Locally Excitatory Globally Inhibitory Oscillator Networks

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***Abstract*—A novel class of locally excitatory, globally inhibitory oscillator networks (LEGION) is proposed and investigated. The model of each oscillator corresponds to a standard relaxation oscillator with two time scales. In the network, an oscillator jumping up to its active phase rapidly recruits the oscillators stimulated by the same pattern, while preventing other oscillators from jumping up. Computer simulations demonstrate that the network rapidly achieves both synchronization within blocks of oscillators that are stimulated by connected regions and desynchronization between different blocks. This model lays a physical foundation for the oscillatory correlation theory of feature binding and may provide an effective computational framework for scene segmentation and figure/ground segregation in real time.**

I. Introduction

The ability to group elements of a perceived scene or sensory field into coherent clusters (objects) is a fundamental aspect of perception. This ability underlies perceptual processes such as figure/ground segregation, object identification, and separation of multiple objects, and it is generally known as scene segmentation or perceptual organization. Although humans perform it with apparent ease, the general problem of scene segmentation remains unsolved in the engineering of sensory processing, such as computer vision and auditory processing.

Crucial to scene segmentation is the grouping of similar sensory features and the segregation of dissimilar ones. Theoretical studies of brain functions and feature binding point to the mechanism of temporal correlation as a representational framework [14], [16]. In particular, the correlation theory of von der Malsburg [14] asserts that an object is represented by the temporal correlation of the firing activities of the scattered cells coding different features of the object. A natural way of encoding temporal correlation is to use neural oscillations, whereby each oscillator encodes some feature (maybe just a pixel) of an object. In this scheme, each segment (object) is represented by a group of oscillators that shows synchrony (phase-locking) of the oscillations, and different objects are represented by different groups whose oscillations are desynchronized from each other. We refer to this form of temporal correlation as oscillatory correlation. The theory of oscillatory correlation has received direct experimental support from cell recordings in the visual cortex [3], [4] and other brain regions. The discovery of 40 Hz synchronous oscillations in the visual cortex has triggered much interest in simulating the experimental results and in exploring oscillatory correlation to solve the problem of scene segmentation (see among others [19], [6], [11], [12], [7], [10], [18]). While several demonstrate synchronization in a group of oscillators using local (lateral) connections [6], [10], [17], most of these models rely on long-range (all-to-all) connections to achieve phase synchrony. It has been pointed out that local connections for reaching synchrony may play a fundamental role in scene segmentation since long-range connections would lead to indiscriminate segmentation [12], [18].

The theory of oscillatory correlation has two aspects: 1) synchronization within an oscillator group representing the same object, and 2) desynchronization between oscillator groups representing different objects. Despite intensive studies on the subject, few studies have addressed the question of desynchronization. One of these few is by von der Malsburg and Schneider [16], who introduced a common inhibitor to desynchronize two segments (see also [15]). Their method of synchronization, however, depends on long-range connections, and thus suffers the deficiency mentioned earlier. Schillen and König [9] introduced desynchronizing lateral connections with fixed delays to segment two objects. These desynchronizing connections have a short range, and two objects cannot be segmented if their distance is not very small. Also it is not clear how these models perform with a scene of more than two objects. The lack of an effective mechanism for desynchronization greatly limits the utility of oscillatory correlation to perceptual organization. In this letter, we propose a new class of oscillatory networks, LEGION (locally excitatory, globally inhibitory oscillator networks), and show that it can rapidly achieve both synchronization within an oscillator group representing each object and desynchronization among a number of oscillator groups representing multiple simultaneously presented objects. LEGION is composed of the following elements: 1) a new model of a single oscillator, 2) local excitatory connections to produce phase synchrony within an oscillator group representing each object, and 3) a global inhibitor that receives input from the entire network and feeds back with inhibition to produce desynchronization of the oscillator groups representing different objects. In other words, the mechanism of LEGION consists of local cooperation and global competition, thus encoding both aspects of oscillatory correlation. This surprisingly simple neural architecture may provide an elementary approach to scene segmentation and a computational framework for perceptual organization.

II. Model Description

The building block of LEGION, a single oscillator i, is defined in the simplest form as a feedback loop between an excitatory unit x_i and an inhibitory unit y_i

$$\frac{dx_i}{dt} = 3x_i - x_i^3 + 2 - y_i + \rho + I_i + S_i \tag{1a}$$

$$\frac{dy_i}{dt} = \epsilon[\gamma(1 + \tanh(x_i/\beta)) - y_i] \tag{1b}$$

where ρ denotes the amplitude of a Gaussian noise term. I_i represents external stimulation to the oscillator, and S_i denotes coupling from other oscillators in the network. The noise term is introduced both to test the robustness of the system and to actively desynchronize different input patterns. The parameter ϵ is chosen to be small, and in this case, (1), without any coupling or noise, corresponds to a standard relaxation oscillator. The x-nullcline of (1) is a cubic curve, while the y-nullcline is a sigmoid function, as shown in Fig. 1. If $I > 0$, these curves intersect along the middle branch of the cubic, and (1) is oscillatory. The periodic solution alternates between silent and active phases of near steady-state behavior. The parameter γ is introduced to control the relative times that the solution spends in these two phases. If $I < 0$, then the nullclines of (1) intersect at a stable fixed point along the left branch of the cubic. In this case (1) produces no oscillations (inactive). The parameter β specifies the

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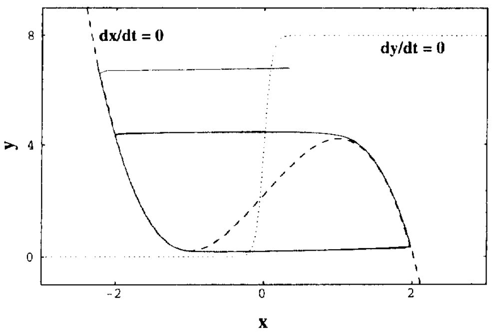

Fig. 1. Nullclines and periodic orbit of a single oscillator as shown in the phase plane. The x-nullcline ($dx/dt = 0$) is shown by the dashed curve and the y-nullcline ($dy/dt = 0$) is shown by the dotted curve. In a simulation when the oscillator starts at a randomly generated point (upper middle position in the figure) in the phase plane, it quickly converges to the stable trajectory of a limit cycle, which alternates between the left branch (silent phase) and the right branch (active phase) of the cubic. The parameters for this simulation are $I = 0.2$, $\rho = 0.02$, $\epsilon = 0.02$, $\gamma = 4.0$, $\beta = 0.1$.

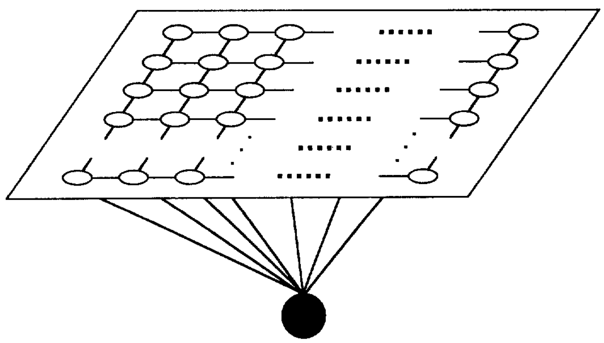

Fig. 2. Architecture of a two dimensional LEGION with nearest neighbor coupling. The global inhibitor is indicated by the black circle.

steepness of the sigmoid. The oscillator model (1) may be interpreted as a model for the spiking behavior of a single neuron or the mean field approximation to a network of excitatory and inhibitory neurons.

The LEGION we study here is two dimensional. Our results, however, can easily be extended to other dimensions. Each oscillator in the LEGION is connected to only its four nearest neighbors, thus forming a two-dimensional (2-D) grid. This is the simplest form of local connections. The global inhibitor receives excitation from each oscillator on the grid and in turn inhibits each oscillator. This architecture is shown in Fig. 2. The intuitive reason why the LEGION does scene segmentation is the following. When multiple objects (each of which corresponds to a connected region) are mapped onto the grid, local connectivity on the grid will group together the oscillators covered by each object. This grouping will be reflected by phase synchrony within the oscillator group representing each object. The global inhibitor serves to desynchronize the oscillatory responses to different objects.

The coupling term S_i in (1) is given by

$$S_i = \sum_{k \in N(i)} W_{ik} S_\infty(x_k, \theta_x) - W_z S_\infty(z, \theta_{xz}) \tag{2}$$

where

$$S_\infty(x, \theta) = \frac{1}{1 + \exp[-K(x - \theta)]} \tag{3}$$

W_{ik} is a connection (synaptic) weight from oscillator k to oscillator i, and $N(i)$ is the set of the adjacent oscillators that connect to i. In this model, $N(i)$ is four immediate neighbors on the two-dimensional (2-D) grid, except on the boundaries where $N(i)$ may be either two or three immediate neighbors. θ_x is a threshold [see the sigmoid function of (3)] above which an oscillator can affect its neighbors. W_z (positive) is the weight of inhibition from the global inhibitor z, whose activity is defined as

$$\frac{dz}{dt} = \phi(\sigma_\infty - z). \tag{4}$$

Here $\sigma_\infty = 0$ if $x_i < \theta_{zx}$ for every oscillator, and $\sigma_\infty = 1$ if $x_i \geq \theta_{zx}$ for at least one oscillator i. Hence θ_{zx} represents a threshold. If the activity of every oscillator is below this threshold, then the global inhibitor will not receive any input. In this case $z \to 0$, and the oscillators on the grid will not receive any inhibition from z. If, on the other hand, the activity of at least one oscillator is above this threshold, then the global inhibitor will receive input. In this case $z \to 1$, and every oscillator on the grid receives inhibition from z when z is above the threshold θ_{xz} [see (2)]. The parameter ϕ determines the rate at which the inhibitor reacts to such stimulation. In (3), K is a positive parameter.

In summary, once an oscillator is in the active phase, it triggers the global inhibitor. This then inhibits the entire network as described in (2). On the other hand, an active oscillator spreads its activation to its nearest neighbors, again through (2), and from them to its further neighbors. Thus, the entire dynamics of LEGION is a combination of local cooperation through excitatory coupling among neighboring oscillators and global competition via the global inhibitor.

Besides boundaries, the oscillators on the grid are basically symmetrical. Boundary conditions may cause certain distortions to the stability of synchronous oscillations. Recently, Wang [17], [18] proposed a mechanism called dynamic normalization to ensure that each oscillator, whether it is in the interior or on the boundary, has equal overall effective connection weights from its neighbors. The dynamic normalization mechanism is adopted in the present model to form effective connection weights. For binary images (each pixel being either zero or one), an effective connection is established between two oscillators if and only if they are neighbors and both of them are activated by external stimulation. The outcome of dynamic normalization is that the weights of all effective connections to one stimulated oscillator are normalized to a constant.

III. Computer Simulation

To illustrate how LEGION is used for scene segmentation, we have simulated a 20×20 LEGION as defined by (1)–(4). We arbitrarily selected four objects (patterns): two **O**'s, one **H**, and one **I**, and they form the word **OHIO**. These patterns were simultaneously presented to the network as shown in Fig. 3(a). Each pattern is a connected region, but no two patterns are connected to each other.

All the oscillators stimulated (covered) by the objects received an external input $I = 0.2$, while the others had $I = -0.02$. Thus the oscillators under stimulation become oscillatory, while those without stimulation remain inactive. The amplitude ρ of the Gaussian noise was set to 0.02, which was a 10% noise as compared to the external input. Dynamic normalization resulted in that only two neighboring oscillators stimulated by a single pattern had an effective connection. The differential equations were solved numerically with the following parameter values: $\epsilon = 0.02$, $\gamma = 6.0$, $\beta = 0.1$, $K = 50$, $\theta_x = -0.5$, $\theta_{zx} = \theta_{xz} = 0.1$, and $\phi = 3.0$. The total effective connections were normalized to 6.0. The results described below were robust to considerable changes in the parameter values. The phases (positions

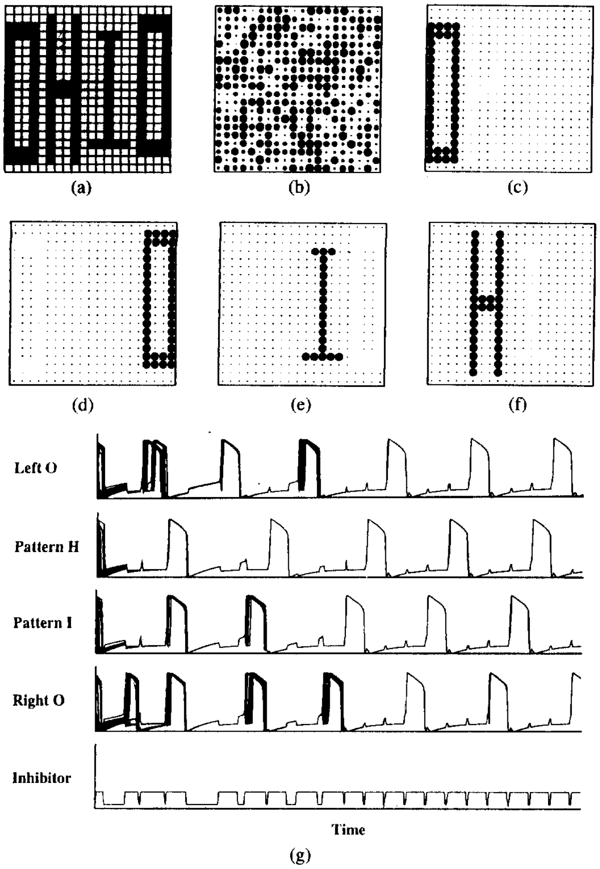

Fig. 3. (a) A scene composed of four patterns which were presented (mapped) to a 20×20 grid of oscillators, (b) A snapshot of the activities of the oscillator grid at the beginning of dynamic evolution, (c) A snapshot taken shortly after the beginning, (d) Another snapshot taken shortly after (c), (e). Another snapshot taken shortly after (d), (f). Another snapshot taken shortly after (e), (g). The upper four traces show the combined temporal activities of the oscillator blocks representing the four patterns, respectively, and the bottom trace shows the temporal activity of the global inhibitor. The ordinate indicates the normalized x activity of an oscillator calculated in the same way as in (b)–(f) (see text). The simulation took 8,000 integration steps.

in the phase plane) of all the oscillators on the grid were randomly initialized.

Fig. 3(b)–(f) shows the instantaneous activity (snapshot) of the network at various stages of dynamic evolution. The diameter of each black circle represents the x activity of the corresponding oscillator. Specifically, if the range of x values of all the oscillators are given by $x_{\min}$ and $x_{\max}$, then the diameter of the black circle corresponding to one oscillator is set to be proportional to $(x - x_{\min})/(x_{\max} - x_{\min})$. Fig. 3(b) shows a snapshot of the network a few steps after the beginning of the simulation. In Fig. 3(b), the activities of the oscillators of the network were largely random. Fig. 3(c) shows a snapshot after the system had evolved for a short time period. One can clearly see the effect of grouping and segmentation: all the oscillators belonging to the left **O** were entrained and had large activities. At the same time, the oscillators stimulated by the other three patterns had very small activities. Thus the left **O** was segmented from the rest of the input. A short time later, as shown in Fig. 3(d), the oscillators stimulated by the right **O** reached the active phase and were separated from the rest of the input. Fig. 3(e) shows another snapshot after Fig. 3(d). At this time, pattern **I** had its turn to be activated and separated from the rest of the input. Finally, in Fig. 3(f), the oscillators representing **H** were in the active phase and the rest of the scene remained inactive. This successive "pop-out" of the objects continued in an approximately periodic fashion. To provide a complete picture of dynamic evolution, Fig. 3(g) shows the temporal evolution of every oscillator. Since the oscillators receiving no external input were inactive during the entire simulation process, they were excluded from the display in Fig. 3(g). The activities of the oscillators stimulated by each object are combined into a single trace in the figure. Thus, if they are synchronized, they appear like a single oscillator. In Fig. 3(g), the four upper traces represent the activities of the four oscillator blocks, and the bottom trace represents the activity of the global inhibitor. The synchronized oscillations within each object are clearly shown within just three cycles of dynamic evolution.

The LEGION simulated above can readily be applied to segmentation of binary images. The exact shapes and positions of the patterns in Fig. 3(a) do not matter for segmentation. In fact, this 2-D LEGION provides a general solution to segmentation of planar connected regions. For gray-level images, where each pixel takes a value within a certain range, the following modification suffices to make the network applicable. An effective connection is established between two oscillators if and only if they are neighbors and the difference of their corresponding pixel values is below a certain threshold.

IV. Discussion

We have formally analyzed LEGION. The analysis is done using singular perturbation theory with ϵ considered as a small parameter [13]. In summary, LEGION exhibits a mechanism of selective gating, whereby an oscillator jumping to its active phase quickly recruits the oscillators of the same block due to local excitatory connections. At the same time, global inhibition prevents the oscillators representing different blocks from jumping up. With the selective gating mechanism, the network rapidly achieves both synchronization within blocks of oscillators that are stimulated by connected regions and desynchronization between different blocks. In particular, the time the system takes to segment a scene of N objects is no greater than N cycles, as illustrated in Fig. 3. We regard this as particularly important not only because speed is critical for real time scene segmentation but also for the considerations of neural plausibility of the oscillatory correlation theory. It is known that humans can identify an object in a visual scene in less than 100 ms [1]. This suggests that both synchrony and desynchrony must be achieved in just a few cycles if 40 Hz oscillations are taken to be the underlying mechanism.

In addition to biological plausibility, oscillatory correlation has a unique feature as a computational approach to the engineering of scene segmentation and figure/ground segregation. Due to the nature of oscillations, no single object can dominate and suppress the perception of the rest of the scene for a long time. The intrinsic dynamics embedded in oscillatory correlation provides a natural and reliable representation of multiple segmented patterns. The global inhibitor (see Fig. 2) exerts control to the entire oscillator network, and it may be regarded as an attentional control unit. Crick [2] has suggested that part of the thalamus, which sends projections to and receives input from almost the entire cortex, plays a critical role in attentional control. Based on structural and functional similarities, the global inhibitor in our model may correspond to a neuronal group located in the thalamus.

The basic principles of selective gating are established for LEGION with lateral connections beyond nearest neighbors. Indeed, in terms of synchronization, more distant connections even facilitate phase entrainment. In this sense, synchronization with all-to-all connections is a special case of our system. With nearest-neighbor connectivity (Fig. 2), any isolated part of an image is considered as a segment. In a noisy image with many tiny regions, segmentation would result in too many small fragments. More distant connections could provide a solution to this problem. Lateral connections may be designed so that the connection strength between two oscillators falls off exponentially. Since global inhibition is superimposed to local excitation, two oscillators positively coupled may still be desynchronized if global inhibition is strong enough. Thus, it is unlikely that all objects in an image form a single segment as the result of extended connections. Undoubtedly many questions in scene segmentation are left unaddressed in this letter. Issues like object occlusion, noise in a scene, top-down influence from memory, etc., must be addressed by future research. Nevertheless, we believe that LEGION provides a computational framework in which these important questions can be addressed and tested.

Due to its critical importance for computer vision, scene segmentation (perceptual organization, as it is known in computer vision) has been studied quite extensively. Many techniques have been proposed in the past [5], [8]. Despite these techniques, as pointed out by Haralick and Shapiro [5], there is no underlying theory of image segmentation, and the techniques tend to be ad hoc and emphasize some aspects while ignoring others. Compared to the traditional techniques for segmentation, the oscillatory correlation approach offers many unique advantages. The dynamical process is inherently parallel. While conventional computer vision algorithms are based on descriptive criteria and many adhoc heuristics, LEGION as exemplified in this paper performs computations based on only connections and oscillatory dynamics. The organizational simplicity renders LEGION particularly feasible for very large scale integration (VLSI) implementation. Also, continuous-time dynamics allows real time processing, desired by many engineering applications.

References

[1] I. Biederman, "Recognition-by-components: A theory of human image understanding," *Psychol. Rev.*, vol. 94, pp. 115–147, 1987.

[2] F. Crick, "Function of the thalamic reticular complex: The searchlight hypothesis," in *Proc. Natl. Acad. Sci. USA*, vol. 81, 1984, pp. 4586–4590.

[3] R. Eckhorn, *et al.*, "Coherent oscillations: A mechanism of feature linking in the visual cortex?" *Biol. Cybern.*, vol. 60, pp. 121–130, 1988.

[4] C. M. Gray, P. König, A. K. Engel, and W. Singer, "Oscillatory responses in cat visual cortex exhibit inter-columnar synchronization which reflects global stimulus properties," *Nature*, vol. 338, pp. 334–337, 1989.

[5] R. M. Haralick and L. G. Shapiro, "Image segmentation techniques," *Comput. Graphics Image Process.*, vol. 29, pp. 100–132, 1985.

[6] P. König and T. B. Schillen, "Stimulus-dependent assembly formation of oscillatory responses: I. Synchronization," *Neural Comput.*, vol. 3, pp. 155–166, 1991.

[7] T. Murata and H. Shimizu, "Oscillatory binocular system and temporal segmentation of stereoscopic depth surfaces," *Biol. Cybern.*, vol. 68, pp. 381–390, 1993.

[8] S. Sarkar and K. L. Boyer, "Perceptual organization in computer vision: a review and a proposal for a classificatory structure," *IEEE Trans. Syst. Man Cybern.*, vol. 23, pp. 382–399, 1993.

[9] T. B. Schillen and P. König, "Stimulus-dependent assembly formation of oscillatory responses: II. Desynchronization," *Neural Comput.*, vol. 3, pp. 167–178, 1991.

[10] D. Somers and N. Kopell, "Rapid synchronization through fast threshold modulation," *Biol. Cybern.*, vol. 68, pp. 393–407, 1993.

[11] H. Sompolinsky, D. Golomb, and D. Kleinfeld, "Cooperative dynamics in visual processing," *Phys. Rev. A*, vol. 43, pp. 6990–7011, 1991.

[12] O. Sporns, G. Tononi, and G. M. Edelman, "Modeling perceptual grouping and figure-ground segregation by means of active reentrant connections," in *Proc. Natl. Acad. Sci. USA*, vol. 88, 1991, pp. 129–133.

[13] D. Terman and D. L. Wang, "Global competition and local cooperation in a network of neural oscillators," *Physica D*, to appear.

[14] C. von der Malsburg, "The correlation theory of brain functions," Internal Report 81-2, Max-Planck-Institut for Biophysical Chemistry, Göttingen, FRG, 1981.

[15] C. von der Malsburg and J. Buhmann, "Sensory segmentation with coupled neural oscillators," *Biol. Cybern.*, vol. 67, pp. 233–246, 1992.

[16] C. von der Malsburg and W. Schneider, "A neural cocktail-party processor," *Biol. Cybern.*, vol. 54, pp. 29–40, 1986.

[17] D. L. Wang, "Modeling global synchrony in the visual cortex by locally coupled neural oscillators," in *Proc. 15th Ann. Conf. Cognit. Sci. Soc.*, 1993, pp. 1058–1063.

[18] D. L. Wang, "Emergent synchrony in locally coupled neural oscillators," *IEEE Trans. Neural Networks*, to appear.

[19] D. L. Wang, J. Buhmann, and C. von der Malsburg, "Segmentation in associative memory," *Neural Comput.*, vol. 2, pp. 94–106, 1990.