Reliable Control System Design Despite Byzantine Actuators

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Abstract

Sensor-actuator networks are increasingly being used in distributed control of large scale systems. Often these applications are mission-critical and are required to maintain satisfactory performance in the presence of component failures. On the one hand, sensor-actuator network components are becoming inexpensive but they also tend to be unreliable, especially when deployed in harsh or unpredictable environments. The various component failures can manifest themselves in the form of arbitrary actuator behavior in which case their effect on the underlying systems can be severe. In this paper we focus our attention on applications of sensor networks in control of linear systems and show how to deal with Byzantine faults of actuators. We first describe a fault-tolerant control scheme using locally redundant actuators. We then relax the requirement on actuators to be at the same location and design a fault-tolerant scheme where the actuator redundancies are further reduced as well. We demonstrate our methodologies using a beam vibration control application as a case study.

1 Introduction

Fueled in part by recent advances in MEMS and communication technologies, sensor networks are increasing in popularity. Thus far most of the applications of sensor networks have focussed on observation. Examples include habitat monitoring and area surveillance applications where the sensors gather a variety of information and this information is processed centrally or in a distributed manner. That said, it is widely believed that the number of applications of wireless sensor networks will increase manifold when they also perform actuation and control.

Some actuation based applications do exist currently. Actuators such as sound and radio are being used to solve problems such as localization. Mobility is another form of actuation which is being applied to distributed pursuer-evader applications using sensor networks [18]. Sensor-actuator networks are being prototyped in the control of distributed parameter systems such as flexible structures. A specific example is the vibration control of a fairing during payload launch using embedded MEMS components based sensor-actuator networks [1, 20]. Since MEMS based sensor-actuator devices are potentially cheap, a large number of these devices can be embedded on flexible structures and combinations of these sensors can be used to obtain the required mode vibration information and then the output from these combinations can be used to provide adequate distributed control. Similar applications arise in the control of chemical plants and nuclear reactors.

It is also important to note that although most of the literature on sensor networks focusses on wireless networks, many of these control applications are better suited to wired networks. Wired networks have higher network bandwidth and provide better network reliability compared to wireless networks and this is crucial to guarantee stability and performance in control systems.

Yet the constraint in most distributed control applications is that of mission critical stability, and despite the access to more resources in wired networks this is a challenge. Distributed control systems have applications in space missions and nuclear plants where degradation of systems performance may even compromise human safety. Hence satisfactory performance in the presence of faults is a requirement for these systems. But in our experiences with deploying and using large sensor networks [2], one of the key learnings has been that these networks are unreliable in many ways. Sensor-actuator network based control systems typically comprise of embedded sensors and actuators, microprocessor-based controllers (central or distributed) and an underlying network that provides information processing services to the controllers such as controller group synchronization, communication, (re)parameterization, reconfiguration, etc. Each of the above subsystems are subject to faults: there are hardware faults and these will increase when subject to harsh and unpredictable environments, there are faults in the underlying software and middleware services such as information loss, delay and corruption, and there are configuration faults which given the scale of these networks this will increase even more. Early experiments conducted on a vibration control system of a fairing show that the effect of faults on the stability and performance of control systems can be particularly severe [10]. This leads us to focus on fault-tolerant distributed control systems.

One of the methodologies for the design of fault-tolerant control systems involves real-time fault detection, isolation and control system reconfiguration [4, 5, 8, 11, 14]. An appropriate action is taken after the diagnosis of the faults. Another methodology in fault-tolerant system design is to use redundancy and voting to achieve tolerable performance in the presence of faults. Incorrect data generated by faults in control software and sensor failures can be tolerated by voting based schemes which estimate or filter the correct data by using multiple redundant inputs [12, 16].

But these methods still leave the following challenges. The hardware can be faulty causing the actuators to fail-stop and offer no control or debond from their surface causing them to offer incorrect control. The underlying fault detection service is itself vulnerable to faults in the middleware. It is sometimes not feasible to integrate the fault detection, diagnosis and reconfiguration in dynamical systems particularly when the available reaction time is limited. In the voting based schemes, faults in underlying middleware services can affect each of the redundant component in the same way and then the voting fails. For example a network error such as delay or dropping of data is likely to affect each redundant component. Also, the voter itself is subject to faults [16].

Thus the faults in the hardware and underlying software services can cause the actuator to behave in a nondeterministic and potentially malicious manner. This suggests a Byzantine model for the actuator faults. A Byzantine actuator can produce an arbitrary control input to the plant at all times. The behavior is non-deterministic and it can even be the worst possible value at all times. In this paper we focus on designing systems that maintain asymptotic stability in the presence of Byzantine actuators that apply arbitrary control input to the plant.

Problem statement

> *Assuming that a bounded number of network actuators can exhibit incorrect (and potentially arbitrary) behavior, how can distributed control be designed to be provably stable?*

Specifically, in this paper we describe a distributed, local, output feedback control system and use that to design two control schemes that maintain asymptotic stability in the presence of a given number of actuators that are Byzantine. We demonstrate our methodologies using a beam vibration control application [3, 15] as a case study.

Related Work A control system designed to tolerate failures in system components while maintaining closed loop system stability and performance has been defined as a reliable control system [19]. Such systems are also called systems possessing integrity against component failures. Redundancy is a key ingredient in all such reliable control systems. A basic difference between robust control techniques and reliable control is that the former deals with small parameter variations and system model uncertainties while the latter handles more drastic changes in the control system configuration. There exist several reliable control schemes [6, 7, 17, 19, 21, 22] that provide stability in the presence of a set of failed actuators and sensors that are non responsive. However, in this paper we design control schemes that guarantee stability in the presence of malfunctioning actuators which continuously offer detrimental input and thereby can lead the system to instability.

Outline of the paper In Section 2 we describe the system and fault model and provide a sufficient condition for the stability of the system in the absence of faults. In Section 3, we first design a reliable control scheme using redundant colocated actuators and then design a reliable control scheme for a second order system, where the redundant actuators are not colocated and the redundancy is further decreased. In Section 4, we extend our results for the latter scheme to higher order systems. In Section 5, we demonstrate our methodology using a beam vibration control application [3, 15] as a case study.

2 System and Fault Model

In this section we describe the system and fault model and derive sufficient conditions for the asymptotic stability of the system without faults.

2.1 System Model

Consider a marginally stable linear time-invariant multivariable system S with m sensor-actuator pairs, described by the following equations and control law.

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

where x is an n-dimensional state vector $[x_1, x_2, \cdots, x_n]^T$, u is an m-dimensional actuator vector, B is an $n \times m$ dimensional matrix and the individual sensor-actuator pairs are colocated. We assume that the system is controllable and observable from individual locations. Since S is marginally stable, A has eigenvalues on the imaginary axis. Since the individual pairs of sensors and actuators are colocated, we have the following condition.

$$B = C^T \tag{3}$$

Starting at any state, without any control being applied the system maintains its energy as it is marginally stable. We apply the following local on-off output feedback control law to stabilize the system.

$$u_i = \alpha \times sign(y_i), \quad i = 1....m \tag{4}$$

where α is less than zero. Further u_i equals zero when y_i is zero. Thus a correct actuator can have 3 possible control values 0, $-\alpha$ and α. We choose $|\alpha|$, the magnitude of the actuator force, to be the maximum force that an actuator can apply and assume that this is the same across all actuators. $\blacksquare$

2.2 Asymptotic Stability Without Faults

We now analyze and prove the stability properties of S in the absence of faults.

Theorem 2.1 *If $m \geq n$ and the matrix B is of rank n, the system S is asymptotically stable.*

Proof We use the Lyapunov approach to prove stability. Now, let us define function V as

$$V = x^T M x \tag{5}$$

where M is a symmetric, positive definite $n \times n$ matrix. The Lyapunov derivative can then be written as

$$\dot{V} = x^T (A^T M + MA)x + 2x^T MBu \tag{6}$$

Since A is marginally stable, we can transform A to be skew symmetric and $A^T + A$ equals zero. Thus M can be the identity matrix.

$$\dot{V} = 2x^T Bu \tag{7}$$

Let B_i denote the i^{th} column of matrix B. For the system described in Eq. 1, we have

$$\begin{aligned}
\dot{V} &= 2 \times \alpha(\sum_{i=1}^{m}(x^T).(B_i) \times sign(y_i)) && (8)\\
&= 2 \times \alpha(\sum_{i=1}^{m}(x^T).(C_i^T) \times sign(y_i)) && (9)\\
&= 2 \times \alpha(\sum_{i=1}^{m}(y_i) \times sign(y_i)) && (10)\\
&= 2 \times \alpha(\sum_{i=1}^{m}|(x^T).(B_i)|) && (11)
\end{aligned}$$

Note that we can use the magnitude of the dot product $(x^T).(B_i)$ because we see from Eq. 10 that $(y_i) \times sign(y_i)$ is always positive. Since m is at least equal to n and B is of rank n, the state x can be orthogonal to at most $n - 1$ actuators. Hence the Lyapunov derivative is strictly negative. Thus the system is asymptotically stable. ∎

2.3 Fault Model

We now describe the fault model acting on system S. We start with the definition of a *Byzantine* actuator.

> **Definition** A Byzantine actuator q is one that can generate arbitrary value of u_q in the range $-\alpha$ to α at all times.

We note that a Byzantine actuator behavior also captures the case of an actuator fail-stopping ($u_q = 0$, and an actuator debonding form its surface thereby applying a fraction of the control force ($0 \leq u_q \leq \alpha$). In our fault model, k out of the m actuators are Byzantine in system S.

We will prove that the system remains asymptotically stable even when the Byzantine actuators behave in the worst possible way at all times. This is described below. Let $u_{ci}(t)$ be the correct actuator value at any time t for actuator i. Let $u_{fi}(t)$ be the corresponding value generated if the actuator is Byzantine. We then have the following conditions.

$$W1: \quad u_{ci}(t) \neq 0 \quad \Rightarrow u_{fi}(t) = -u_{ci}(t) \tag{12}$$

$$W2: \quad u_{ci}(t) = 0 \quad \Rightarrow u_{fi}(t) = \pm\alpha \tag{13}$$

Note: If the system S is in equilibrium and is acted upon by a Byzantine actuator, then the system is subject to perturbation and the energy of the system increases. We do not consider this case in our fault model. We are interested in maintaining the asymptotic stability of S in the presence of Byzantine actuators.

3 Reliable Control System Design

In this section, we design two reliable control schemes that maintain asymptotic stability of the System S in the presence of Byzantine actuators.

3.1 Reliable Control System Using Redundant Colocated Actuators

In this scheme we place multiple actuators at each location. Thus the effect of each redundant actuator on the control stays the same.

Theorem 3.1 *A sufficient condition to tolerate k Byzantine actuators at each location and guarantee asymptotic stability in the system S is to have $2k+1$ actuators at each of the m locations, where $m \geq n$ and the B matrix formed by the m distinct locations is of rank n.*

Proof Since there are $2k+1$ actuators at each location, the Lyapunov derivative in Eq. 11 can be written as follows

$$\dot{V} \quad = \quad 2 \times \alpha(\sum_{i=1}^{m}((2k+1) \times |(x^T).(B_i)|)) \tag{14}$$

First of all, we see from Eq. 14 that if the actuators are not Byzantine, the redundant actuators still keep the energy derivative negative. We now analyze the effect of Byzantine actuators at each location. Without loss of generality let us consider the q^{th} location and assume that k actuators at this location are Byzantine. We consider the 2 conditions $W1$ and $W2$, described in the fault model.

When condition $W1$ of the fault model applies, the energy derivative term corresponding to the q^{th} actuator location can be written as follows.

$$\begin{aligned} \dot{V}_q &= 2 \times \alpha((k+1) \times |(x^T).(B_q)| - (k) \times |(x^T).(B_q)|) && (15) \\ &= 2 \times \alpha(|(x^T).(B_q)|) && (16) \end{aligned}$$

Thus we see that the energy derivative corresponding to the q^{th} location still stays negative. This can similarly proved for all locations.

Now consider condition $W2$. If $u_{cq}(t) = 0$, it implies that $y_q(t) = 0$, i.e the local output is zero. Thus the current state $x(t)$ is orthogonal to the vector C_q. Since the actuators are colocated, the current state $x(t)$ is also orthogonal to the vector B_q. Thus, the term $x^T.(B_q)$ is equal to zero no matter what force the Byzantine actuator applies.

Hence the system S with $2k+1$ actuators at each of the m locations, is asymptotically stable in the presence of k Byzantine actuators at each location. ∎

Note that this scheme tolerates k Byzantine actuators per location. If the expected reliability ratio of the actuators are known, then we can design for the number of actuators required at each location.

Given a reliability ratio for the actuators (greater than 0.5), denoted as ρ, we can choose a k such that the system is reliable against Byzantine faults.

$$\frac{k}{(2k+1)} > (1-\rho) \tag{17}$$

Note However it should be pointed out that placing the redundant actuators at the same location may not be feasible in all control systems. Moreover the redundancy rapidly increases as n increases because the actuators are replicated at each location.

We now describe a reliable control scheme where the colocation of redundant actuators is not required and given that k actuators are Byzantine we add redundant controllers to the system as a whole thus decreasing the redundancy required.

3.2 Reliable Control System Without Using Colocated Actuators

We first state the minimum number of actuators to be added to the system which ensures that the energy derivative of Eq. 11 is than zero at all times.

Lemma 3.2 *For the energy derivative of Eq. 11 to be less than zero at all times in the presence of k Byzantine actuators, we require $m >= 2k + n$*

Proof Let the number of actuators in the system be $2k + n - 1$. The state x can be orthogonal to at most $n - 1$ actuators. Let all of these be non-Byzantine actuators. Thus the energy derivative terms corresponding to these actuators is zero. There are $2k$ actuators left. Without loss of generality assume that in the presence of any k Byzantine actuators belonging to set of $2k$ actuators, the energy derivative is less than zero. Then for the same state x, if the remaining set of actuators had been Byzantine the energy derivative would be greater than zero. Thus we need at least $2k + n$ actuators for the energy derivative of Eq. 9 to be less than zero at all times in the presence of k Byzantine actuators. ∎

However, finding an actuator configuration that satisfies such a lower bound for any k and n is a complex problem. We now focus our attention on second order systems, i.e $n = 2$ and show that $3k + 1$ is an upper bound on the number of actuators needed.

Definition An *m-uniform configuration* of the system is the actuator configuration of the system in which each of the m columns of the B matrix has the same amplitude and are uniformly distributed in the state space of n dimensions such that the column vectors of B are pairwise equi-angular and the angle between consecutive pairs of vectors is equal to $\frac{\pi}{m}$.

A second order system with 4-uniform and 7-uniform configuration is depicted in Fig. 1. For simplicity, Let α be equal to 1. Thus given the actuator locations, each actuator vector can either be equal or opposite to the direction shown depending on the current state of the system.

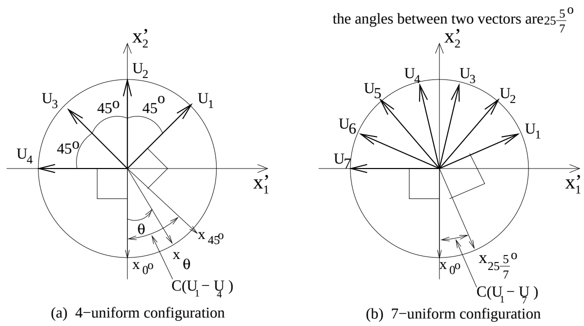


Fig. 1. 4-uniform and 7-uniform configurations for the second-degree system

Let a unit state vector x_θ form an angle θ with the vertical axis as shown in the figure. When an actuator is behaving correctly, the actuator vector would be such that its dot product with the state vector is less than or equal to zero. This is because the system is observable from each location and each actuator applies control in a direction opposite to that of the local output. The inner product would be equal to zero when the actuator vector is orthogonal to the current state.

Thus, the 4-uniform configuration shown in the figure is the proper actuator configuration when θ is between $0°$ and $45°$. Note that the dot product of the state vector with each actuator vector is less than or equal to zero. If θ is between $45°$ and $90°$, everything would be identical to the former except that first actuator changes its direction so that $\bar{U_1}$ is the new actuator vector. In the case of 4-uniform configuration, 4 normal actuators $U_1, \cdots, U_4$ keep the current control vector directions for the negative energy derivative if θ between $0°$ and $45°$. Thus the whole configuration is rotated by 45° in the clockwise direction. Thus in an $m - uniform$ configuration, if all the actuators are correct, then the actuator vectors remain pairwise equi-angular at all times.

Therefore while showing that a particular $m - uniform$ configuration is sufficient to guarantee asymptotic stability in the presence of Byzantine faults, it is enough to consider the case that the unit state vector x_θ is located in the *basic range* $[0.0°, 45.0°]$. In general, the basic range of m-uniform configuration system is $[0.0, \frac{\pi}{m}]$. Also note that it is enough to consider unit state vectors because all the actuator vectors are of same magnitude and the total dot product depends only on the angle.

> **Definition** In an m-uniform system of second-degree($m = 3k + 1$), let $S(k, \theta)$ denote the set of k Byzantine faulty actuators such that, for a unit state vector x_θ, the corresponding energy derivative becomes maximized among all possible k subsets of actuators. Let $ED(k, \theta)$ be the corresponding energy derivative.

For example, for 4-uniform configuration system, $S(1, 0°)$ and $S(1, 45°)$ are $\{U_2\}$ and $\{U_3\}$, respectively.

$$
\begin{aligned}
ED(1, 0°) &= x_{0^\theta} \cdot (U_1 - U_2 + U_3 + U_4) \\
&= (\cos 135° - \cos 180° + \cos 235°) \\
&= -0.4142
\end{aligned}
$$

Likewise, $ED(1, 45°)$ turns out to be equal to -0.4142. Thus, in the boundary angles of the basic range $[0.0°, 45.0°]$, the system is asymptotically stable due to the negative values of $ED(1, 0°)$ and $ED(1, 45°)$.

It is seen that for any m-uniform configuration, when the state vector is at the boundary of the basic range, one of the actuator vectors is orthogonal to the state vector and offers no control. Thus if $ED(k, 0°)$ and $ED(k, \frac{\pi}{m})$ are both negative, the system is asymptotically stable in the presence of k Byzantine faults. We now write down the expressions for $ED(k, 0°)$ and $ED(k, \frac{\pi}{m})$ in any m-uniform configuration.

$$\phi = \pi/m = \pi/(3k+1) \tag{18}$$

$$ED(k,\phi) = \sum_{i=1}^{k} \cos(\frac{\pi}{2} + i \cdot \phi) - \sum_{i=k+1}^{2k} \cos(\frac{\pi}{2} + i \cdot \phi) + \sum_{i=2k+1}^{3k+1} \cos(\frac{\pi}{2} + i \cdot \phi) \tag{19}$$

$$ED(k,0) = \sum_{i=0}^{k-1} \cos(\frac{\pi}{2} + i \cdot \phi) - \sum_{i=k}^{2k-1} \cos(\frac{\pi}{2} + i \cdot \phi) + \sum_{i=2k}^{3k} \cos(\frac{\pi}{2} + i \cdot \phi) \tag{20}$$

$$ED(k) = min(ED(k,0), ED(k.\phi)) \tag{21}$$

Upon numerical analysis of $ED(k)$ for a large spectrum of values for k from 1 to 1000, it turns out to be that all values of $ED(k)$ are negative as shown in the figure below. Thus an m-uniform configuration of actuators is sufficient to guarantee asymptotic stability of a second order system in the presence of k Byzantine actuators when $m = 3k + 1$.

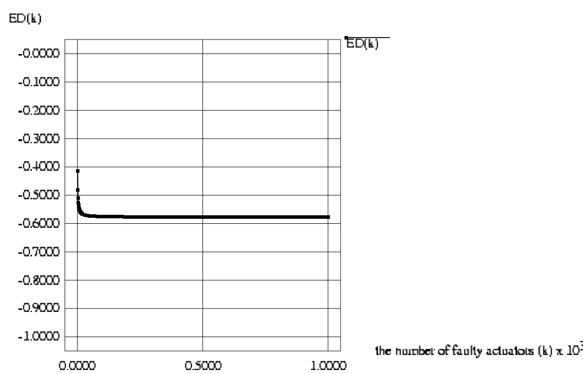


Fig. 2. The maximum energy derivative $ED(k)$ in m-uniform configuration system

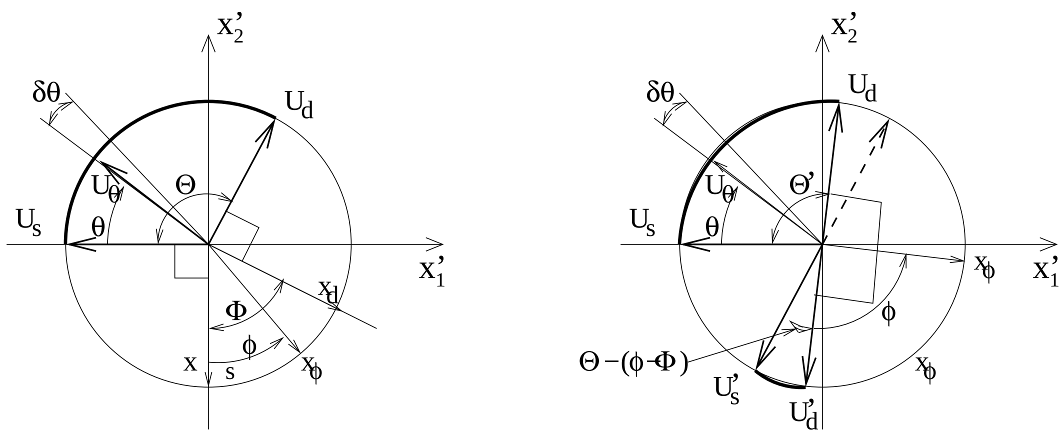
Remark Note that the case of $n = 2$ and $k = 1$, where we need 4 actuators to guarantee asymptotic stability satisfies the lower bound $2k+n$. Further, an upper bound on the redundancy required to tolerate k faults for higher order systems can be found in a related technical report.

4 Reliable Control Design for Higher Dimensions

Befor proceeding to derive a sufficient condition to guarantee asymptotic stability in the presence of faults for higher dimension systems, we first present a useful concept of an *(∞, Θ)-uniform* configuration. From now on, let's assume that angles of vectos are measured counter-clockwise from the negative x-axis.

> **Definition** *(∞, Θ)-uniform* configuration is one in which an infinite number of actuators are uniformly distributed within an angle Θ.

An *(∞, Θ)-uniform* configuration is depicted in Fig. 3(a). Thus, for an angle θ, the ratio of the number of actuators between subrange $[0, \theta]$ and total range $[0, \Theta]$ is $\frac{\theta}{\Theta}$. Note that $(\infty, 180°)$-uniform configuration is identical to k-uniform configuration in the previous section for which k is very large.



(a) (b)

$(\infty$,$\Theta)$-uniform configuration

For a unit state vector x_ϕ, if ϕ is in $[0, \Theta]$, then the correct actuator vectors are distributed in the range $[0, \Theta]$ as shown in Fig. 3(a). If ϕ is greater than Θ, then the vectors are distributed as in Fig. 3(b). In both the cases every actuator generates a negative energy derivative with the state vector.

Then, for a given unit state vector x_ϕ, the energy derivative for the set of actuators distributed in the range $[\theta, \theta + \delta\theta]$ is $x_\phi \cdot U_\theta \cdot \delta\theta$. If all the actuators are normal, then total energy derivative becomes $\int_{\theta=0}^{\Theta} x_\phi \cdot U_\theta \cdot \delta\theta = \int_{\theta=0}^{\Theta} \cos(\theta + \phi + \frac{\pi}{2})\delta\theta$.

Definition The Byzantine ratio γ is defined to be the fraction of Byzantine actuators amongst all actuators.

Given the infinite number of actuators, we enumerate this ratio in terms of the total angle formed by the set of vectors corresponding to the Byzantine actuators. Thus the Byzantine actuators cumulatively form an angle $\gamma \cdot \Theta$.

Let $ED(\Theta, \gamma, \phi)$ denote the maximum possible energy derivative formed by a unit state vector x_ϕ with a set of Byzantine actuators given that the Byzantine ratio γ.

$ED(\Theta, \gamma, \phi)$ depends on the distribution of γ within the set of all actuators. For example, consider the case that $\phi = 0$ and $\Theta \geq \frac{\pi}{2} + \frac{\gamma\Theta}{2}$. In this case when Byzantine actuators are distributed in the range $[\frac{\pi}{2} - \frac{\gamma\Theta}{2}, \frac{\pi}{2} + \frac{\gamma\Theta}{2}]$, the energy derivative is the maximum.

Let $ED(\Theta, \gamma)$ denote the maximum among all values of $ED(\Theta, \gamma, \phi)$ over all the state vector angles ϕ.

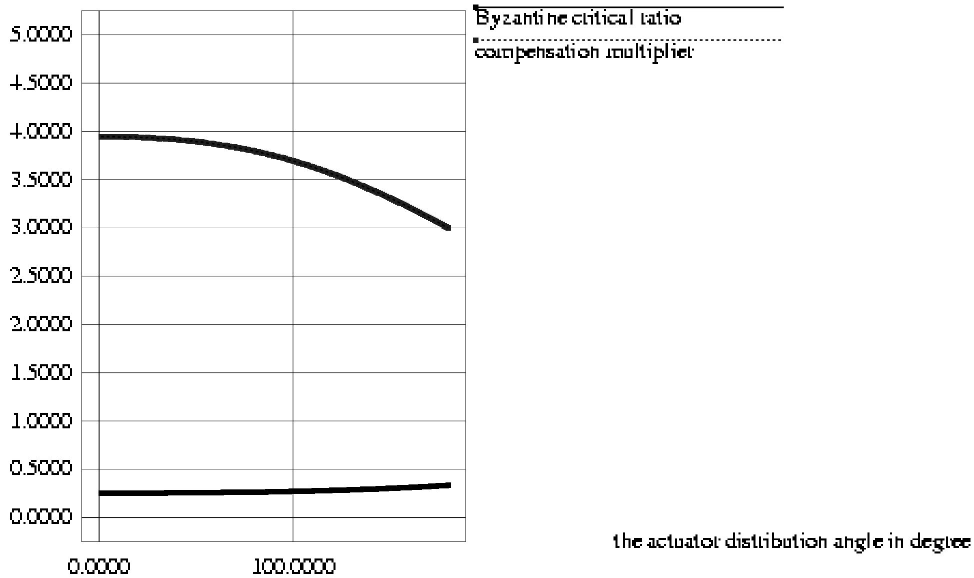
As the value of γ increases, $ED(\Theta, \gamma)$ increases.

Definition The Byzantine Tolerance Ratio, denoted as $\gamma_{tolerance}$ is defined as the value of Byzantine Ratio such that $ED(\Theta, \gamma_{tolerance}) = 0$.

Then, for a given value of Θ, the system with (∞, Θ)-uniform configuration is asymptotically stable in the presence of Byzantine actuators if and only if the Byzantine ratio γ is less than $\gamma_{tolerance}$.

Definition The *Compensation multiplier* τ is the inverse of the Byzantine Tolerance Ratio, i.e $\frac{1}{\gamma}$

Numerically for a 2-dimesional system S, we compute the Byzantine tolerance ratio $\gamma_{tolerance}$ and compensation multiplier τ for all angles of Θ in the range $[1°, 180°]$. This is shown in Fig. 4.



Byzantine tolerance ratio $\gamma_{tolerance}$ and compensation multiplier

From the figure, we can observe several aspects of the fault-tolerant characteristics about second-degree (∞, Θ)-uniform configuration system.

First, in $(\infty, 180^\circ)$-uniform configuration, the corresponding compensation multiplier τ is 3.0, matching with the condition in the previous section in which $3k + 1$ was shown as a sufficient condition to guarantee asymptotic stability in the $m - uniform$ configuration.

Second, in any uniform distribution where $0 \leq \Theta < 180^\circ$, the compensation multiplier increases as Θ decreases. Thus distribution of the actuator vectors with a smaller angular seperation results in more redundancy required.

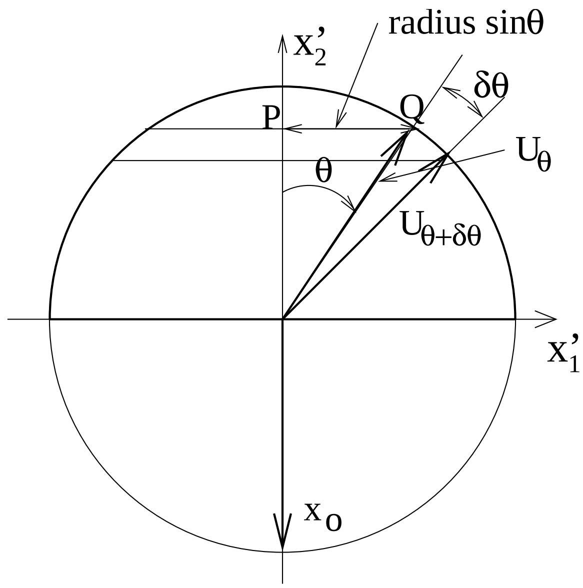
Third, even if the actuators are distributed within a very small angular range, the compensation multiplier τ does not increases significantly. In particular, in (∞, Θ)-uniform configuration with $\Theta = 1^\circ$, it is about 4.0, increasing τ by 33% in comparison with that for the perfectly uniform contribution.

We now use the infinite uniform distribution to analyze and determine an upper bound on the compensation multiplier for higher dimension systems.

4.1 Byzantine Tolerance Ratio for Higher Order Systems($n > 2$)

In this section, we develop the compensation multiplier for $(\infty, 180°, n)$-uniform configuration for n-th order system. We show the computation details for a third-order system, i.e. $n = 3$ which are then generalized for higher dimensions. We also consider only (∞, Θ)-uniform configurations in which $\Theta = 180°$.

Let (∞, n)-p-uniform configuration denote the (∞, Θ)-uniform configuration for the n^{th} dimension system in which $\Theta = 180°$.



$(\infty, 3)$-p-uniform configuration

Fig. 5 depicts an $(\infty, 3)$-p-uniform configuration. Since all the actuator vectors are of the same magnitude, in our analysis it is sufficient to consider only unit vectors. For a given state vector x_o, the corresponding $(\infty, 3)$-p-uniform configuration is shown in the figure. The angle $\delta\theta$ is infinitesimally small as typically used in differential and integral calculus. The energy derivative for the correct actuator vectors distributed between two angles θ and $\theta+\delta\theta$ is $-\cos\theta \cdot (2\pi \sin\theta) \cdot \delta\theta$ where $-\cos\theta$ is the vector inner product between unit state vector x_o and unit actuator vector U_θ, $(2\pi \sin\theta)$, is the circumference of the circle that is generated by rotating the line $\bar{PQ}$ arround the vertical axis, and $\delta\theta$ is the length of the circumferential piece between two unit actuator vectors U_θ and $U_{\theta+\delta\theta}$.

In the $(\infty, 3)$-p-uniform configuration, we see that the actuator vectors are uniformly distributed on the surface of a unit hemi-sphere. The area of the sub-surface between angular range $[0, \alpha]$ is $\int_{\theta=0}^{\alpha}(2\pi \sin\theta)\delta\theta = 2\pi(1 - \cos\alpha)$. Let us now calculate the compensation multiplier given the Byzantine ratio *gamma*.

In the worst case, the Byzantine actuators are all distributed in such a way that they maximize the energy derivative. We also see that the area of the subsurface formed by the actuators is maximised when α is small. Therefore we let the Byzantine actuators to lie in a range starting from 0. Let $\alpha_\gamma = \frac{\alpha}{\pi}$. Then, the ratio of the surface area formed by Byzantine actuator vectors and the whole area is given by the following equation.

$$\gamma = \frac{2\pi(1 - \cos\alpha_\gamma)}{2\pi} = (1 - \cos\alpha_\gamma). \tag{22}$$

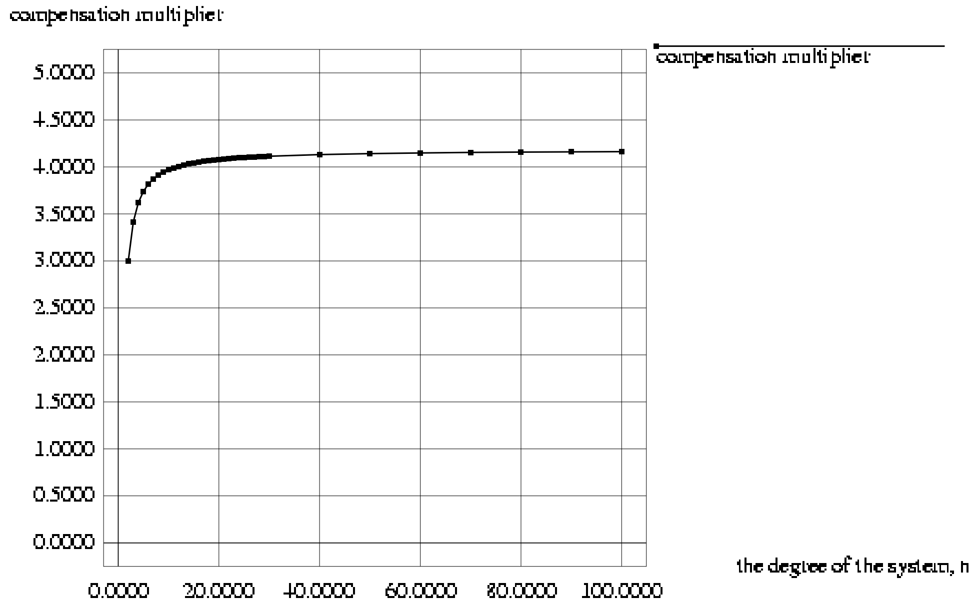
Now, we compute the total energy derivative corresponding to the worst case situation denoted as $ED(3, \infty, \gamma)$ with Byzantine ratio γ.

$$\begin{aligned} ED(3, \infty, \gamma) &= \int_{\theta=0}^{\alpha_\gamma} \cos\theta \cdot (2\pi \sin\theta) \cdot \delta\theta - \int_{\theta=\alpha_\gamma}^{180^\circ} \cos\theta \cdot (2\pi \sin\theta) \cdot \delta\theta && (23) \\ &= -\pi \cdot \cos(2\alpha_\gamma) && (24) \end{aligned}$$

The Byzantine tolerance ratio $\gamma_{tolerance}$, for which $ED(3, \infty, \gamma_{tolerance})$ becomes zero, requires the condition that $\cos(2\alpha_{\gamma_{tolerance}}) = 0$, i.e. $\alpha_{\gamma_{tolerance}} = 45^\circ$. By substituting this value into Eq. 22, the Byzantine tolerance ratio is computed as $\gamma_{tolerance} = 1 - \cos 45^\circ \simeq 0.2928932$. Therefore, the compensation multiplier for the third-degree system, τ, is 3.4142.

In general, for n-th degree system under (∞, n)-p-uniform configuration, similar computations are preformed and we get the followign results. Fig. 6 depicts the compensation multipliers for the higher-degree system under (∞, n)-p-uniform configuration.

The rate of increase of the compensation multiplier decreases as n increases. A 1000 dimensional system has a compensation multiplier of 4.1815.



Compensation multipliers for the higher-degree system under (∞, n)-p-uniform configuration

5 Application to Beam Vibration Control System

We now apply our reliable control system designs on a local output feedback control scheme to a beam vibration control system. Given is a uniform beam of unit length, unit mass, and unit stiffness factor, that is restricted by pins at both ends and subjected to an initial disturbance. The beam has no dampening factor so that it may vibrate endlessly. The beam has colocated velocity sensors and actuators to reduce the vibration. For simplicity, we consider two fundamental modes of vibration.

The two fundamental vibration modes, denoted as M_1 and M_2, are derived [13] as follows:

$$M_1 \quad : \quad 1.4142 \sin \pi z, \quad \lambda_1 = \omega_1^2 = 97.41 \tag{25}$$

$$M_2 \quad : \quad 1.4142 \sin 2\pi z, \quad \lambda_2 = \omega_2^2 = 1558.55 \tag{26}$$

where $z \in [0.0, 1.0]$ denotes the position in the beam spatial axis and λ_i and ω_i, $i = 1, 2$, represent the eigenvalues and the frequencies of i-th modes, respectively.

Since each mode is governed by a second-degree differential equation, the state vector for the system contains four variables $x = [x_1, x_2, x_3, x_4]^T$. $x_1(x_2)$ and $x_3(x_4)$ denote the vertical displacement and velocity of first (second) vibration mode, respectively. Then, the system matrix A in Eq. **??** is denoted as

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -97.41 & 0 & 0 & 0 \\ 0 & -1558.55 & 0 & 0 \end{pmatrix}$$

Note that we use a velocity feedback control. So the control input does not have any effect on the states $x1$ and $x2$. The actuation is used to control the velocity states $x3$ and $x4$. We will assume that the beam cannot be deformed permanently. Thus when the velocity of the beam comes to zero, the displacement is also zero. Thus in this specific example although the number of states is 4, the control affects only the 2 velocity states.

We first show that using 2 sensor-actuator pairs that form a B matrix of rank 2, we can asymptotically stabilize the system. We choose the following B matrix.

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1.4142 \\ 1.3066 & 1 \end{pmatrix}$$

The Fig. 3(a) shows the energy of the system staring from an arbitrray initial state going down to zero in the absence of faults. The energy of the system at time t is calculated as $x^T(t) \times X(t)$, where $x(t)$ is the state of the system.

[Energy of the system with 2 Actuators, No faults] [Energy of the system with 3 actuators at Each Locat

Fig. 3. Energy of the Beam Vibration System - 2 Actuator Locations

We now show that 1 Byzantine actuator at each location can be tolerated and asymptotic stability can be maintained by having 3 actuators at each location. The Fig. 3(b)shows the energy of the system staring from an arbitrray initial state when one actuator at each location is Byzantine.

We now show that when $k = 1$, we can asymptotically stabilize the system using 4 actuators that are distributed according to the $4-uniform$ configuration. We choose 4 pairs of colocated sensors and actuators such that the columns of B matrix have equal magnitude and successive column vectors are seperated by an angle $\frac{\pi}{4}$.

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.5754 & 0.1715 & 0.8179 & 0.9852 \\ -0.8179 & -0.9852 & 0.5754 & 0.1714 \end{pmatrix} \qquad (27)$$

The following graphs show the states of the $4 - uniform$ configuration system staying asymptotically stable in the presence of no actuator faults and one actuator failing.

[Energy of the 4-uniform configuration - No Faults] [Energy of the 4-uniform configuration - 1 Byzantine

Fig. 4. Energy of the 4-Uniform Configuration Beam Vibration System

6 Conclusions and Future Work

In this paper we designed two reliable control schemes using a local output feedback control system that maintain asymptotic stability in the presence of Byzantine actuators that continuously generate erroneous control inputs. The first scheme was designed using redundant actuators that were colocated. However, it may not be feasible to collocate actuators in all systems. The second scheme does not require the actuators to be colocated. The other advantage with the second scheme is that the required redundancy is reduced. But in this scheme the restrictions in the choice of actuator locations increased. The design of the system becomes more complex when the number of state dimensions of the system increases. We gave an application of both the control schemes in stabilizing a beam subjected to an initial perturbation.

We plan to extend our results on tolerating faulty actuators to systems that use centralized and decentralized state feedback. An interesting topic for future study is also to design reliable control schemes based on adaptive control laws using state feedback. Extending some heuristic studies in this area [9] to sufficient conditions is a subject of ongoing work. We would also like to work on verification techniques for the fault-tolerance of a system given a set of faulty actuators.

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