Mathematical Set Notation

Set Theory

- A mathematical model that we will use often is that of ***mathematical sets***
- A (finite) set can be thought of as a collection of zero or more ***elements*** of *any* other mathematical type, say, T
  - T is called the ***element type***
  - We call this math type ***finite set of*** T

Math Notation for Sets

- The following notations are used when we write mathematics (e.g., in contract specifications) involving sets
- Notice two important features of sets:
  - There are no ***duplicate*** elements
  - There is no ***order*** among the elements

The Empty Set

- The ***empty set***, a set with no elements at all, is denoted by $\{ \}$ or by **`empty_set`**

Denoting a Specific Set

- A particular set can be described by listing its elements between `{` and `}` separated by commas
- Examples:

```
{ 1, 42, 13 }
{ 'G', 'o' }
{ }
```

Denoting a Specific Set

- A particular se
its elements b
by commas
- Examples:

`{ 1, 42, 13 }`

`{ 'G', 'o' }`

`{ }`

A ***finite set of integer*** value whose elements are the ***integer*** values *1*, *42*, and *13*; equal to the set *{ 1, 13, 42 }*.

Denoting a Specific Set

- A particular set c… its elements bet… by commas
- Examples:

  { 1, 42, 13 }

  { 'G', 'o' }

  { }

A ***finite set of character*** value whose elements are the ***character*** values *'G'* and *'o'*; this is *not* the same as the ***string of character*** value *< 'G', 'o' > = "Go"*.

Denoting a Specific Set

- A particular se
  its elements b
  by commas
- Examples:

```
{ 1, 42,   }
{ 'G'   'o' }
{ }
```

Now it can be seen that this notation for **`empty_set`** is a special case of the set literal notation.

Membership

- We say `x` ***`is in`*** `s` iff `x` is an element of `s`
- Examples:

  `33` ***`is in`*** *`{ 1, 33, 2 }`*

  `'G'` ***`is in`*** *`{ 'G', 'o' }`*

  `33` ***`is not in`*** *`{ 5, 2, 13 }`*

  `5` ***`is not in`*** *`{ }`*

Membership

- We say `x` ***is in*** `s` iff `x` is an element of `s`
- Examples:

  33 ***is in*** *{ 1, 33, 2 }*

  'G' ***is in*** *{ 'G', 'o' }*

  33 ***is not in*** *{ 5, 2, 13 }*

  5 ***is not in*** *{ }*

The usual mathematical notation for this is $\in$.

Union

- The ***union*** of sets `s` and `t`, a set consisting of the elements that are in either `s` or `t` or both, is denoted by `s` **`union`** `t`
- Examples:

```
{ 1, 2 } union { 3, 2 } = { 1, 2, 3 }
{ 'G', 'o' } union { } = { 'G', 'o' }
{ } union { 5, 2, 13 } = {5, 2, 13 }
{ } union { } = { }
```

Union

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- Examples:

The usual mathematical notation for this is ∪.

`} = { 1, 2, 3 }`

`= { 'G', 'o' }`

`= {5, 2, 13 }`

`{ }` **`union`** `{ } = { }`

Intersection

- The ***intersection*** of sets `s` and `t`, a set consisting of the elements in both `s` and `t`, is denoted by `s` **`intersection`** `t`
- Examples:

  `{ 1, 2 }` **`intersection`** `{ 3, 2 } = { 2 }`

  `{ 'G', 'o' }` **`intersection`** `{ } = { }`

  `{ 5, 2 }` **`intersection`** `{ 13, 7 } = { }`

  `{ }` **`intersection`** `{ } = { }`

Intersection

- The ***intersection*** of sets `s` and `t`, a set consisting of the elements in both `s` and `t`, is denoted by `s` ***`intersection`*** `t`
- Examples:

The usual mathematical notation for this is ∩.

`{ 3, 2 } = { 2 }`

`ion` `{ } = { }`

`{ 13, 7 } = { }`

`{ }` ***`intersection`*** `{ } = { }`

Difference

- The ***difference*** of sets `s` and `t`, a set consisting of the elements of `s` that are not in `t`, is denoted by `s \ t` (or by `s - t`)
- Examples:

```
{ 1, 2, 3, 4 } \ { 3, 2 } = { 1, 4 }
{ 'G', 'o' } \ { } = { 'G', 'o' }
{ 5, 2 } \ { 13, 5 } = { 2 }
{ } \ { 9, 6, 18 } = { }
```

Difference

- The ***difference*** of sets `s` and `t`, a set consisting of the elements of `s` that are not in `t`, is denoted by `s \ t` (or by `s - t`)
- Examples:

```
2 } = { 1, 4 }
'G', 'o' }
{ 2 }
{ } \ { 9, 6, 18 } = { }
```

This may be pronounced
“s without t”.

Subset

- We say `s` ***`is subset of`*** `t` iff every element of `s` is also in `t`
  - `s` ***`is proper subset of`*** `t` does not allow `s = t`

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The usual mathematical notations are
$\subset$ (for proper) and $\subseteq$;
we say ***`is not ...`*** for the negation of each.

Size (Cardinality)

- The ***size*** or ***cardinality*** of a set `s`, i.e., the number of elements in `s`, is denoted by `|s|`
- Examples:

```
|{ 1, 15, -42, 18 }| = 4
|{ 'G', 'o' }| = 2
|{ }| = 0
```

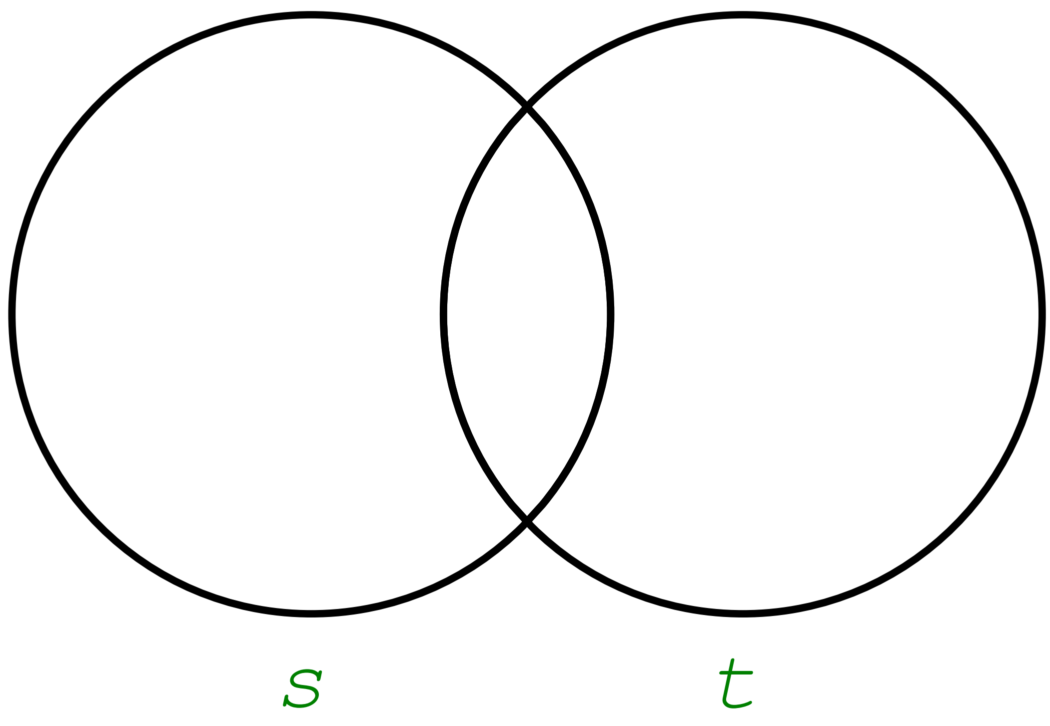
Entries of a String

- The set whose elements are exactly the entries of a string *s* (i.e., the string's entries without duplicates and ignoring order) is denoted by ***entries****(s)*
- Examples:

entries*(< 2, 2, 2, 1 >) = { 1, 2 }*

entries*(< >) = { }*

Venn Diagrams



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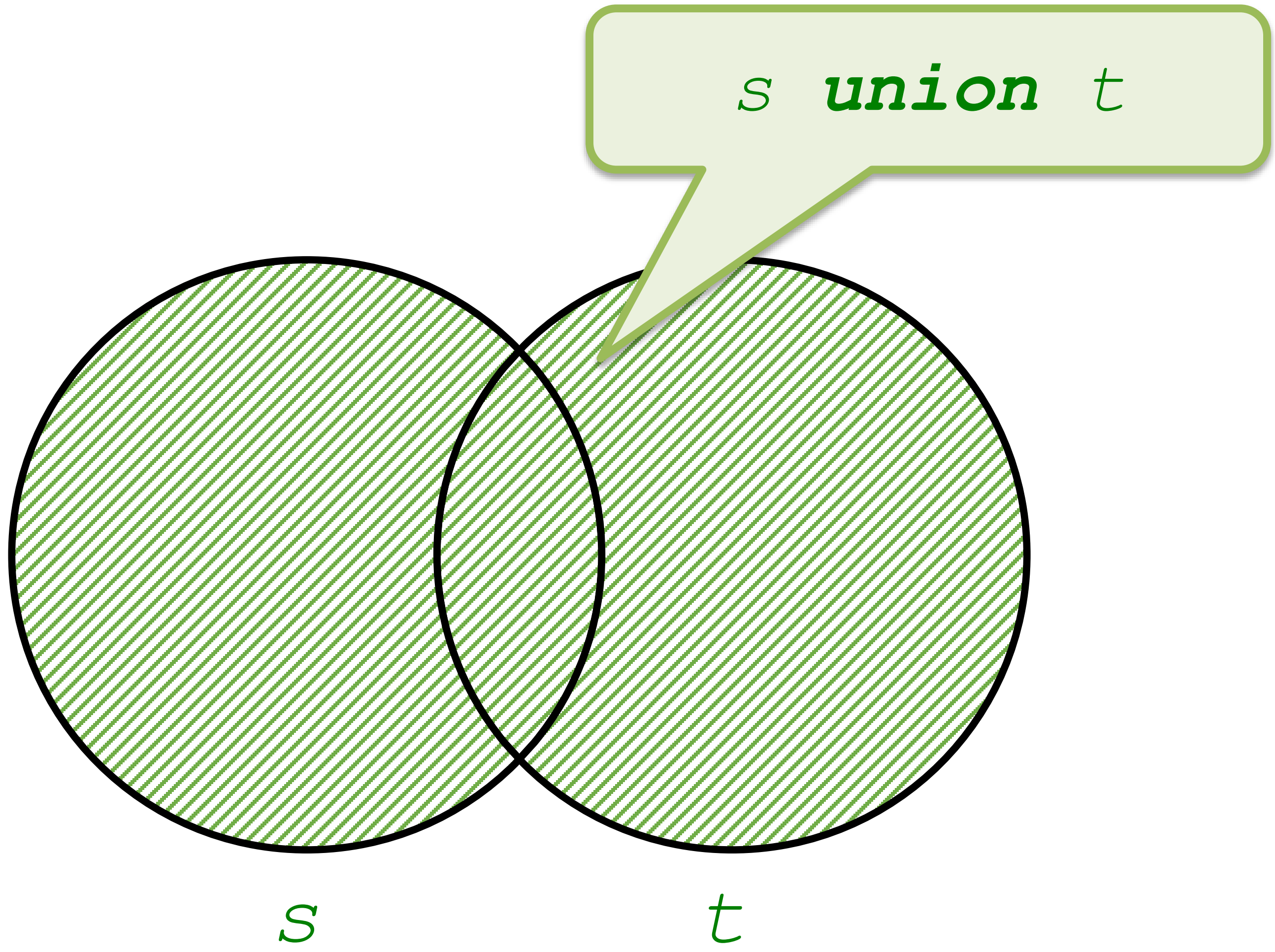

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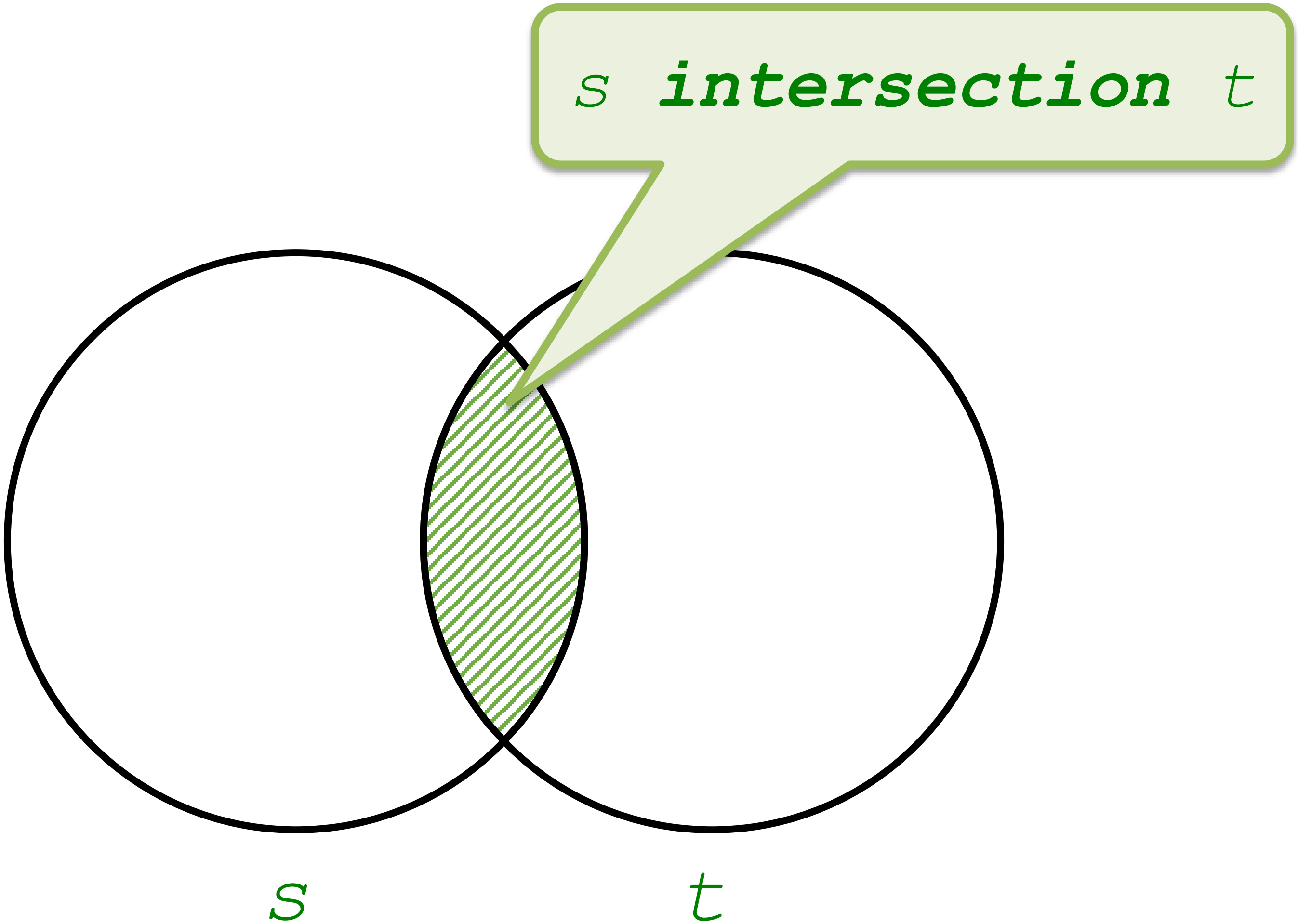

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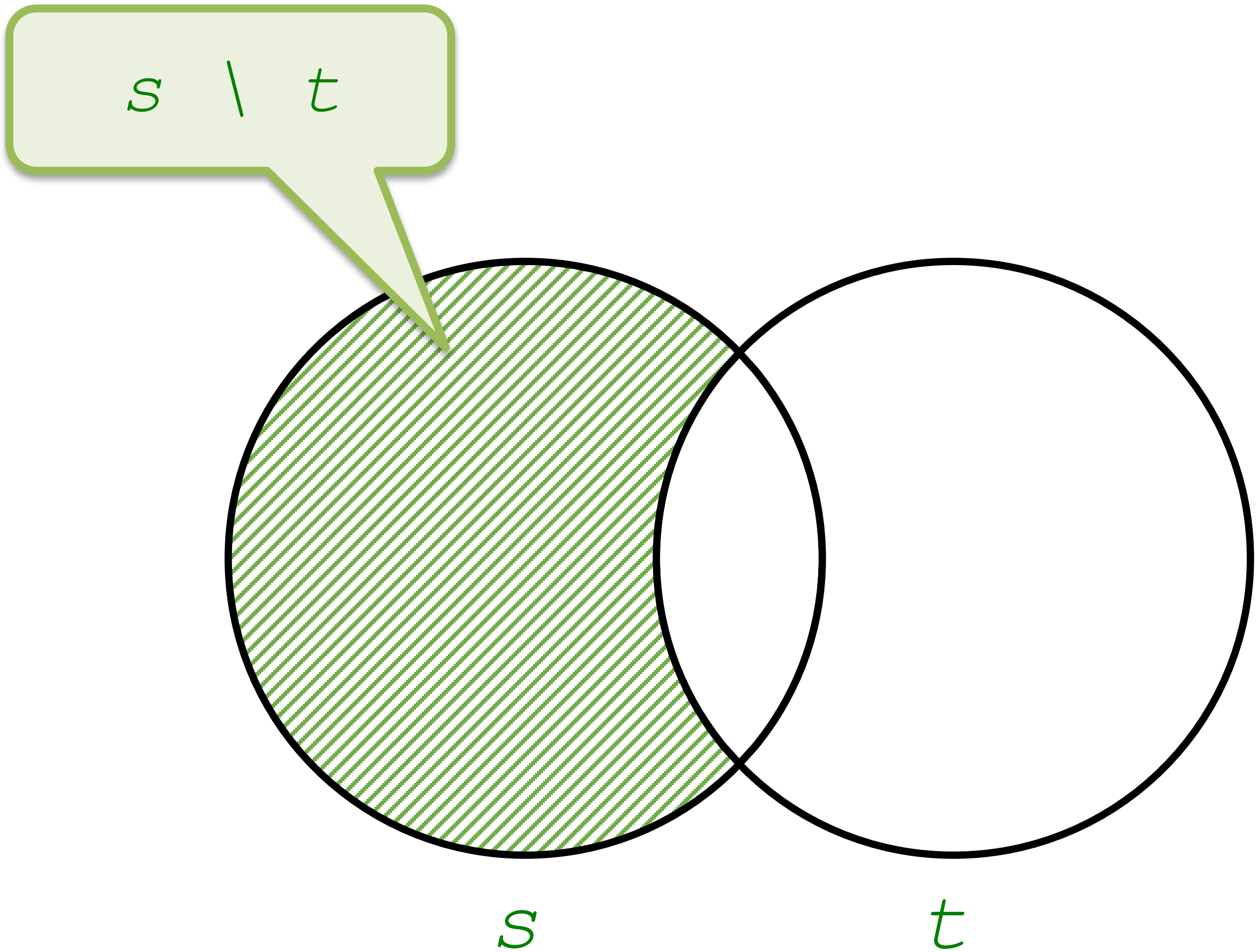

Venn Diagrams

`s` **`is proper subset of`** `t`