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Indexes for Databases with Missing Data

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Abstract Incomplete databases, that is, databases that are missing data, are present in many research and industry domains. It is important to derive techniques to access these databases efficiently. We first show that known indexing techniques for multi-dimensional data search break down in terms of performance when indexed attributes contain missing data. This paper utilizes two popularly employed classes of indexing techniques, bitmaps and quantization, to correctly and efficiently answer queries in the presence of missing data. Query execution and interval evaluation are formalized for the indexing structures based on the traditional query semantics of whether missing data is considered to be a query match or not for each attribute. Query selectivity in the presence of missing data when different subsets of attributes may be required to contain data is explored. The performance of bitmap indexes and quantization based indexes is evaluated and compared over a variety of analysis parameters for real and synthetic data sets. Insights into the conditions and applications for which to use each technique are provided.

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1 Introduction

Real world applications using databases with missing data are common. Databases with missing data occur in a wide range of research and industry domains. Some examples of these are:

1. A census database that allow null values for some attributes
2. A survey database where answers to one question cause other questions to be skipped
3. A medical database that relates human body analyte (a substance that can be measured in the blood or urine) measurements to a number of diseases, or patient risk factors to a specific disease

In each of these cases, there can be valid reasons for a record to contain missing data and the fact that data is missing for a given attribute value may be relevant to database users. As users and researchers in many domains are faced with the issue of querying databases with missing data, the goal of this paper is to provide techniques that access databases efficiently in the presence of missing data while preserving the knowledge that the data is missing.

There are a variety of reasons why databases may be missing data. The data may not be available at the time the record was populated or it was not recorded because of equipment malfunction or adverse conditions. Data may have been unintentionally omitted or the data is not relevant to the record at hand. The allowance for and use of missing data may be intentionally designed into the database. In some application domains, the missingness of data is not important to answering the queries and missingness is ignorable. In some applications, the data can be ‘completed” using regression or other statistical models and treated as if it was never missing. However, if the data are missing as a function of some other variable, a complete treatment of missing data would have to include a model that accounts for missing data. Consider the example of the analyte-disease database where diseases are the records and analyte ranges are the attributes. This database contains values for analyte ranges if they are relevant for a specific disease, or null values if the analyte readings are not important in the diagnosis of that disease. We may query such a database with a patient's analyte readings to get a list of potential diagnoses. We do not want to discount diseases that do not have a value for an analyte included in the query, because the act of taking an analyte's measurement has no bearing on if a patient has a disease that is not relevant to that particular analyte. So in this case, missing data should be interpreted as a query match for that attribute. Alternatively, the intent of a query may not be to return records that *could* match query criteria, but to only return records that definitely match query criteria. In this case any missing data for a record that occurs in an attribute specified by the query search key means that the record does *not* match the query. An example of this is a survey results query where the query asks for a count of respondents that answered question 5 with answer "A" and question 8 with answer "C".

This paper deals with data where missingness is not ignorable, in other words whether a data value is missing or not is important and we want to be able to distinguish between the real values and the absence of such values. In order to achieve this, we could assign a specific value for missing fields that is not in the domain of that particular attribute. For example, if the domain of an attribute is the positive integers, a value of -1 may be used to denote missing data. Then the transformed, complete multi-dimensional database could be indexed using traditional hierarchical multi-dimensional indexing techniques. However, this solution for indexing databases with missing data experiences significant performance issues when applied to hierarchical indexing techniques. To illustrate this point, we performed a set of experiments on two-dimensional data sets that are identical except that they vary with respect to their percentage of missing data. We built an R-tree index on the different datasets and executed 2-dimensional queries with a global selectivity of 25%. Figure 1 shows the effect on query execution time as missing data probability varies.

The graph shows time performance of a query using an R-tree built on the different data sets, normalized to the time to perform the query on a complete data set. This graph shows that even for a data set and index that is only two dimensions, we get far worse performance when the database contains missing data. Even when there is only 10% missing data for each attribute, the time performance is 23 times worse than if the data set were complete.

Multi-dimensional indexing techniques work best when records are mapped to non-overlapping hypercubes. When

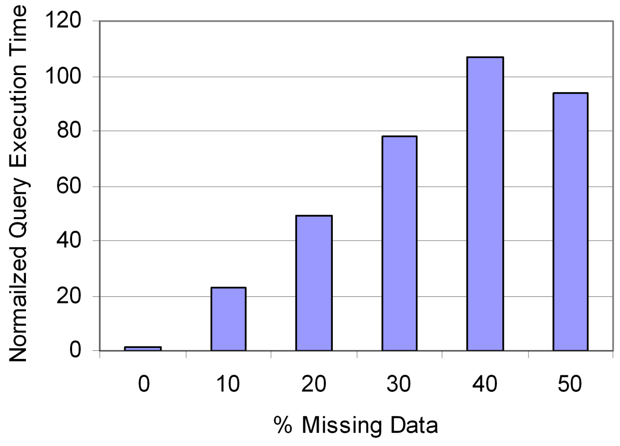


Fig. 1 Normalized Query Execution Time versus Percent Missing Data, Query Selectivity = 25%, 2-D Data Set

missing data are mapped to a single value, the overlaps associated with the index structure increase. One technique to deal with this issue is to somehow randomize the values assigned to missing data so pruning potential results when traversing the index structure is not compromised. However, it becomes necessary to transform the initial query involving k attributes into 2^k subqueries. This is because there are 2^k possible combinations of missing and non-missing values among the k attributes in the search key. Therefore there are 2^k subspaces where query matching data can reside, and all of them must be searched. This fact causes query execution performance to become exponentially worse with respect to query dimensionality. Lastly, as described in [14] all hierarchical multi-dimensional index structures break down after a certain number of dimensions indexed.

Space partitioning multi-dimensional indexing techniques would also suffer from the same weaknesses in the presence of missing data. Records with missing data values would get mapped to lesser-dimensioned spaces, and the full benefit of data space partitioning would not be realized. Again, partitioning the data space beyond a certain number of dimensions has limitations as discussed in [14].

Data repositories need techniques for indexing multi-dimensional data that work well in the presence of missing data. Further benefit is derived if the techniques also work for databases with higher dimensionality than can be achieved effectively using hierarchical or data partitioning indexes. The objective of this paper is to facilitate efficient access to and define query execution for databases with missing data in a way that even works well when the database dimensionality is high. The techniques introduced are evaluated in terms of performance against a number of parameters including database dimensionality, missing data frequency, query selectivity, and query semantics (whether missing data indicates a query match or not).

Contributions of this paper include the following:

1. Demonstrates that missing data not only causes semantic problems but also degradation in the performance of queries.
2. Introduces techniques to efficiently index databases with missing data using variations of bitmaps and VA-Files.
3. Formalizes query processing operations for the proposed techniques in the presence of missing data.
4. Provides insights into the environments and applications appropriate for each proposed technique. Although bitmaps and quantization (VA-Files) have been extensively studied, and their applications are similar, we know of no work that compares and contrasts them.
5. Covers a variety of query level semantics when a query answer set can contain missing data. At a query attribute level this entails treating missing data as either a query match for that attribute or a non-match for the attribute. At a query level, this includes the combinatorial possibilities of the attribute level semantics.
6. Provides methods for query selectivity estimation for the possible query level semantics when the database includes missing data.

The rest of this paper is organized as follows: Section 2 discusses related work, Section 3 defines the problem addressed in this paper, Section 4 describes the proposed solutions and Section 5 presents query level semantics and techniques for query selectivity estimation. Section 6 describes and provides experimental results. Finally, we conclude in Section 7 and provide directions for future work.

2 Related Work

Missing Data. Although in practice databases commonly contain missing data, relatively little work has been performed for this topic. Formal definitions for imperfect databases, of which databases with missing data is a subset, and database operations are provided in [20]. Two techniques for indexing databases with missing data are introduced and evaluated in [11]. This is the only paper we are aware of that focuses on indexing missing data. These are the bitstring augmented method and the multiple one-dimensional one-attribute indexes technique, called MOSAIC.

For the bitstring-augmented index, the average of the non-missing values is used as a mapping function for the missing values. The goal is to avoid skewing the data by assigning missing values to several distinct values. However, by applying this method it becomes necessary to transform the initial query involving k attributes into 2^k subqueries, making the technique infeasible for large k. MOSAIC is a set of B+-Trees where missing data is mapped to a distinguished value. Similarly to the previous method, it becomes necessary to transform the initial query involving k attributes into $2k$ subqueries, two subqueries for each attribute.

What makes MOSAIC perform better than the Bitstring-Augmented index for point queries is that it uses independent indexes for each dimension. However, by using several B+-Trees the query has to be decomposed and intersection and union operations need to be performed to obtain the final result. Queries that could gain a greater performance benefit by utilizing multiple-dimension indexes would not achieve it using this technique. Therefore, this method may not be useful for multiple-dimension range queries, or other queries where the number of matches associated with a single dimension is high.

We introduce and evaluate techniques that do not suffer the same weaknesses as the techniques in [11]. In our approach the query need not be transformed into exponential number of queries and no extra expensive computation, such as set operations, needs to be performed in order to obtain the final result set. Moreover, even though VA-File is not a hierarchical index it benefits from pruning multiple dimensions in one pass through the structure. In addition, our solution using bitmaps and VA-Files is also scalable with respect to the data dimensionality.

Bitmaps. The topic of bitmap indexes was introduced in [9]. Several bitmap encoding schemes have been developed, such as equality [9], range [5], interval [5], and workload and attribute distribution oriented [8]. Several commercial database management systems use bitmaps [10,3,6]. Numerous performance evaluations and improvements have been performed over bitmaps [4,16,12,7,17, 18,5,19]. While the fast bitwise operations afforded by bitmaps are perhaps their biggest advantage, a limitation of bitmaps is the index size. Several compression techniques have been proposed [2,15,1,12] to reduce the bitmap index size. Some of the most popular compression techniques such as Byte-Aligned Bitmap Code (BBC) [2] and Word Aligned Hybrid (WAH) code [15], use a hybrid between the run-length encoding and the literal scheme to compress the bitmap.

VA-Files. The motivation for VA-files is introduced in [14]. This paper showed theoretical limitations for the classes of data and space partitioning indexing techniques with respect to dimensionality. Since reading all database pages becomes unavoidable when the number of indexed dimensions is high, the authors suggest reading a much smaller approximate version, or vector approximation (VA), of each record in the database. An initial read approximately answers queries, and actual database pages are read to determine the exact query answer. VA-files are more thoroughly described in [13].

3 Problem Definition

Let D be a database with a schema of the form $(A_1, A_2, \ldots, A_d)$. D is said to be incomplete if tuples in it are allowed to have missing attribute values. Without loss of generality, assume the domain of the attribute values is the integers from 1 to C_i, where C_i is the cardinality of attribute A_i, i.e. the number of distinct non-null values among all records for attribute A_i.. We assume that data retrieval is based on a k-dimensional search key, where k is less than or equal to d.

In range queries, bounds are specified for each attribute in the search key. Each interval in the query is represented as $v_1 \leq A_i \leq v_2$, where v_1 and v_2 are between 1 and C_i. The query is said to be a point query if all lower bounds are equal to the corresponding upper bound for each attribute in the search key.

Given a range query Q with a k-dimensional search key, we have two ways to compute the results for each of the k dimensions of Q. A tuple t in the database is considered to be a query match if every attribute a of t matches the query criteria. When missing data is considered to be a query match for an attribute a, the attribute matches the query if the attribute value for that tuple falls within the query range or is missing. When missing data is not considered to be a query match for an attribute a, the attribute matches the query only if the attribute's value falls in the range specified by the query search key for that attribute.

The performance of a query can be characterized by the time it takes to perform the query and the accuracy of the result. For this work we only consider techniques that provide accurate query results. The time it takes to perform a query when an index is used is made up of the time to read the index (if the index does not already reside in memory), the time to execute the query over the index, and the time to read the database pages indicated by the index. The goal of this work is to propose indexing techniques that exhibit better performance than existing techniques and sequential scan when the database attributes that are specified in a search key have missing data.

When measuring query performance we consider two metrics: index size and query execution time. Index size is simply measured as the size of the requisite index files on disk. It is indicative of the time required to initially load the index structures. Although this metric is not as critical for static read-only databases with ample disk-space available, it becomes important as database updates become more frequent or available disk space becomes limited. Query execution time is measured in milliseconds for a query set. Given that the indexes are in memory, this measurement indicates the time required to process a set of queries and arrive at a set of pointers to records in the database that match the query criteria.

4 Proposed Solutions

Our proposed solutions are to apply the techniques of bitmap indexes and vector-approximation (VA) files modified appropriately to account for missing data and to execute the query according to the query's semantics. The reason is that we want to independently index each dimension and execute queries efficiently without needing to perform expensive operations to obtain the final result. Bit operations for bitmaps provide fast computation and VA-Files provide pruning in multiple dimensions at the same time using cheap comparisons.

4.1 Bitmap Indexes

We base one solution for the efficient access of incomplete databases on bitmap indexes. In the bitmap index context, records are represented by a bit string. Each attribute A_i would be represented by at most C_i bits of the string where C_i is the cardinality of A_i. A bitmap is a column wise representation of each position of the bit string. Each bitmap would have n bits where n is the number of records in the dataset. Given a dataset D = $(A_1, A_2, \ldots, A_d)$ for each A_i attribute we build a certain number of bitmaps depending on C_i. To handle missing data using bitmaps, we map missing values to a distinct value, i.e. 0. By doing this we are increasing the number of bitmaps for each attribute with missing data by 1. While mapping missing data to a distinct value fails for multi-dimensional indexes, it is acceptable for bitmaps because the attributes are indexed independently and we are not creating an exponential number of subspaces that must be searched to answer a query.

Let's denote the bitvectors or bitmap vectors for attribute A_i by $B_{i,j}$ where $0 \leq j \leq C_i$ if A_i has missing values and $1 \leq j \leq C_i$ otherwise. $B_{i,0}$ represents the bitvector for missing values. Let's denote by $B_{i,j}[x]$ where $1 \leq x \leq n$ the bit value for record x in the bitmap for attribute A_i and value j. Using bitmap indices, queries are executed by performing bit operations over the relevant bitmaps. OR, XOR, AND and NOT are commonly used.

An important aspect of a bitmap index is the type of encoding of the records. We explore two alternatives: equality and range encoding.

4.2 Bitmap Equality Encoding (BEE)

Using equality encoded bitmaps, bit $B_{i,j}[x]$ is 1 if record x has value j for attribute A_i and 0 otherwise. Using this encoding, if $B_{i,j}[x] = 1$ then $B_{i,k}[x] = 0$ for all $k \neq j$. If attribute A_i has missing values, we add the bitmap $B_{i,0}$ that behaves in the same manner explained above. Figures 2 and 3 show a sample equality encoded bitmap representation as modified to handle missing data.

Bitmap Vector	Value
$B_{1,0}$	0001000010
$B_{1,1}$	0000001000
$B_{1,2}$	0100000001
$B_{1,3}$	0010000100
$B_{1,4}$	0000100000
$B_{1,5}$	1000010000

Fig. 3 Bitmap indexes

Adding an extra bitmap for each attribute with missing data is not a major burden with few records or few dimensions, but when we consider 1,000,000 records with 100 dimensions we are effectively adding 100,000,000 bits to our index which correspond to approximately 12 MB in size. An intuitive solution that could be used to encode missing data without adding an extra bitmap would be to use different encodings depending on whether missing data means an attribute match or not. In this alternative, when missing is a match we make $B_{i,j}[x] = 1$ for all j if record x has missing data in attribute A_i; and when missing data means a non-match, we make $B_{i,j}[x] = 0$ for all j if record x has missing data in attribute A_i.

However, there are problems associated with this approach. We will need to perform more bitmap operations when we use the NOT operator. The reason is that when we negate a bitmap when missing data is considered to be a query match, the resulting bitmap would have 0's for the missing records. In order to recover the records with missing data we will need to AND together two bit columns. We then need to OR that result with the original negated bitmap to arrive at a correct final result. When missing data does not imply a query match, we would need to OR together two bit columns to ensure we are eliminating the records with missing values and then AND this result with the negated bitmap to get correct results. Using this approach, it would also be impossible to distinguish between missing values and a real value when the cardinality of the attribute is 1. In addition, by making all bits 1 for the attribute when missing data means a match we interrupt the runs of 0s and compression decreases dramatically for the attribute bitmaps.

Empirically, we realized that after compression using WAH, the addition of an extra bitmap to handle missing data did not introduce much overhead. For the same example of 1,000,000 records with 100 dimensions, and assuming 10,000 records with missing data, each bitmap for missing values would have a compression ratio of approximately 0.47 and overall the compression ratio for the dataset would also improve.

Query Execution With equality encoded bitmaps a point query is executed by ANDing together the bit vectors corresponding to the values specified in the search key. Bitmap Equality Encoded are optimal for point queries [5]. However, when missing data means a query match we need to use two bitmaps instead of one to answer the query, i.e. the bitmap corresponding to the value queried and the one for missing values.

Range queries are executed by first ORing together all bit vectors specified by each range in the search key and then ANDing the answers together. If the query range for an attribute queried includes more than half of the cardinality then we execute the query by taking the complement of the ORed bitmaps that are not included in the range query.

We execute the query differently depending on whether missing data is a query match or not. Figure 4(a) shows how a query interval for one attribute is evaluated when missing data implies a query match. Figure 4(b) shows the same evaluation when missing data is not a match. The query execution time is a function of the number of bitvectors used to answer the query. The number of bitvectors used in the worst case to evaluate a single interval in the query is equal to $\min(AS_i, 1 - AS_i) * C_i + 1$ where AS_i is the attribute selectivity of attribute A_i for this query.

4.3 Bitmap Range Encoding (BRE)

For range encoded bitmaps, bit $B_{i,j}[x]$ is 1 if record x has a value that is less than or equal to j for attribute A_i and 0 otherwise. Using this encoding if $B_{i,j}[x] = 1$ then $B_{i,k}[x] = 1$ for all $k > j$. In this case the last bitmap B_{i,C_i} for each attribute A_i is all 1s. Thus, we drop this bitmap and only keep $C_i - 1$ bitmaps to represent each attribute. If attribute A_i has missing values we add the bitmap $B_{i,0}$ which has $B_{i,0}[x] = 1$ if record x has a missing value for attribute A_i. Also in this case $B_{i,j}[x] = 1$ for all j. We are treating missing data as the next smallest possible value outside the lower bound of the domain, in our case, the value 0. In total the set of bitmaps required to represent attribute A_i with missing values is C_i. Figures 6 and 7 show a sample range encoded bitmap representation as modified to handle missing data.

Another kind of encoding was considered instead of making missing data the smallest value we evaluate the extra bitmap as a flag indicating whether the data is missing. In this alternative, if record x has a missing value for attribute A_i, $B_{i,0}[x] = 1$ and $B_{i,j}[x] = 0$ for all $j > 0$. However, by making $B_{i,C_i}[x] = 0$ when x has a missing value for attribute A_i, we can no longer drop it. This will effectively increase the number of bitmaps for attribute A_i to C_i+1, and will not provide any advantage to the query evaluation logic.

Query Execution For range encoded bitmaps, the bitvectors used and the operations performed to execute a query depend on the range being queried. We identify three scenarios, depending on whether the range includes the minimum value, or includes the maximum value, or

Record	Value	$B_{1,0}$	$B_{1,1}$	$B_{1,2}$	$B_{1,3}$	$B_{1,4}$	$B_{1,5}$
1	5	0	0	0	0	0	1
2	2	0	0	1	0	0	0
3	3	0	0	0	1	0	0
4	missing	1	0	0	0	0	0
5	4	0	0	0	0	1	0
6	5	0	0	0	0	0	1
7	1	0	1	0	0	0	0
8	3	0	0	0	1	0	0
9	missing	1	0	0	0	0	0
10	2	0	0	1	0	0	0

Fig. 2 Equality encoded with missing data

$$v_1 \le A_i \le v_2 =$$

$$\begin{cases} (\bigcup_{j=v_1}^{v_2} B_{i,j}) \vee B_{i,0} & \text{if} \quad \mathrm{v}_2 - \mathrm{v}_1 \le \lfloor \mathrm{C}_\mathrm{i}/2 \rfloor \\ \overline{\bigcup_{j=1}^{v_1-1} B_{i,j} \vee \bigcup_{j=v_2+1}^{C_i} B_{i,j}} & \textit{otherwise} \end{cases}$$

(a) Missing Data is a Match

$$\begin{cases} (\bigcup_{j=v_1}^{v_2} B_{i,j}) & \text{if} \quad \mathrm{v}_2 - \mathrm{v}_1 \le \lfloor \mathrm{C}_\mathrm{i}/2 \rfloor \\ \overline{\bigcup_{j=1}^{v_1-1} B_{i,j} \vee \bigcup_{j=v_2+1}^{C_i} B_{i,j}} \oplus B_{i,0} & \textit{otherwise} \end{cases}$$

(b) Missing Data is not a Match

Fig. 4 Interval Evaluation for Bitmap Equality Encoding

$$v_1 \le A_i \le v_2 =$$

$$\begin{cases} B_{i,1} & \textit{if} \quad v_2 = v_1 = 1 \\ B_{i,v_1} \oplus B_{i,v_1-1} \vee B_{i,0} & \textit{if} \quad 1 < v_1 = v_2 < C_i \\ \overline{B_{i,C_i-1}} \vee B_{i,0} & \textit{if} \quad 1 < v_1 = v_2 = C_i \\ \overline{B_{i,v_1-1}} \vee B_{i,0} & \textit{if} \quad 1 < v_1 < C_i, \quad v_2 = C_i \\ B_{i,v_2} & \textit{if} \quad v_1 = 1, \quad 1 < v_2 < C_i \\ B_{i,v_2} \oplus B_{i,v_1-1} \vee B_{i,0} & \textit{otherwise} \end{cases}$$

(a) Missing Data is a Match

$$\begin{cases} B_{i,1} \oplus B_{i,0} & \textit{if} \quad v_2 = v_1 = 1 \\ B_{i,v_1} \oplus B_{i,v_1-1} & \textit{if} \quad 1 < v_1 = v_2 < C_i \\ \overline{B_{i,C_i-1}} & \textit{if} \quad 1 < v_1 = v_2 = C_i \\ \overline{B_{i,v_1-1}} & \textit{if} \quad 1 < v_1 < C_i, \quad v_2 = C_i \\ B_{i,v_2} \oplus B_{i,0} & \textit{if} \quad v_1 = 1, \quad 1 < v_2 < C_i \\ B_{i,v_2} \oplus B_{i,v_1-1} & \textit{otherwise} \end{cases}$$

(b) Missing Data is not a Match

Fig. 5 Interval Evaluation for Bitmap Range Encoding

Bitmap Vector	Value
$B_{1,0}$	0001000010
$B_{1,1}$	0001001010
$B_{1,2}$	0101001011
$B_{1,3}$	0111001111
$B_{1,4}$	0111101111

Fig. 7 Range encoded bitmap indexes

is within the domain and includes neither the minimum or maximum.

Figures 5(a) and 5(b) show how the interval is evaluated for a single query attribute when missing data implies a match or does not imply a match respectively.

The first three conditions in Figures 5(a) and 5(b) refer to point queries. The other three refer to range queries.

In the presence of missing data, range encoded bitmaps are more efficient for range queries than equality encoded bitmaps in all but extreme cases.

In the case where missing data is a query match, we will need to access between 1 and 3 bitvectors per query dimension. In databases without missing data, we would need to access between 1 and 2 bitvectors per query dimension. We introduce some overhead to deal with the missing data case.

In the case where missing data is not a match, we need to access between 1 and 2 bitvectors per query dimension. This is also true for databases without missing data, but there are two conditions, specifically the conditions where the query range includes the minimum domain value, that require 1 extra bitvector access. This is due to the fact that missing values are encoded as 1's in all bitmaps and a XOR operation is required to eliminate missing data from the result set.

Record	Value	$B_{1,0}$	$B_{1,1}$	$B_{1,2}$	$B_{1,3}$	$B_{1,4}$	$B_{1,5}$
1	5	0	0	0	0	0	1
2	2	0	0	1	1	1	1
3	3	0	0	0	1	1	1
4	missing	1	1	1	1	1	1
5	4	0	0	0	0	1	1
6	5	0	0	0	0	0	1
7	1	0	1	1	1	1	1
8	3	0	0	0	1	1	1
9	missing	1	1	1	1	1	1
10	2	0	0	1	1	1	1

Fig. 6 Sample data using Range encoding

4.4 Bitmap Compression

One of the biggest disadvantages of bitmap indices is the amount of space they require. Several compression techniques have been developed in order to reduce bitmap size and at the same time maintain the advantage of fast operations [2,15,1,12].

The two most popular compression techniques are the Byte-aligned Bitmap Code (BBC) [2] and the Word-Aligned Hybrid (WAH) code [15]. BBC stores the compressed data in Bytes while WAH stores it in words. WAH is simpler because it only has two types of words: literal words and fill words. The most significant bit indicates the type of word we are dealing with. Let w denote the number of bits in a word, the lower (w-1) bits of a literal word contain the bit values from the bitmap. If the word is a fill, then the second most significant bit is the fill bit, and the remaining (w-2) bits store the fill length. WAH imposes the word-alignment requirement on the fills. This requirement is key to ensure that logical operations only access words.

We chose WAH over BBC because the bit operations over the compressed WAH bitmap file are faster than BBC (2-20 times) [15]. However, we do sacrifice space since BBC gives better compression ratio. When we perform logical operations over intermediate compressed bitmaps, we get compressed bitmaps as a result and do not need to uncompress the representations.

4.5 VA-Files

For traditional VA-files, data values are approximated by one of 2^b strings of length b bits. A lookup table provides value ranges for each of the 2^bpossible representations. For each attribute A_i in the database we use b_i bits to represent 2^{b_i} bins that enclose the entire attribute domain. In general $b_i \ll \lg C_i$ when the cardinality is high. We made $b_i = \lceil \lg(C_i + 1) \rceil$. For our purposes, we use $2^b - 1$ possible representations for data values and we use a string of b 0's to represent missing data values. A VA-file lookup table relates attribute values to the appropriate bin number. For VA-files we make a modification to the query based on the query semantics. For a range query where missing data is not a query match, we look for matches over the range of bins returned by the lookup table. In the case where missing data means a query match, we also include those records in the all 0's bin as a query match.

Record Number	Data Value	VA-File Representation
1	6	11
2	1	01
3	3	10
4	missing	00

Fig. 8 Database and VA-File representations.

VA-File Representation	Range
00	missing
01	1-2
10	3-4
11	5-6

Fig. 9 VA-file representations and data ranges.

Figures 8 and 9 show a simple example of a VA-file using our missing data modification. If we perform a query "return all records where value is 4 or 5", our VA-file technique will return the records in bins 00, 10, 11 as approximate answers in the case where missing data is a match. A filtering step would verify that record 1 does not answer the query. In the case where missing data is not a match, only the records in bins 10 and 11 would be returned in the first step.

Query translation is simple. When missing data implies a match, a range query in the form $v_1 \leq A_i \leq v_2$ is converted to $(VA(v_1) \leq VA(A_i) \leq VA(v_2)) \vee (VA(A_i) = 0^b)$, where $VA(x)$ is a function that converts values to their representative VA-file bit representation and b is the number of bits used to define an attribute.

These techniques are easy to apply and require little or no modification of the queries or query processing. As shown using empirical experiments, they are also scalable in terms of the number of data dimensions.

5 Query Level Semantics and Query Selectivity Estimation

At the attribute level, we have described two basic query semantics with respect to missing data. Missing data means the attribute matches the query for that attribute or it does not. At the query level with k query attributes, the query semantics can entail all 2^k combinations of these 2 attribute-level semantics. In this section we describe some query level semantics and potential applications. We also explored query selectivity estimation in the context of missing data. Query selectivity estimates are used to provide online aggregation estimates or to develop query plans. A certain level of accuracy is needed for these estimates in order for them to be useful. When databases contain missing data and the missing data can be a match, the traditional methods for estimating query selectivity are no longer valid due to the effects of the missing data. For each query level semantic, we provide methods to estimate the query selectivity using both simplified equations when certain assumptions hold, and also equations for the general case.

5.1 Query Level Semantics - Maximum Allowed Missing

In the first described query level semantic, the user can define a maximum number or percentage of attributes that can be missing and still be a query match, assuming all the other attributes which are not missing match the query criteria.

Formally this query semantic can be described by the algorithm presented in Figure 10

```
Input: Record Set $S$,
       Query $Q$,
       Allowable attributes with missing data $AM$
Output: Set of Matching Records $M$
 $M = \emptyset$

for each Record $R$ in $S$
   boolean match = true
   int count = 0
   for each Attribute $A$ in $Q$
      if $R$s attribute value is missing
         count++
      else if $R$s attribute value for $A$ not in range of $A$
         match = false
         break
   if $count \leq AM$ AND match
      $M = M \cup R$
```

Fig. 10 Query Processing Algorithm for Maximum Allowable Attributes Missing Query Level Semantic

This type of query semantic can be applied to a situation where the user is trying to accumulate at least a certain number of candidates and progressively relaxes the constraint on the allowable missing data until the number of candidate answers is reached. For example, consider a clinical trial that is trying to find a certain number of participants. Candidates are typically found by reviewing answers on forms filled out by patients or doctors. The dataset of form data is susceptible to the presence of missing data, either by design of the form or the patient not completing the fields. A first set of clinical trial candidates can be found by requiring no query fields be missing. This constraint can be iteratively relaxed until the candidate set is large enough. This process will tend to target the most likely candidates first.

Because this alternate query semantic is structured in much the same way as VA-files are processed, the enhanced VA-file is a natural choice to use to answer this type of query. VA-file operations are more flexible to allow greater expressiveness. Given a number of maximum allowed attributes that can be missing, we simply compute query matches the same way that we did for the query semantics where missing attributes are a query match for the attribute. We accumulate the number of attributes for which the reason that it matches is because the data is missing (e.g. bucket number is 0). If this count is greater than the number of allowed attributes that can be missing, the data object does not match the query.

While bitmaps provide fast bit operations when the query semantics are relatively simple, they do not have the expressive power to easily compute results for this more complicated query semantic. In order to compute the query matches for this particular semantic using bitmaps without modification, the level of allowable missing attributes would need to be converted to a potentially complicated series of bitmap operations that reflect the level of missingness. For a query of length q where n attributes are allowed to be missing, we need a combinatorial formula using just AND and OR operations to verify that not more than n attributes are missing. In the worst case, the length of this formula can be exponentially dependent on q, even after all possible simplifications.

For this query semantic, we can estimate the query selectivity for the case where the probability of missing data and attribute selectivity are the same across the attributes, and the data and data missingness are not correlated across attributes using the following formula:

$$QS_m = \sum_{i=0}^{m} (n)!/((i)!(n-i)!) * (AS * (1 - PM))^{n-i} * PM^i$$

where QS_m is the query selectivity for a query that matches the query semantic for the case where missing data is a match and the number of attributes for which the query is a match because the data is missing is no greater than m, AS is the attribute selectivity (the selectivity for the attribute where data is not missing), n is the number of attributes in the query, and PM is the ratio of missing data.

For a given combination of k missing and n non-missing attributes where the attribute selectivity and

Allowed Missing	Actual Size	Predicted Size
0	104	105
1	337	338
2	529	533
3	601	605
4	612	615

Fig. 11 Query Set Size as Number of Allowed Missing Attributes Vary, PM = 0.1, AS = 0.2, k = 4

ratio of missing data is not the same over different attributes we can estimate the query selectivity as:

$$QS = \prod_{i=1}^{n} AS_i * (1 - PM_i) \prod_{j=1}^{k} PM_j$$

We can then estimate the query selectivity for a limit of m attribute matches as a result of missing data by aggregating all possible combinations of k and n where $k \leq m$. The accuracy of the estimation is affected by correlations of data and missingness between attributes.

We performed tests in order to gain intuition about the effect on performance and number of query matches as we vary the user defined allowable level of missing attributes for both synthetic and real data.

With respect to time performance, the enhanced VA-file method of handling this alternate query semantic took the same amount of time as the case where we consider missing attributes to be a query attribute match. Figure 11 and Figure 12 show samples of the variation in the number of query matches as we vary the user defined allowable level of missing attributes. Figure 11 is for the synthetic data with cardinality 10, a probability of missing data of 10% per attribute, a query length of 4 attributes, and an attribute selectivity level of 0.2. Figure 12 is for the synthetic data set with cardinality 10, probability of missing data of 50%, a query length of 8 attributes, and an attribute selectivity of 0.1. The first column shows the number of attributes for each query that are allowed to match because they contain missing data. The second column is the average number of query results over a set of 100 randomly generated queries. The last column shows the predicted number of results computed using the formula for uniform attribute selectivity and missing data ratio.

Figure 13 shows the effect of relaxing the level of allowed missingness using our real set of Spanish census data. For this experiment the cardinality and percent of data missing for each dimension is highly variable, the query length is 8 attributes, and the attribute selectivity is variable due to data distribution, but the percent of values that are covered by each query attribute range is 40%. The last column in this table is the average predicted query result set size using the attribute specific attribute selectivity and missing data ratio formula.

These results show that we have some degree of control over the size of the query result set. This has important implications with respect to estimating query

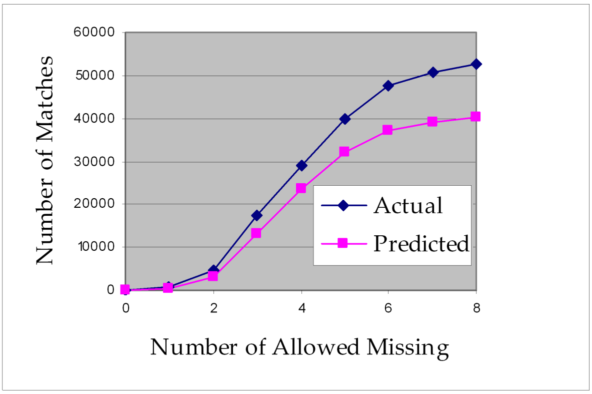


Fig. 13 Query Set Size as Number of Allowed Missing Attributes Varies, AS = variable, k = 8

selectivity in the presence of missing data. For uniform attribute selectivity and missing data ratio, we can accurately predict the average query result set size, and could use this information to select an appropriate maximum allowed missing attribute value, m that will return at least the given number of maximally filled results.

5.2 Query Level Semantics - User Specified Allowed Missing Attributes

A second version of query level semantics in the presence of missing data allows the user to specify which attributes are allowed to be considered to be a query match when their attribute values are missing, and which can not. This is analogous to database schema definition, where attributes are marked as allowing null values or not. In this case the allowance of null, or missing, values is built into the query itself, lending more overall flexibility to the database design. This type of query semantics would be applicable in any situation where there are hard constraints about the presence of data with some portion of attributes, while not on others.

As this query semantic can be easily translated into a logical function, it can be easily addressed by both the proposed VA-file and bitmap indexing solutions. In order to apply this query semantic, the query language itself would need some mechanism in order to denote if an attribute is required to contain data. The queries could be executed using either of the proposed solutions in multiple ways. One way would be to compute the result set allowing missing data, R_{all}, then compute the result set of only those data objects that contain missing data in the attributes that require data, R_{miss}. The query answer is the difference, $(R_{all} - R_{miss})$.

Alternatively, we could process attributes appropriately based on the indication if missing data is considered a query match for the attribute. For the VA-file solution, as we process records, if the query attribute is not allowed to contain missing data for a query match then

Allowed Missing	Actual Size	Predicted Size
0	0	0
1	0	0
2	0	0
3	0	0
4	3	3
5	26	25
6	134	134
7	447	447
8	839	837

Fig. 12 Query Set Size as Number of Allowed Missing Attributes Vary, PM = 0.5, AS = 0.1, k = 8

the attribute is processed as being a match only if the attribute value is within the range of buckets that match the query range for the attribute. If missing data can indicate a query match for that attribute, the attribute is processed as being a match if the attribute values quantization is in the range of buckets that match the query range for the attribute, or the attribute values quantization is the missing data bucket.

Since each attribute is evaluated separately in bitmaps, we can apply the appropriate logical formula to obtain results based on the query semantics for each attribute. For the case where missing data can be a query match, we can use the formulas presented in Figures 2a and 3a for equality encoded and range encoded bitmaps, respectively. For attributes where missing data is not a match, we can use the formulas from figures 2b and 3b for equality encoded and range encoded bitmaps, respectively. We then AND together the results we generate for individual attributes.

In order to calculate the query selectivity using this query level semantic, we take the product of each of the attribute selectivitites. For those attributes that allow missing data, we use the attribute selectivity that accounts for the missing data. For those that do not, we use the attribute selectivity that does not count missing data. The formula is:

$$QS = QS_m * QS_n$$

where QS_m is the missing data is a match query selectivity, and QS_n is the missing data is not a match query selectivity. In databases that contain missing data, the value of QS_m for a query with j attributes that are allowed to contain missing data can be estimated as:

$$QS_m = \prod_{i=1}^{j} (AS_i * (1 - PM_i) + PM_i)$$

and the value of QS_n for a query k attributes that are not allowed to contain missing data can be estimated as:

$$QS_n = \prod_{i=1}^{k} AS_i * (1 - PM_i)$$

P_A	Actual Size	Predicted Size
0.0	612	615
0.1	530	531
0.2	455	457
0.3	390	391
0.4	331	332
0.5	278	280
0.6	233	234
0.7	195	195
0.8	160	160
0.9	130	130
1.0	104	105

Fig. 14 Query Set Size as Probability Missing Data is Allowed (P_A) varies, PM = 0.1, AS = 0.2, k = 4

Accuracy of the estimates will depend on the correlation of the data and missingness between queried attributes.

In our experiments for this query semantic, we randomly determine the semantic for each attribute in the query. If a random number generated for an attribute in a query is below some threshold value, then we compute the result set for that particular attribute using the missing data is a match semantic. When the attribute selectivity and ratio of missing data is the same across the attributes, we can compute the query selectivity using:

$$QS = \sum_{i=0}^{k} (k)!/((i)!(k-i)!) * (P_A * AS * (1 - PM))^i$$
$$* ((1 - P_A) * ((1 - PM) * AS + PM))^{k-i}$$

where k is the query dimensionality, and P_A is the threshold ratio for allowing missing data per attribute.

Figure 14 and Figure 15 show how query result set size varies as the probability that an attribute will be required to contain data is varied. Figure 14 is for the synthetic data with cardinality 10, a probability of missing data of 10% per attribute, a query length of 4 attributes, and an attribute selectivity level of 0.2. Figure 12 is for the synthetic data set with cardinality 10, probability of missing data of 50%, a query length of 8 attributes, and an attribute selectivity of 0.1.

The query selectivity for a given query where m attributes are allowed to contain missing data in a query

P_A	Actual Size	Predicted Size
0.0	839	837
0.1	389	391
0.2	169	168
0.3	65	66
0.4	22	23
0.5	7	7
0.6	1	2
0.7	0	0
0.8	0	0
0.9	0	0
1.0	0	0

Fig. 15 Query Set Size as Probability Missing Data is Allowed (P_A) varies, PM = 0.5, AS = 0.1, k = 8

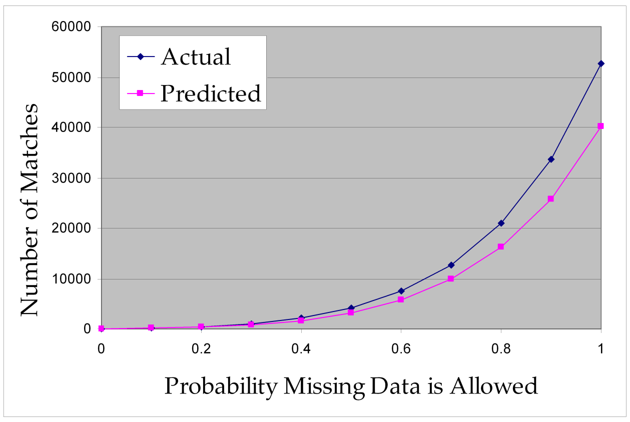


Fig. 16 Query Set Size as Probability Missing Data is Allowed (P_A) varies, PM,AS = variable, k = 8

answer and n attribute are not, where the ratio of missing data and attribute selectivity can vary among attributes can be estimated using:

$$QS = \prod_{i=1}^{m}(AS_i(1 - PM_i)) * \prod_{j=m+1}^{m+n}(AS_j(1 - PM_j) + PM_j)$$

Accuracy of the estimation is dependent on correlation of the data and missingness among attributes. Estimate for each individual combination, weighted by their probability of occurrence, can be aggregated to estimate an overall query selectivity for a query using those attributes and given a ratio that each attribute may or may not allow missing data in a query match.

Figure 16 shows the effect of varying the probability that each attribute in a query will allow data to be missing for a query match using our real set of Spanish census data. Again, for this real data set the cardinality and percent of data missing of each dimension is highly variable, query length is 8 attributes, and the attribute selectivity is variable due to data distribution, but the percent of values that are covered by each query attribute range is 40%. The last column in this table is the average predicted query result set size using the attribute specific attribute selectivity and missing data ratio formula.

The predicted values are based on an assumption that the missingness of data is independent between dimensions. For real data sets, there is likely a correlation between missing values, and therefore actual query selectivity will likely differ from predicted similarity. Actual values turn out to be higher than the predicted values, but the overall shape of the results is similar.

6 Comparative Experiments and Results

6.1 Experimental Framework

We performed experiments to compare the performance of the bitmaps and VA-file approaches using both synthetic and real datasets as we vary analysis parameters. By using the synthetic data set we could control analysis parameters individually and gain insights into the behavior of the indexing techniques. We applied the techniques to a real data set to verify the effectiveness of the techniques on real scenarios.

For the synthetic data, we generated a uniformly distributed random dataset set with 450 attributes and 100,000 records. For the set of attributes we varied the cardinality and percent of missing data. Cardinality varied among 2, 5, 10, 20, 50, and 100 values and percent of missing data among 10, 20, 30, 40, and 50 percent.

The real data is census data with 48 attributes and 463,733 records. The attribute cardinalities widely vary from 2 to 165 (average of 37) and percent of missing data varies from 0% to 98.5% (average of 41%). Table 1 details the distribution for the synthetic and the real dataset.

We implemented query executors for both bitmaps and VA-Files in Java. We ran 100 queries for each type of experiment. Queries were executed in both scenarios when missing data is a query match and when missing data is not a query match. Since the graphs look very similar in both scenarios we present only results for queries executed where missing data is a match.

Given that we used the same precision (100%) for our implementations we compared bitmap indices and VA-Files in terms of:

– **Index Size.** Index Size is an important factor in any indexing technique. We are interested in indices that can fit into memory to ensure fast query execution without the overhead introduced when reading from disk.

– **Query Execution Time.** Query Execution Time is the time required to produce a query result set. We assume the indexes are in memory and do not consider time to read the indexes.

6.2 Index Size

In this section we evaluate how the attribute cardinality and the percentage of missing data affects index size.

6.2.1 Attribute Cardinality

For cardinality less than 10 there is not much room for compression and the index size is equal for both types of bitmap encoding and is not sensitive to the percent of missing data. For equality encoded bitmaps, as the attribute cardinality increases the compression ratio improves considerably, however, at the same time, bitmaps index size increases linearly with cardinality. For VA-Files the index grows very slowly with cardinality given our current quantization strategy. Index sizes are presented for attributes with 10% missing data in Figure 17(a). As can be seen, BRE does not benefit from WAH compression.

With real data, compression rate is highly variable with respect to attribute cardinality. Since real data can be far from uniform, an attribute that has low cardinality but frequently has one value can acheive high compression ratios. With our set of real data, those attributes which have cardinalities of between 1 and 10 and are not missing any data have a compression ratio between 0.002 and 1.03 using equality encoding and between 0.001 and 0.82 using bitmap range encoding. The wide range is attributable to the bit density (ratio of 1's) in the bit columns. As the bit density approaches 1 or 0, the compression ratio improves. Therefore, if one particular value is frequent, then the bit density for that value's column is close to 1 yielding good compression ratio for that column and the bit density for all other bit columns is close to 0, which results in good compression ratio for them.

6.2.2 Percent of Missing Data

For equality encoded bitmaps, as the percent of missing data increases the compression ratio decreases making the index smaller. Range encoding does not get significant compression using WAH code. VA-File is not sensitive to the presence of missing data and its size is independent of it. In any case the index size for VA-Files is much smaller than bitmaps. Index sizes are presented for cardinality 50 in Figure 17(b).

Good compression is also obtained on the real dataset when an attribute has a high occurrence of missing data. The missing data bit column has a bit density close to 1 and all other columns are close to 0. This leads to very good compression ratios for equality encoded bitmaps (between 0.01 and 0.09 for each of the 8 attributes in our real data set which have more than 90% missing data) and decent compresison ratios for range encoded bitmaps (between 0.11 and 0.44). Overall, this real data set had an equality encoded bitmap compression ratio of 0.17 and a range encoded bitmap compression ratio of 0.70.

6.3 Query Execution Time

To measure the effect of the various parameters over the query execution time of the 100 queries we needed to have control over the global query selectivity, i.e. the number of records that match the given query. The following formula relates Global Selectivity (GS), Attribute Selectivity ($AS = (v_2 - v_1 + 1)/C_i$) and Percent of Missing Data (P_{m_i}) of all the attributes involved in the queries:

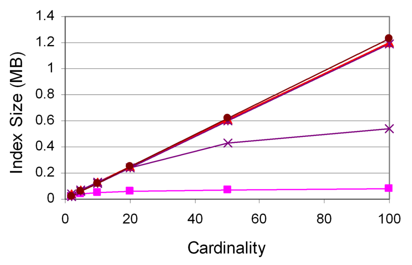
$$GS = \prod_{i=1}^{k}((1 - P_{m_i})AS_i + P_{m_i})$$

, where k is the number of dimensions involved in the query. In order to simplify this formula we assume equal attribute selectivity on all the attributes in the query. By doing this, individual attribute selectivities are easy to compute but we lose some precision on the global query selectivity. To measure query execution time we fixed the global query selectivity to 1 percent. Plugging in different values for the parameters into $GS = [(1-P_m)AS+P_m]^k$ we compute the attribute selectivity for each attribute in the query. Note that the granularity of attribute selectivity is limited by C_i. In general, our estimate was very close to 1 percent but sometimes the actual global query selectivity went up to 3 percent. Note that when we make the global selectivity constant and increase the percent of missing data, the attribute selectivity decreases. We tested the effect of attribute selectivity, percent of missing data, and query dimensionality against query execution time.

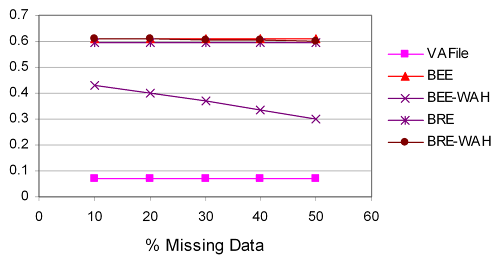
6.3.1 Attribute Cardinality

Figure 18(a) shows the query execution time of 100 queries over attributes with 10 percent missing data and various cardinalities. Also in this case the execution time for BRE and VA-Files remains somewhat constant with BRE being faster than VA-Files. For BEE, the execution time is linear since the number of bitmaps used to answer the queries depends on the cardinality of the attribute and its selectivity.

The proposed techniques also compared favorably to using a number of single dimension indexes, such as done by the MOSAIC technique. We achieved speedups of between 5 and 40 times using our techniques over 8 attribute point queries. The relative difference in times increased as the query ranges per attribute increased. This is because the intermediate answer sets are larger as more results are returned by the single dimension indexes.

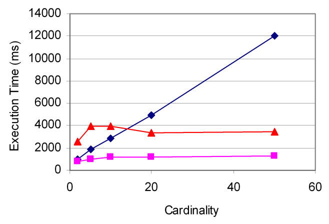


(a)

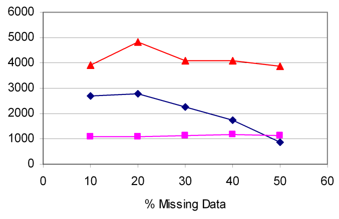


(b)

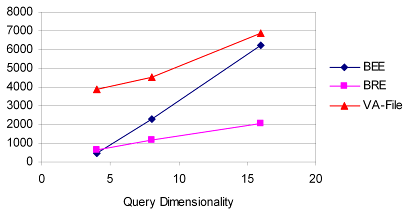
Fig. 17 Index Size Versus (a) Cardinality and (b) Percent of Missing Data



(a)



(b)



(c)

Fig. 18 Query Execution Time Versus (a) Cardinality, (b) Percent of Missing Data, and (c) Query Dimensionality

6.3.2 Percent of Missing Data

Figure 18(b) shows the results of these experiments for attributes with cardinality 10. For equality encoded bitmaps, the execution time decreases when the percent of missing data increases. This is because when we make the global selectivity constant and increase the percent of missing data, the attribute selectivity decreases and the number of bitmaps used in the query execution depends on the attribute selectivity for this kind of encoding. For range encoded bitmaps, the execution time remains somewhat constant. The small variations are due to the possibility of using between 1 and 3 bitmaps per dimension over the query execution. It turns out that as the percent of missing data increases the number of bitmaps used per dimension gets closer to 3. For VA-Files, the execution time is also somewhat constant. The variations are due to the actual global selectivity for cardinality 10 and 8 dimensions in the query. For cardinality 10 and 50 % missing data the global selectivity is 0.84%, for 30 and 40 is 1.28%, but for 20 is 1.7%. In general, BRE executes range queries faster than the other two. The only case in which BEE performs better than BRE is at 50% missing when the attribute selectivity is 10% and the range query becomes a point query.

6.3.3 Query Dimensionality

Figure 18(c) shows the query execution of 100 queries over attributes with cardinality 10 and 30 percent of missing data. For all indices the execution is linear in the number of query dimensions. BRE grows very slowly since we are only using between 1 and 3 bitmaps per query dimension. BEE grows much faster since as we increase the number of dimensions with this percent of missing data the attribute selectivity get closer to 50 %. For smaller percents of missing data and same cardinality the attribute selectivity is greater than 50 %, around 70 % so effectively we only access the 30 % of the bitmaps and therefore the execution time does not increase linearly. For VA-Files the execution time also increases with the query dimensionality.

6.3.4 Results on Real Data

Experiments using this real data set yielded several conclusions. For this data set, the bitmap solutions were significantly faster than the VA-File solution (3 to 10 times faster). This was because the skewness of this particular data set allowed for very good compression of the bitmaps and while the VA-file implementation had to operate over about 500,000 vector approximations of the records, the bitmap implementations performed bit operations over substantially fewer words. The average compression ratio for the equality encoding bitmaps was 0.17 (with 23 attributes compressing to less than 0.1 times their original size). The average compression ratio for the range encoding bitmaps was 0.7 (with 18 attributes compressing to less than 0.5 times their original size and only 3 attributes not compressing at all).

Also of note is that whereas the presence of missing data can introduce a degradation of a couple of orders of magnitude in hierachical multiple-dimension indexes as shown in the motivating example, there is not a large degradation associated with the presence of missing data using these techniques.

In our experiments with real data, the range encoded bitmaps performed faster than the equality encoded bitmaps. In these experiments we used range queries over 20% of the queried attribute possible values and would expect this result since range encoded bitmaps are tailored for range queries.

7 Conclusions

In this paper we demonstrate that missing data not only causes semantic problems, but also that indexing such data while maintaining the knowledge that data is missing causes significant performance problems. We show that performance using current multi-dimensional index structures degrades (e.g. by a factor of 23 for a 2-D R-tree with 10% missing data) and introduce techniques for indexing missing data that are scalable with respect to query dimensionality.

The techniques presented in this paper are easy to apply and allow the effective indexing of missing data. As opposed to traditional hierarchical indexing structures and previously proposed missing data indexing techniques, these techniques exhibit linear performance for query execution time with respect to database and query dimensionality. This is done by essentially indexing attributes independently. Our solutions take advantage of this independence by handling missing data for each attribute, and still maintain the linear performance associated with respect to dimensionality that bitmaps and VA-files have been known for.

These techniques exhibit a tradeoff between execution time and indexing space. The bit operations used to evaluate queries for bitmaps are fast, but the space required to represent an exact bitmap can be much higher than a corresponding exact VA-file.

The range encoded bitmaps typically offer the best time performance but, at least using the techniques we used, can not be compressed as much as equality encoded bitmaps. They typically perform faster because there is a limit on the number of bit operations that must be performed to evaluate a query for each dimension. Equality encoded bitmaps perform a maximum of C/2+1 bit operations per query dimension and can perform faster than range encoded bitmaps for point queries or range queries with small ranges. Equality encoded bitmaps can be compressed much more than range encoded bitmaps.

VA-files offer the least size to represent the same information offered by bitmaps, but the operations performed are not bit operations, they usually do not operate as fast as the range encoded bitmaps.

Query level semantics can cover any possible combination of atrtibute level semantic. These can be applied to enhance the flexibility of the query language and find answer sets that reflect a likelihood of matching the query. Query selectivity when the database contains missing data can be relatively accurately estimated if the missingness of the data is not correlated.

A characteristic of the range encoded bitmaps is the inability to compress them. We would like to explore techniques such as BBC compression and row reordering in order to achieve more compression of these bitmaps.

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Table 1 Synthetic and Census Datasets Distribution

Synthetic Dataset						
	% of Missing Data					Total
Card	10	20	30	40	50	Columns
2	10	10	10	10	10	50
5	10	10	10	10	10	50
10	20	20	20	20	20	100
20	20	20	20	20	20	100
50	20	20	20	20	20	100
100	10	10	10	10	10	50
Total	**90**	**90**	**90**	**90**	**90**	**450**

Census Dataset						
	% of Missing Data					Total
Card	0	≤10	≤50	≤90	>90	Columns
<10	11	0	2	2	0	15
10-50	7	2	3	5	4	21
51-100	2	0	1	2	2	7
>100	0	0	1	2	2	5
Total	**20**	**2**	**7**	**11**	**8**	**48**

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