} \leq \alpha$
  - 4:    $H_c \leftarrow H_c \cup \{v_{k'}\}$ ,  $k \leftarrow k'$
  - 5: **until**  $k = n$
- 

The time complexity of the above procedure is  $O(n)$ . The following proposition states that the size of a core in terms of the number of vertices on it is independent of the size of the hole, and is only determined by  $\alpha$ , which directly follows from the fact that the sum of the exterior angles of any convex polygon is  $2\pi$ .

**Proposition 3.1.** *The size of the core of a convex hole obtained by  $\alpha$ -approximation is bounded by  $\lfloor 2\pi/\alpha \rfloor$ .*

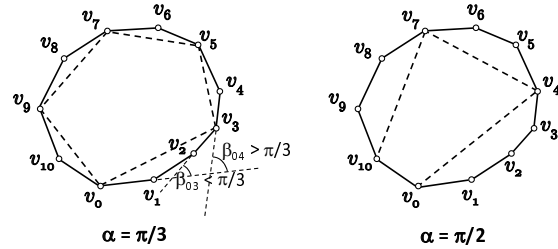


Figure 3. The approximation of convex holes.

##### 3.2.2 Holes with non-Convex Boundary

Consider a routing hole  $H$  bounded by a non-convex polygon  $\partial H$  with vertices  $v_0, v_1, v_2, \dots, v_{n-1}$  ( $n \geq 4$ ) sorted in counterclockwise order.  $\partial H$  can be divided into multiple disjoint interleaving convex and concave chains defined as follows.

**Definition 3.1. Convex chain, concave chain:** *A sequence of vertices  $C_{ij} = (v_i, v_{i+1}, \dots, v_j)$  (without repetition) of a polygon  $P$  is a convex (resp. concave) chain if every vertex in the chain is a convex (resp. concave) vertex, and the chain can not be extended further to maintain the above property.*

<sup>1</sup>In this section, all the arithmetic operations on subscripts are modulo  $n$  operations, and  $v_n := v_0$ .

The approximation of a non-convex  $\partial H$  works as follows.

1. Apply  $\alpha$ -approximation to every convex chain of  $\partial H$  starting at one end of the chain, with the additional requirement that the line segments used to replace the chain must lie in the core computed so far. Name the resulting polygon  $P$ .
2. Simplify every concave chain of  $P$  (discussed below) to get  $H_c$ .

There are two things to be noted. First, a line segment  $\overline{v_i v_j}$  lies in a polygon  $P$  iff (1)  $\overline{v_i v_j}$  is disjoint from any edges of  $P$  except possibly at  $v_i$  and  $v_j$  and (2) for any point  $x$  in the segment other than  $v_i$  and  $v_j$ ,  $x$  lies in  $P$ . Both conditions can be checked in  $O(n)$  time. Therefore, the time complexity of the above procedure is  $O(n^2)$ . Second, step 1 and step 2 may be applied alternately more than once to reduce the size of  $H_c$ .

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**Algorithm 3** Simplification of a concave chain  $(v_i, \dots, v_j)$

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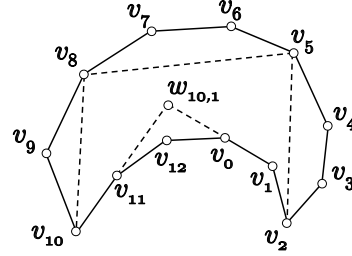
- 1:  $C \leftarrow \{v_{i-1}\}, k \leftarrow i - 1$
  - 2: **repeat**
  - 3:   find the largest  $k'$  s.t.  $k < k' \leq j + 1$  and both segments  $\overline{v_k w_{kk'}}$  and  $\overline{w_{kk'} v_{k'}}$  lie in the polygon computed so far
  - 4:    $C \leftarrow C \cup \{w_{kk'}, v_{k'}\}, k \leftarrow k'$
  - 5: **until**  $k = j + 1$
- 

To simplify concave chains, we recall that the concave vertices of a core are not part of the shortest-path map, and therefore a concave chain could be simplified without considering the error criterion – the worst case route-stretch in our scenario. Consider a concave chain  $C_{ij}$ . Let  $w_{kk'}, k' \geq k + 1$  denote the intersection of line  $v_k v_{k+1}$  and line  $v_{k'-1} v_{k'}$  (see Figure 4). Algorithm 3 embodies the similar idea of  $\alpha$ -approximation and outputs  $C$ , the simplification of  $C_{ij}$ . See Figure 4 for reference. The running time of the algorithm is  $O(|C_{ij}|n)$  and the total time complexity of the approximation of a routing hole with non-convex boundary of size  $n$  is therefore  $O(n^2)$ .

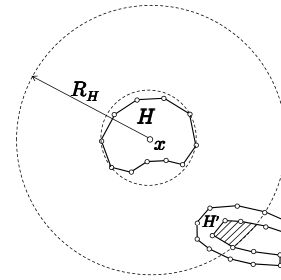
Notice that for any concave chain  $C_{ij}$ ,  $v_{i-1}$  and  $v_{j+1}$  must exist and must be convex vertices. Furthermore,  $v_{i-1}$  and  $v_{j+1}$  will not be removed by the approximation algorithm, which implies the following two properties about cores.

**Property 3.1.** *The vertex set of a convex (resp. concave) chain of  $H_c$  is a subset of the vertex set of a convex (resp. concave) chain of  $H$ .*

**Property 3.2.** *For any shortest path  $f$  computed using cores, if  $f$  is disjoint from a core  $H_c$ , then either  $f$  is disjoint from the corresponding hole  $H$ , or  $f$  intersects a single convex chain of  $H$ .*



**Figure 4.** The approximation of a routing hole with one convex chain and one concave chain.  $\alpha = \pi/2$  for the convex chain. The hole is simplified to the polygon with vertices  $v_1, v_2, v_5, v_8, v_{10}, w_{10,1}$ .



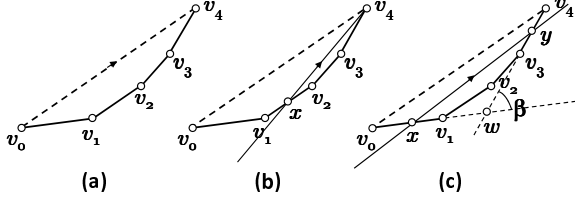
**Figure 5.** Controlled hole advertisement. The small circle centered at  $x$  is the minimum bounding circle of hole  $H$ . The core of  $H$  is advertised within the large circle also centered at  $x$  with radius  $R_H$ . Besides flooding, to reach the shaded region, the message will also traverse  $\partial H'$ .

### 3.3 Controlled Hole Advertisement

After  $H_c$  is computed, the coordinator of  $H$  computes  $C_H$ , the *minimum bounding circle* of  $H$ , which can be done in linear time [11]. Suppose  $C_H$  is centered at  $x$ . The coordinator then sends a message containing all the vertices of  $H_c$  sorted in counterclockwise order. The message is flooded to all the nodes within a big circle  $C'_H$  centered at  $x$  with radius  $R_H = \lambda p_H$ , where  $\lambda \geq 1$  is a constant and  $p_H$  is the perimeter of  $H$ , which equals to the size of  $H$  in the discrete domain. The impact of different choices of  $\lambda$  on the worst case route-stretch will be discussed in Section 4.2. To reach obstructed regions within  $C'_H$  (such as the shaded region in Figure 5), the flooding messages also hug the boundary of holes intersecting with  $C'_H$ .

## 4 Bounded Worst Case Stretch in Continuous Domain

In this section, we analyze the worst case route-stretch of HBR in the continuous domain, assuming routing holes are



**Figure 6.** A line segment in  $f'$  vs. the corresponding segment in  $f$ . The convex chain  $(v_0, v_1, v_2, v_3, v_4)$  is approximated by the line segment  $\overline{v_0v_4}$ . (a)  $\overline{v_0v_4}$  vs.  $(v_0, v_1, v_2, v_3, v_4)$ . (b)  $\overline{xv_4}$  vs.  $(x, v_2, v_3, v_4)$ . (c)  $\overline{xy}$  vs.  $(x, v_1, v_2, v_3, y)$ .

approximated using Algorithms 2 and 3. In Section 4.1, we assume all the cores are advertised to the *entire* network, and show that the worst case route-stretch is only determined by  $\alpha$ . The impact of controlled advertisement on route stretch is considered in Section 4.2.

#### 4.1 Hole Approximation

Consider an arbitrary source-destination pair  $(s, t)$ . Let  $f$  denote the path from  $s$  to  $t$  found by HBR with  $\alpha$ -approximation and  $f_{opt}$  denote the shortest path. We have the following theorem.

**Theorem 4.1.**  $|f| \leq |f_{opt}|/\cos(\alpha/2)$ .

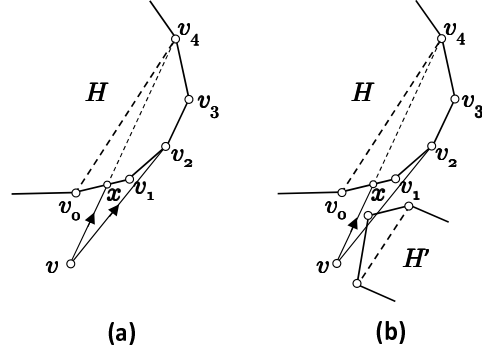
*Proof.* Let  $f'$  denote the shortest path planned using cores. First, we have  $|f'| \leq |f_{opt}|$  since every core is contained in the corresponding routing hole. We show that  $|f| \leq |f'|/\cos(\alpha/2)$ , which implies the theorem.

It is known that  $f'$  is composed of contiguous line segments ending at convex vertices of cores (we can view  $s$  and  $t$  as cores with only one vertex). By Algorithm 1 and Properties 3.1 and 3.2,  $f$  is also composed of a sequence of segments, and every such segment  $\omega$  corresponds to a line segment  $\omega'$  in  $f'$ . Furthermore, there are only three cases where  $\omega$  and  $\omega'$  are different, as shown in Figure 6. It is sufficient to show that  $|\omega| \leq |\omega'|/\cos(\alpha/2)$  for each  $\omega$ .

Without loss of generality, consider the case (c) in Figure 6, where  $\omega = (x, v_1, v_2, v_3, y)$  is part of a convex chain and  $\omega' = \overline{xy}$ . By geometric argument, we have  $|\omega| \leq |\overline{xv_1}| + |\overline{v_1v_2}| + |\overline{v_2v_3}| + |\overline{v_3y}| \leq |\omega'|/\cos(\beta/2) \leq |\omega'|/\cos(\alpha/2)$ .  $\square$

#### 4.2 Controlled Hole Advertisement

In this section, we consider the impact of controlled advertisement on route-stretch. For an arbitrary source-destination pair  $(s, t)$ , we still use  $f$  to denote the path from  $s$  to  $t$  found by HBR, with both  $\alpha$ -approximation and controlled advertisement applied, and use  $f_{opt}$  to denote the



**Figure 7.** Convex chain  $(v_0, v_1, \dots, v_4)$  is approximated by  $\overline{v_0v_4}$ . A packet is forwarded by  $v$  towards  $v_4$ . (a) The optimization of HBR by early bypassing.  $(v, x, v_1, v_2, v_3, v_4)$  and  $(v, v_2, v_3, v_4)$  are paths found by HBR without and with optimization, respectively. (b) The optimization can not be applied if the extreme node is not visible.

shortest path. Let  $h$  denote the number of routing holes whose core intersects  $f'$  but is not used in route planning. We have the following theorem.

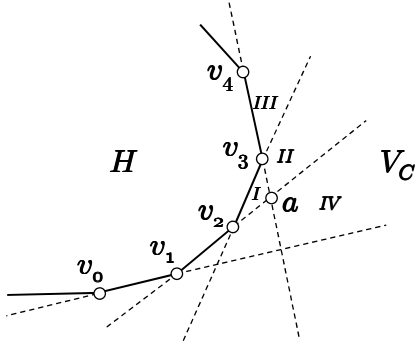
**Theorem 4.2.**  $|f| \leq |f_{opt}|(1/\cos(\alpha/2) + h/(\lambda - 1))$ .

*Proof.* Let  $f'$  denote the shortest path planned using cores. First, we have  $|f'| \leq |f_{opt}|$  since only a subset of cores are used to make routing plans and every core is contained in the corresponding routing hole. Therefore, it is sufficient to show  $|f| \leq |f'|/\cos(\alpha/2) + h/(\lambda - 1)$ . Let  $\mathcal{H}$  denote the routing holes that intersect  $f'$  but not used in route planning.  $|\mathcal{H}| = h$ . Since each  $H \in \mathcal{H}$  increases the route by at most  $p_H$ , combining with Theorem 4.1, we have  $|f| \leq |f'|/\cos(\alpha/2) + \sum_{H \in \mathcal{H}} p_H$ . For every  $H \in \mathcal{H}$ , since  $f'$  intersects  $H$  and  $H$  is not known by the last route planning node before  $\partial H$  is touched, we have  $|f'| > \lambda p_H - r_H$  according to the advertising protocol, where  $r_H$  is the radius of the minimum bounding circle of  $H$ . Since  $r_H \leq p_H$ , we have  $|f'| > (\lambda - 1)p_H$ . It follows that  $|f| \leq |f'|/\cos(\alpha/2) + h/(\lambda - 1)$ .  $\square$

## 5 An Optimization of HBR

In this section, we consider an optimization of HBR motivated by the following observation. See Figure 7 for reference. Suppose a packet is forwarded by node  $v$  towards  $v_4$  on  $\partial H$ . By Algorithm 1 proposed in Section 2.2, the packet will follow the path  $(v, x, v_1, v_2, v_3, v_4)$  where  $x$  is the intersection of line  $vv_4$  with  $\partial H$ . However, if  $v_2$  is visible to  $v$ , a shorter path would be  $(v, v_2, v_3, v_4)$ .

The example also reveals a typical drawback of most geographic routing protocols that treat holes in a reactive way.



**Figure 8.** EVRs of a convex chain of hole  $H$ . For instance, regions I, II, and III are EVRs of  $(v_2, v_3)$ ,  $(v_2, v_4)$ , and  $(v_3, v_4)$ , respectively.

It shows that even if there is no “local minima”, the simple greedy forwarding strategy may lead to a suboptimal path. To optimize this scenario, we first give the following definitions.

**Definition 5.1. Visible region of a convex chain:** Given a convex chain  $C = (v_i, v_{i+1}, \dots, v_j)$ , each line  $v_k v_{k+1}$  ( $i \leq k < j$ ) induces a half plane  $A_k$  that is disjoint from the chain except at  $\overline{v_k v_{k+1}}$ . The visible region  $V_C$  of  $C$  is defined as  $\bigcup_{i \leq k < j} A_k$ .

**Definition 5.2. Extreme nodes:** Given a convex chain  $C = (v_i, v_{i+1}, \dots, v_j)$  and a point  $s \in V_C$ , there are two vertices  $v_k, v_{k'}$ ,  $i \leq k < k' \leq j$  such that  $v_k$  and  $v_{k'}$  are visible to  $s$  while  $v_{k+1}$  and  $v_{k'-1}$  are not, with respect to  $C$ .  $v_k$  and  $v_{k'}$  are called extreme nodes of  $s$ , with respect to  $C$ .

For instance, in Figure 7(a),  $v_0$  and  $v_2$  are extreme nodes of  $v$ ; in Figure 8,  $v_2$  and  $v_3$  are extreme nodes of  $a$ . By the above definition, we make the following observation.

**Property 5.1.** Given a convex chain  $C$ , the two extreme nodes of a point  $s \in V_C$  are separated by any straight line that goes through  $s$  and intersects  $C$ .

For instance, in Figure 7(a),  $v_0$  and  $v_2$  are separated by line  $vv_4$ .

**Definition 5.3. EVR:** Given a pair of vertices  $v_i$  and  $v_j$  of a convex chain, the connected subregion in the network in which every point has  $v_i$  and  $v_j$  as the extreme nodes is named as an extremely visible region (EVR) relative to  $v_i$  and  $v_j$ .

The above optimization can be more precisely described as follows. Suppose the positions of each pair of vertices in any convex chain have been advertised to its EVR. How this is implemented will be discussed later. Suppose node  $v$  needs to forward a packet to convex vertex  $v'$  of hole  $H$ , the step 3 of Algorithm 1 is modified as follows.

- (a) If  $v$  and  $v'$  are consecutive vertices of a core  $H_c$  for some hole  $H$ ,  $\partial H$  is traversed to reach  $v'$ .
- (b) Let  $v_i, v_j$  denote the extreme nodes of  $v$  with respect to the convex chain where  $v'$  is on. If no such  $v_i$  and  $v_j$  exist, goto (c). Suppose in the chain,  $v_j$  is closer to  $v'$  than  $v_i$ . If  $v_j$  is visible to  $v$ , then  $p$  is forwarded greedily towards  $v_j$ .
- (c) Otherwise,  $p$  is forwarded greedily towards  $v'$ . If a hole  $H$  is touched before reaching  $v'$ ,  $\partial H$  is traversed to reach the node that is closest to the intersection of  $\partial H$  and  $\overline{vv'}$  where  $v'$  is visible, then greedy forwarding continues.

In step (b),  $v$  needs to check whether  $v_j$  is visible. Figure 7(b) explains the reason. If  $\overline{vv_2}$  intersects another hole  $H'$ , then using  $v_2$  does not necessarily give a shorter path. The following proposition states a way for node  $v$  to check whether a convex vertex  $v_j$  is visible.

**Proposition 5.1.** Suppose node  $v$  knows the cores of all routing holes and all the extreme nodes. Then for a given convex vertex  $v_j$  of hole  $H$  and another routing hole  $H'$ ,  $v$  can locally check whether  $\overline{vv_j}$  intersects  $H'$  or not.

*Proof.* First if  $\overline{vv_j}$  intersects  $H'_c$ ,  $v$  can learn that since  $v$  knows  $H'_c$ . Suppose  $\overline{vv_j}$  is disjoint from  $H'_c$  but intersects  $H'$ . By Property 3.2,  $\overline{vv_j}$  must intersect a single convex chain of  $H'$ . Then  $v$  knows the extreme nodes  $a$  and  $b$  on that chain. Furthermore,  $\overline{vv_j}$  must intersect  $\overline{ab}$  by Property 5.1. Therefore,  $\overline{vv_j}$  is disjoint from  $H'$  iff  $\overline{vv_j}$  is disjoint from both  $H'_c$  and  $\overline{ab}$ .  $\square$

## 5.1 Advertising Extreme Nodes

In this section, we describe a protocol that advertises extreme nodes to their EVRs. We first make the following observation.

**Property 5.2.** An EVR is a convex region with at most 4 edges without counting the boundary of the network.

According to this property, an EVR of a pair of vertex  $(v_i, v_j)$  are completely defined by the four edges going through  $v_i$  and  $v_j$ . See Figure 8 for reference. Therefore, a node can determine whether it is in the EVR of  $(v_i, v_j)$  once it knows the two tuples  $\langle v_{i-1}, v_i, v_{i+1} \rangle$  and  $\langle v_{j-1}, v_j, v_{j+1} \rangle$ .

Algorithm 3 outlines the advertisement protocol. Each advertisement contains two tuples defining the EVR where it should be received. Initially, every convex node broadcasts three messages (line 2). For instance, in Figure 8,  $v_3$  will broadcast 3 messages to the subregions I, II, and III, respectively. When a node receives an advertisement, it will

first check whether it is within the specific EVR by examining the two tuples in the message. If it is not, the message is simply dropped (line 5). Otherwise, it stores the two tuples locally and then forwards the advertisement (line 9). A node at the common vertex of three or more EVRs may receive two advertisements. For instance, node  $a$  in Figure 8 will receive the advertisements from both  $v_2$  and  $v_3$ . It can then figure out from the four tuples received that  $v_1$  and  $v_4$  are the extreme nodes of region  $IV$ , and broadcasts a new advertisement (line 8).

---

**Algorithm 4** Local advertisement protocol at node  $v$

---

- 1: **if**  $v = v_i$  is vertex of a convex chain  $C$  **then**
  - 2: Broadcast 3 advertisements:
    - $m_1: (\langle v_{i-2}, v_{i-1}, v_i \rangle, \langle v_{i-1}, v_i, v_{i+1} \rangle)$
    - $m_2: (\langle v_{i-2}, v_{i-1}, v_i \rangle, \langle v_i, v_{i+1}, v_{i+2} \rangle)$
    - $m_3: (\langle v_{i-1}, v_i, v_{i+1} \rangle, \langle v_i, v_{i+1}, v_{i+2} \rangle)$
 where if a desired vertex is not on  $C$ , a null value will be put in the message
  - 3: [ $v$  received a new advertisement  $m$  from a chain  $C$ ]
  - 4: **if**  $v$  is outside the EVR defined by the two tuples in  $m$  **then**
  - 5: drop  $m$
  - 6: **else**
  - 7: **if** four tuples received from  $C$  **then**
  - 8: Broadcast a new advertisement {suppose the extreme nodes of the target EVR are  $v_i$  and  $v_j$ }:
    - $m': (\langle v_{i-1}, v_i, v_{i+1} \rangle, \langle v_{j-1}, v_j, v_{j+1} \rangle)$
  - 9: Broadcast  $m$
- 

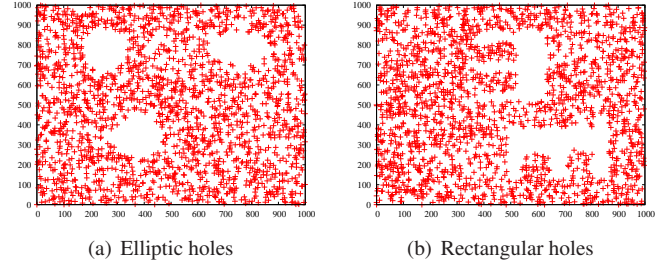
The algorithm may fail if there is no node at the common vertex of multiple EVRs. For instance, in Figure 8, if there is no node at position  $a$ , then region  $IV$  and all the other regions depending on the advertisement from  $a$  will not be covered. To address this problem, the protocol allows each message to be sent to a small number of nodes outside the desired region by a small margin  $\delta$ . The desired value of  $\delta$  depends on the network density. In addition, if a message touches the boundary of a routing hole, it also needs to be forwarded along that boundary.

## 6 Evaluation

### 6.1 Simulation Setup

We evaluated the HBR protocol using *ns2*. We randomly deployed 2000 nodes in  $1000m \times 1000m$  networks. The transmission range is 40m and the average node degree is around 15. Besides the small holes formed randomly in a network, we artificially introduce various number of big holes with different shapes and sizes.

HBR is compared with (1) GPSR [7], which uses greedy forwarding whenever possible and uses perimeter routing when holes are encountered; (2) GLDR [12], a virtual coordinates based routing protocol. GLDR proposes a landmark



**Figure 9.** Two typical simulation scenarios

selection algorithm and a greedy routing protocol based on landmarks. The landmark selection ensures  $r$ -sampling, which ensures that there is at least one landmark within  $r$ -hops for every node. Every landmark then floods the entire network to build the shortest path tree rooted at that landmark. The address of a node is identified by the number of hops to its addressing landmarks. At each step of the routing procedure, a route planning node (called extreme node) selects one of the addressing landmarks of the destination that maximizes the ratio of the distances to this node and to the destination, and forwards packet following the shortest path tree to that landmark until the two distances are equal. Then a new routing plan is made. This greedy rule ensures success in the continuous domain. The IDs of 8 historical extreme nodes are maintained in the packet header to detect loops in the discrete domain. When local minimum happens or a loop is detected,  $L_1$ -norm and then  $L_\infty$ -norm are tried. If the destination is still not reachable, scoped flooding is performed. In our simulation, 25 landmarks are used on average. The routing success rate without flooding is about 95%. The average hop distance from the node where flooding is issued to the destination is about 3 hops.

In all simulations, we generated two types of holes: elliptical holes and rectangular holes with concave regions, as illustrated in Fig.9. The size and shape of a hole is determined as follows. The semimajor and semiminor axes of an elliptical hole are uniformly distributed on  $[a/2, a]$ , and the length and width of a rectangular hole are uniformly distributed on  $[3b/4, b]$  and  $[9b/16, 3b/4]$ , respectively, where  $a$  and  $b$  are parameters. In addition, two concave regions are introduced within each rectangular hole by subtracting two smaller rectangles from the hole. The size of a concave region is  $2l/7 \times w/2$ , where  $l$  and  $w$  are the length and width of the hole respectively.

For each type of holes, we evaluated two cases. We first fixed the number of holes to be 2 and varied the size (a or b) of holes. Then we fixed the size of all holes to be 300, and varied the number of holes. The positions of all holes are randomly selected. Given the number, size, and shape of holes, 5 network scenarios are generated randomly, and the routes between all the  $2000 \times 2000$  pair of nodes are



computed in each scenario. The results are averaged over all these scenarios.

In all simulations,  $\alpha$  is set to  $\pi/2$ ,  $\lambda$  is set so that the cores of the artificially introduced big holes are advertised to the entire network, and  $\delta$  is set to 10 hops.

## 6.2 Route-stretch

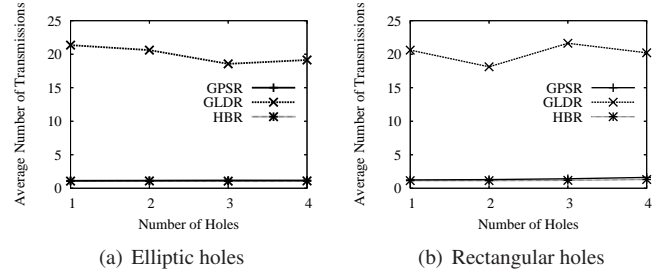
Figure 10 shows the average route-stretches for the three protocols evaluated. We can see that in all the cases, HBR performs much better than GPSR. For elliptic holes, HBR performs better than both GPSR and GLDR. For rectangular holes, GLDR has a relatively stable stretch and performs better than HBR. This is because the performance of GLDR mainly depends on the distribution of landmarks and network density instead of the shapes of holes. However, GLDR achieves the low stretch by paying a relatively high overhead as discussed below. Furthermore, although Figure 10(b) shows that HBR can lead to an increasing stretch with increasing hole size, the stretch is bounded by  $\sqrt{2}$  (since  $\alpha = \pi/2$ ).

## 6.3 Message Overhead

In this section, we compare the message overhead of HBR and GLDR. Figure 11 shows the normalized number of packet transmissions, which is defined as the number of transmissions for routing a single packet using a specific routing protocol divided by that in shortest path routing, averaged over all source-destination pairs. For both GPSR and HBR, the values are the same as their stretches. For GLDR, the value is much higher because scoped flooding is used when destination is not reachable by greedy forwarding. Although this does not happen frequently, the impact is huge, especially for a network with high density.

Besides the high transmission overhead, GLDR also suffers from high overhead for each data packet because a big packet header is used to save the distances to the 10 addressing landmarks of the destination, and IDs of the last 8 extreme nodes visited. In contrast, the packet header size of HBR can be much smaller. In the case where a new routing plan is made at each convex vertex in a path, only the destination and the next convex vertex in the path need to be maintained in the packet header.

Beside the message overhead in the routing stage, both HBR and GLDR require extra control messages for setting up certain roadmaps. For HBR, this includes the messages for hole detection and approximation, and the advertisement of cores and extreme nodes. The former are only needed for nodes on hole boundaries, and only a small constant number of messages per node are required. The latter can be significantly reduced if controlled advertisement is applied. GLDR requires that all the landmarks flood the entire net-



**Figure 11.** Normalized number of packet transmissions in routing procedure

work to build shortest path trees, which in the best case requires  $K$  messages per node for ALL the nodes in network, where  $K$  is the number of landmarks, which is typically greater than 20 for a large network.

## 7 Related Work

Our approach is different from previous works in two key aspects. First, we view routing holes as obstacles and propagate them *proactively* in a *controlled* way. In contrast, most geographic routing protocols treat holes in a reactive way and only try to bypass them when greedy forwarding fails [2, 7, 8, 10], leading to high stretch.

GFG [2] and GPSR [7] guarantee delivery by using perimeter routing, or face routing, to recover from local minima. In GFG and GPSR, nodes forward their packets in greedy mode in the beginning. When a local minimum is encountered, it switches to recovery mode by routing around the hole. GOAFR+ [8] uses similar greedy forwarding and face routing concepts in geographic routing. Unlike GFG and GPSR, GOAFR+ bounds the search on the boundary of holes and therefore achieves asymptotic optimality. GPVFR [10] routes packet greedily to the node whose adjacent edge is closest to the destination on the same face as the sender. GPVFR maintains partial information about the nodes on the same face and uses this information to route packets around the face. However, maintaining complete face information is not a scalable approach.

There do have several recent works that deal with holes explicitly so that packets can bypass holes in advance without getting trapped [1, 6, 15], by filling the nodes in the concave regions of routing holes with higher cost metrics [15], or repositioning these nodes by virtual coordinates [1], or propagating the local hole information to neighboring nodes [6]. However, none of these works ensure a bound on route-stretch and some of them [6, 15] only support “many to one” communications.

Second, all the above works focus on dealing with lo-

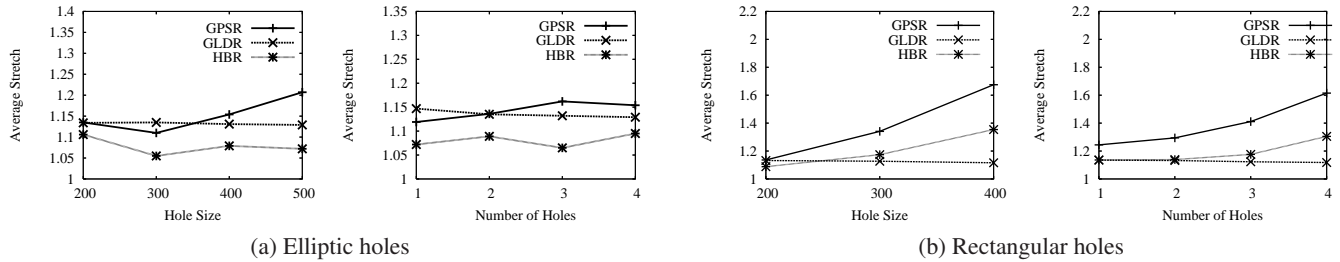


Figure 10. Average route-stretches

cal minima or the non-convex regions of routing holes, and apply greedy forwarding whenever possible. However, we point out that with the presence of holes, even if there is no local minima in the network, greedy forwarding can still be suboptimal. Our approach can be used to improve the greedy forwarding phase of existing protocols.

## 8 Conclusion

Emerging sensor networking scenarios require support for routing between arbitrary pairs of nodes in the network. We propose a distributed shortest path roadmap based routing paradigm that is leveraged to achieve bounded route-stretch with low-overhead for storing and forwarding control information at each node. We demonstrate its application in the context of managing information on routing holes in the most critical regions around the holes to guarantee bounded stretch.

## 9 Acknowledgement

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