Mathematical Set Notation
Set Theory

• A mathematical model that we will use often is that of mathematical sets

• A (finite) set can be thought of as a collection of zero or more elements of any other mathematical type, say, $T$
  – $T$ is called the element type
  – We call this math type finite set of $T$
Math Notation for Sets

• The following notations are used when we write mathematics (e.g., in contract specifications) involving sets

• Notice two important features of sets:
  – There are no *duplicate* elements
  – There is no *order* among the elements
The Empty Set

- The *empty set*, a set with no elements at all, is denoted by \{ \} or by `empty_set`
Denoting a Specific Set

• A particular set can be described by listing its elements between \{ \text{ and } \} separated by commas

• Examples:

\[
\begin{align*}
\{ & 1, \ 42, \ 13 \} \\
\{ & \ 'G', \ 'o' \} \\
\{ & \ } \\
\end{align*}
\]
Denoting a Specific Set

- A particular set can be described by listing its elements between { and }, separated by commas.
- Examples:
  
  \[
  \{ 1, 42, 13 \} \\
  \{ 'G', 'o' \} \\
  \{ \} \\
  \]

A finite set of integer value whose elements are the integer values 1, 42, and 13; equal to the set \{ 1, 13, 42 \}.
Denoting a Specific Set

• A particular set can be described by listing its elements between 

  \{ \text{and} \} \text{ separated by commas}

• Examples:

  \{ 1, 42, 13 \}

  \{ 'G', 'o' \}

  \{ \}

A finite set of character value whose elements are the character values 'G' and 'o'; this is not the same as the string of character value < 'G', 'o' > = "Go".
Denoting a Specific Set

• A particular set can be described by listing its elements between { } separated by commas.

• Examples:

- \{ 1, 42, 63 \}
- \{ 'G', 'o' \}
- \{ \}
- \{ \}

Now it can be seen that this notation for empty set is a special case of the set literal notation.
Membership

• We say \( x \) is in \( S \) iff \( x \) is an element of \( S \)

• Examples:

  33 is in \{ 1, 33, 2 \}
  'G' is in \{ 'G', 'o' \}
  33 is not in \{ 5, 2, 13 \}
  5 is not in \{ \}
Membership

• We say $x$ is in $S$ iff $x$ is an element of $S$.

• Examples:
  33 is in $\{1, 33, 2\}$
  'G' is in $\{ 'G', 'o' \}$
  33 is not in $\{ 5, 2, 13 \}$
  5 is not in $\{ \}$

The usual mathematical notation for this is $\in$. 
Union

• The **union** of sets $s$ and $t$, a set consisting of the elements that are in either $s$ or $t$ or both, is denoted by $s$ **union** $t$

• Examples:

$$\{ 1, 2 \} \text{ **union** } \{ 3, 2 \} = \{ 1, 2, 3 \}$$
$$\{ 'G', 'o' \} \text{ **union** } \{ \} = \{ 'G', 'o' \}$$
$$\{ \} \text{ **union** } \{ 5, 2, 13 \} = \{ 5, 2, 13 \}$$
$$\{ \} \text{ **union** } \{ \} = \{ \}$$
Union

- The **union** of sets $s$ and $t$, a set consisting of the elements that are in either $s$ or $t$ or both, is denoted by $s \cup t$.

- Examples:
  
  - $\{1, 2\} \cup \{3, 2\} = \{1, 2, 3\}$
  - $\{G, o\} \cup \{} = \{G, o\}$
  - $\{} \cup \{5, 2, 13\} = \{5, 2, 13\}$
  - $\{} \cup \{} = \{}$

The usual mathematical notation for this is $\cup$. 

31 August 2017 OSU CSE
Intersection

• The *intersection* of sets \( s \) and \( t \), a set consisting of the elements in both \( s \) and \( t \), is denoted by \( s \ intersect t \)

• Examples:

\[
\begin{align*}
\{ 1, 2 \} \ intersection \ \{ 3, 2 \} &= \{ 2 \} \\
\{ 'G', 'o' \} \ intersection \ \{ \} &= \{ \} \\
\{ 5, 2 \} \ intersection \ \{ 13, 7 \} &= \{ \} \\
\{ \} \ intersection \ \{ \} &= \{ \}
\end{align*}
\]
Intersection

• The **intersection** of sets $s$ and $t$, a set consisting of the elements in both $s$ and $t$, is denoted by $s \text{ intersection } t$

• Examples:

  \[
  \{ 3, 2 \} \text{ intersection } \{ 3, 2 \} = \{ 2 \}
  \]

  \[
  \{ \} \text{ intersection } \{ \} = \{ \}
  \]

  \[
  \{ 3, 2 \} \text{ intersection } \{ 13, 7 \} = \{ \}
  \]

  \[
  \{ \} \text{ intersection } \{ \} = \{ \}
  \]

The usual mathematical notation for this is $\cap$. 

31 August 2017 OSU CSE
Difference

- The **difference** of sets $s$ and $t$, a set consisting of the elements of $s$ that are not in $t$, is denoted by $s \setminus t$ (or by $s - t$).

- Examples:

  $\{ 1, 2, 3, 4 \} \setminus \{ 3, 2 \} = \{ 1, 4 \}$
  $\{ 'G', 'o' \} \setminus \{ \} = \{ 'G', 'o' \}$
  $\{ 5, 2 \} \setminus \{ 13, 5 \} = \{ 2 \}$
  $\{ \} \setminus \{ 9, 6, 18 \} = \{ \}$
Difference

• The **difference** of sets $s$ and $t$, a set consisting of the elements of $s$ that are not in $t$, is denoted by $s \setminus t$ (or by $s - t$)

• Examples:

$\{ 1, 2 \} \setminus \{ 13, 5 \} = \{ 2 \}$

$\{ \} \setminus \{ 9, 6, 18 \} = \{ \}$

This may be pronounced “$s$ without $t$".
Subset

- We say \textit{s is subset of} \textit{t} iff every element of \textit{s} is also in \textit{t}.
  
  \textit{s is proper subset of} \textit{t} does not allow \textit{s} = \textit{t}.
Subset

- We say \( s \) is subset of \( t \) iff every element of \( s \) is also in \( t \)

\[ s \text{ is proper subset of } t \]

\( s = t \)

The usual mathematical notations are \( \subset \) (for proper) and \( \subseteq \); we say \( is not \) ... for the negation of each.
Size (Cardinality)

• The *size* or *cardinality* of a set $s$, i.e., the number of elements in $s$, is denoted by $|s|$

• Examples:

  $|\{ 1, 15, -42, 18 \}| = 4$
  $|\{ 'G', 'o' \}| = 2$
  $|\{ \}| = 0$
Entries of a String

• The set whose elements are exactly the entries of a string \( s \) (i.e., the string’s entries without duplicates and ignoring order) is denoted by \( \text{entries}(s) \)

• Examples:

\[
\text{entries}(\langle 2, 2, 2, 1 \rangle) = \{ 1, 2 \}
\]

\[
\text{entries}(\langle \rangle) = \{ \}
\]
Venn Diagrams

\[ \text{Diagram with circles labeled } s \text{ and } t \]
Venn Diagrams

$S \cup T$
Venn Diagrams

\begin{itemize}
\item $s$ intersection $t$
\end{itemize}
Venn Diagrams

\[ s \setminus t \]

\[ s \quad t \]
$s$ is proper subset of $t$