

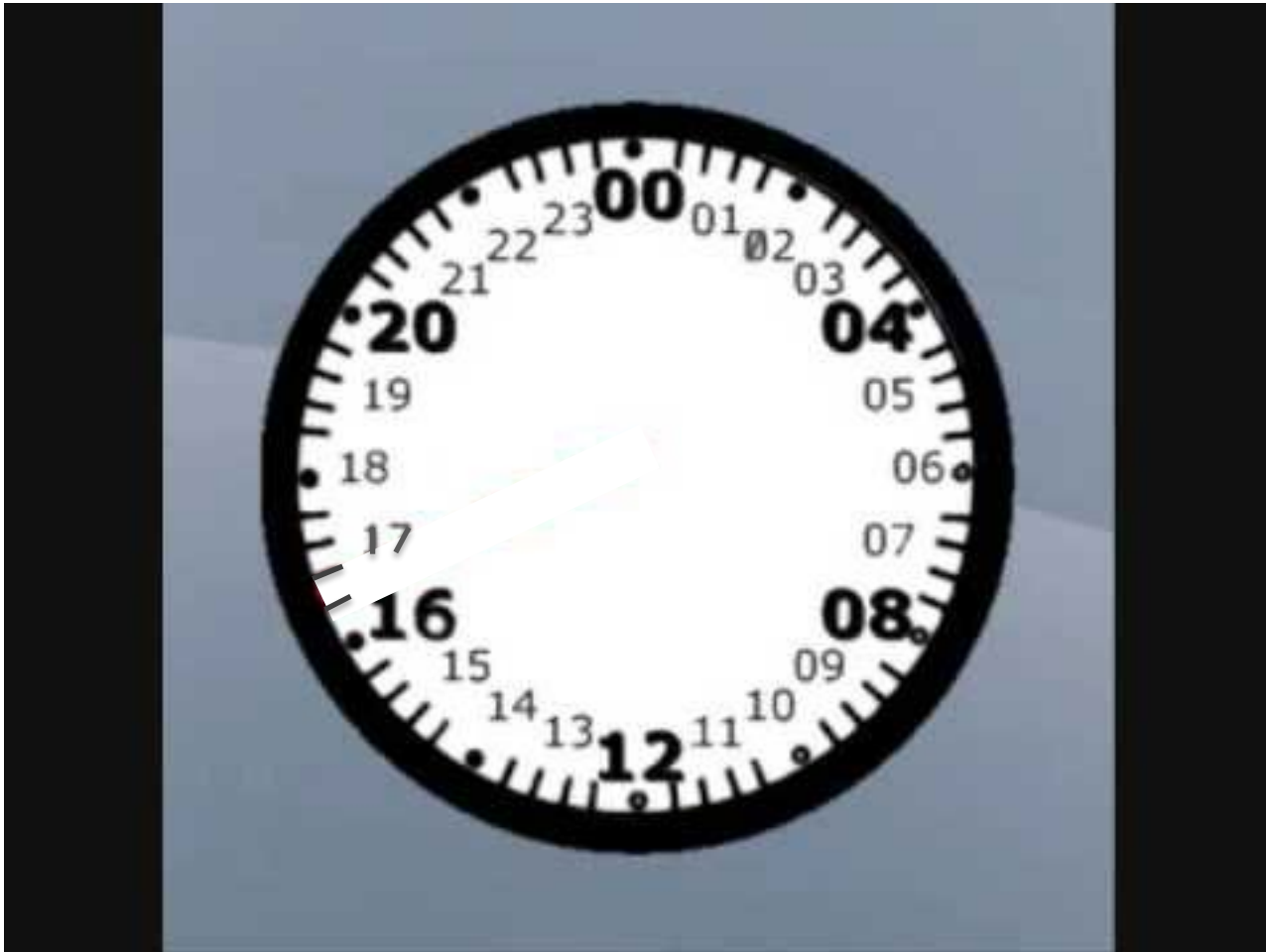
Clock Arithmetic



Mathematical Modulo (“mod”)

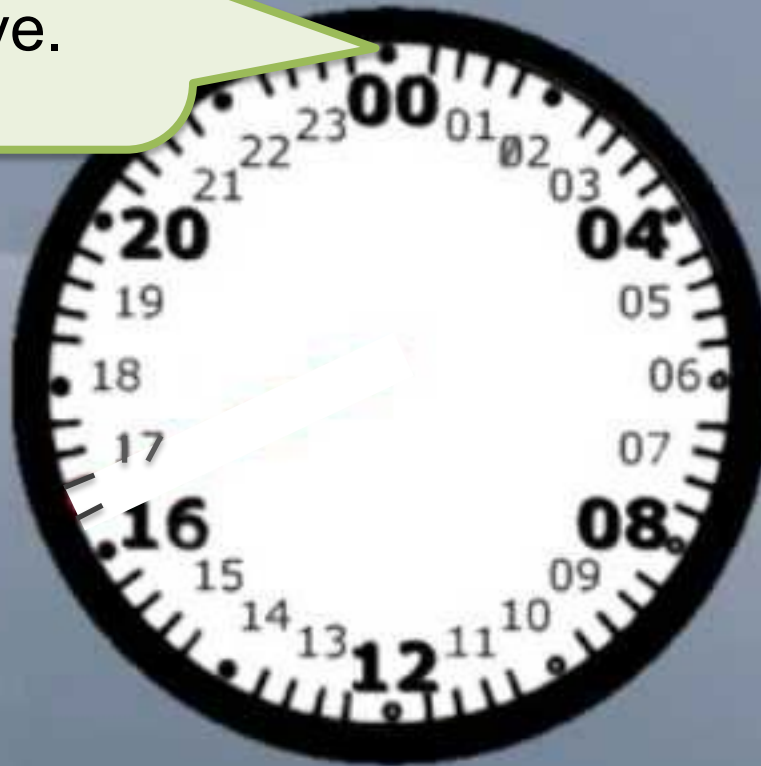
- The value of $a \bmod b$, or $a \bmod b$, where a and b are mathematical *integers* and $b > 0$, is computed by doing *clock arithmetic* on a clock face with b positions
 - If $a > 0$, the “hand” on the clock starts at 0 and moves $|a|$ positions clockwise
 - If $a < 0$, it moves $|a|$ counter-clockwise
 - Where it ends up is the value of $a \bmod b$

Example: 24-hr Clock



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Any integer *mod* 24 is a number between 0 and 23 inclusive.



Example: 24-hr Clock

What is $67 \bmod 24$?

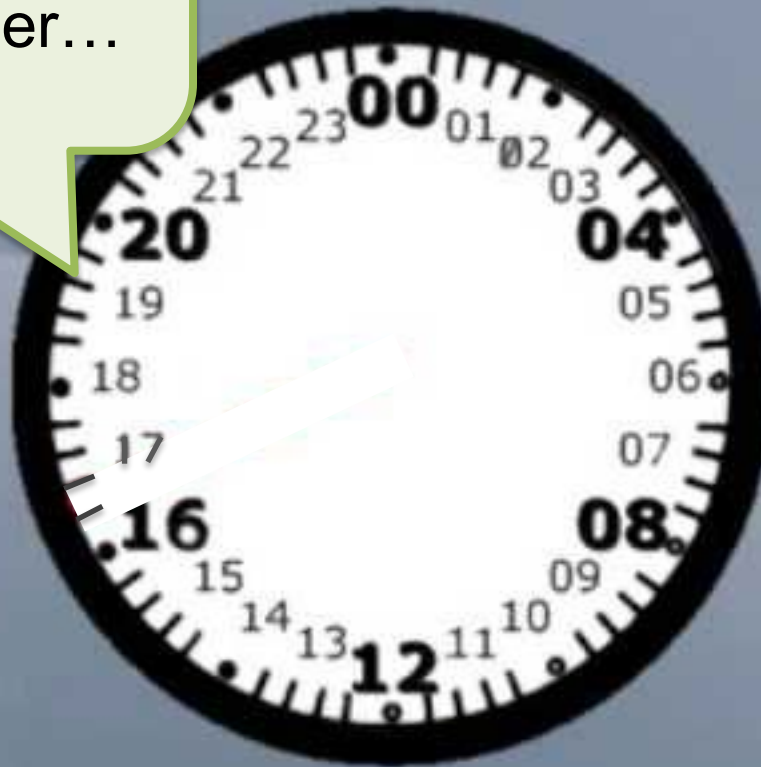
Twice around is 48,
and 19 more makes
67. Hence, 19.



Example: 24-hr Clock

And $(-67) \bmod 24$?

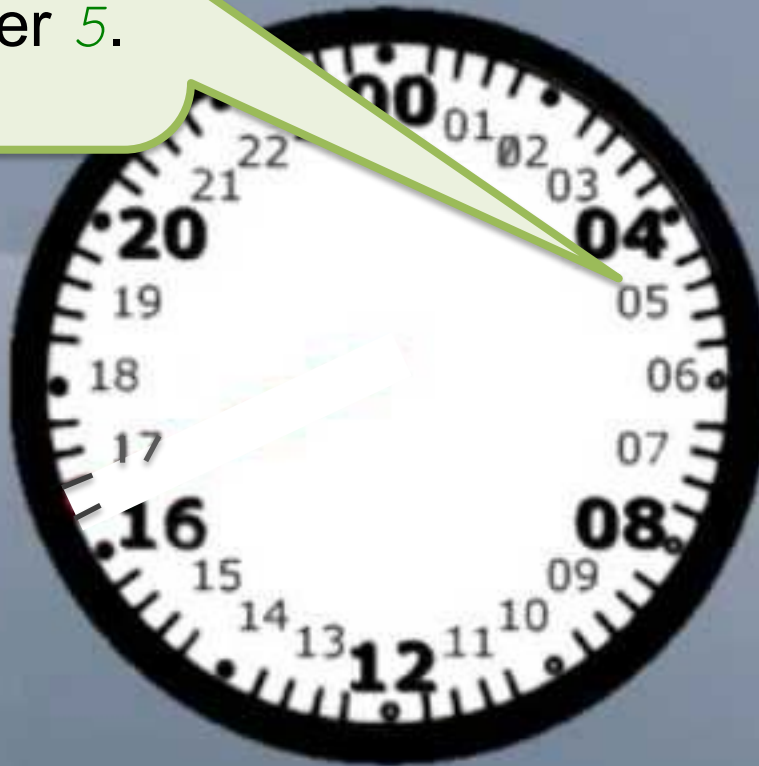
Hint: it is not 19 , it is not -19 , but rather...



Example: 24-hr Clock

And $(-67) \bmod 24$?

Hint: it is not 19 , it is not -19 , but rather 5 .



Modulo \neq Remainder (%)

- What is the **remainder** upon dividing 67 by 24? It is 19.
- What is the **remainder** upon dividing -67 by 24? It is -19 .
 - At least most people would say it is, and indeed this is how Java evaluates the expression:

$$(-67) \% 24$$